TMD Phenomenology:
Recent developments on polarized TMD global analyses

Mariaelena Boglione
Where can we learn about the 3D structure of matter?
Where can we learn about the 3D structure of matter?
Experimental data for TMD studies

Unpolarized and Polarized Drell-Yan scattering

\[ \sigma_{\text{Drell-Yan}} = f_q(x,k_\perp) \otimes f_{\bar{q}}(x,k_\perp) \otimes \hat{\sigma}^{q\bar{q} \rightarrow \ell^+ \ell^-} \]

Allows extraction of distribution functions

Unpolarized and Polarized SIDIS scattering

\[ \sigma_{\text{SIDIS}} = f_k(x) \otimes \hat{\sigma} \otimes D_{h/q}(z) \]

\[ e^+ e^- \rightarrow h_1 h_2 X \]

\[ \sigma_{h_1 h_2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma} \]

Allows extraction of fragmentation functions

Unpolarized and Polarized Drell-Yan scattering

\[ \sigma_{\text{Drell-Yan}} = f_q(x,k_\perp) \otimes f_{\bar{q}}(x,k_\perp) \otimes \hat{\sigma}^{q\bar{q} \rightarrow \ell^+ \ell^-} \]

Allows extraction of distribution functions
Experimental data for TMD studies

Unpolarized and Polarized SIDIS scattering

Allows extraction of distribution and fragmentation functions
Experimental data for TMD studies

Unpolarized and Polarized SIDIS scattering
EIC kinematic coverage

- Current polarized DIS data:
  - CERN
  - DESY
  - JLab-6
  - SLAC

- Current polarized BNL-RHIC pp data:
  - PHENIX
  - STAR 1-jet
  - W bosons

- JLab-12

- Projected CC DIS data:
  - EIC $\sqrt{s} = 141$ GeV

EIC will extend $x$ coverage

- Plot from E. Aschenauer @ SPIN 2016

EIC will extend $Q^2$ coverage

- Current data for Sivers asymmetry:
  - COMPASS $h^p, P_T < 1.6$ GeV, $z > 0.1$
  - HERMES $x^p, K^0, P_T < 1$ GeV, $0.2 < z < 0.7$
  - JLab Hall-A $x^p, P_T < 0.45$ GeV, $0.4 < z < 0.6$

Planned:
  - JLab 12

- $EIC\sqrt{s} = 141$ GeV $0.01 \leq y \leq 0.95$
  - $EIC\sqrt{s} = 45$ GeV $0.01 \leq y \leq 0.95$

Plot from the EIC white book

EIC will extend $x$ coverage
**EIC kinematics coverage**

Higher $\sqrt{s}$ and $Q^2$ values will increase resolution.

Plot from E. Aschenauer @ SPIN 2016
Transverse momentum dependent parton distribution and fragmentation functions
TMD distribution and fragmentation functions

Distribution

\[ f_1^q(x, k_T^2) \]

\[ h_{1T}^q(x, k_T^2) \]

\[ h_{1L}^q(x, k_T^2) \]

\[ f_{1T}^{q\perp}(x, k_T^2) \]

\[ f_{1L}^{q\perp}(x, k_T^2) \]

\[ g_{1L}^q(x, k_T^2) \]

\[ g_{1T}^q(x, k_T^2) \]

\[ N \] @ twist 2

Correlations between spin and transverse momentum (orbital motion)

Fragmentation

\[ D_1^q(z, p_T^2) \]

\[ H_1^{q\perp}(z, p_T^2) \]
Extracting polarized TMDs from SIDIS data: the Sivers function
The Sivers Distribution Function

The Sivers function, particularly interesting, as it provides information on the partonic orbital angular momentum. It embeds correlations between proton spin and quark transverse momentum. The formula for the Sivers function is:

\[ f_{q/p,S}(x, k_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p}^+(x, k_\perp) S \cdot (\hat{p} \times \hat{k}_\perp) \]

\[ = f_{q/p}(x, k_\perp) - \frac{k_\perp}{M} f_{1q}^{1\perp}(x, k_\perp) S \cdot (\hat{p} \times \hat{k}_\perp) \]

The Sivers function is related to the probability of finding an unpolarized quark inside a transversely polarized proton.
Where do we learn about the Sivers function?

Polarized Drell-Yan scattering

\[ \sigma_{\text{Drell-Yan}} = f_q(x, k_\perp) \otimes f_{\bar{q}}(x, k_\perp) \otimes \hat{\sigma}^{\gamma^* \rightarrow \ell^+\ell^-} \]

Allows extraction of distribution functions

Polarized SIDIS scattering

\[ \sigma_{\text{SIDIS}} = f_q(x) \otimes \hat{\sigma} \otimes D_{N\ell}(z) \]

Allows extraction of distribution and fragmentation functions
TMDs have to be defined in a color-gauge invariant way

\[ \Phi_{ij}(x, k_\perp) = \int \frac{d\xi^-}{2\pi} \frac{d^2\xi_\perp}{(2\pi)^2} e^{iP^+\xi^-} e^{-ik_\perp\xi_\perp} \langle P, S_P | \overline{\psi}_j(0) U(0, \xi) \psi_i(\xi) | P, S_P \rangle \bigg|_{\xi^+ = 0} \]

The struck quark propagates in the gauge field of the remnant and forms gauge links.

Gauge links generate initial and final state interactions.
**Sivers function sign change**

**SIDIS**
- The gluon couples to the proton remnant after the quark is scattered
- Attractive final state interaction

**DRELL YAN**
- The gluon couples before the quark annihilates
- Repulsive initial state interaction

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The Sivers function is process dependent: it reverses its sign when measured in SIDIS w.r.t. Drell Yan processes.

\[ \left[ f_{1T}^{q \perp} \right]_{\text{SIDIS}} = - \left[ f_{1T}^{q \perp} \right]_{\text{DY}} \]
First hints of sign change
**Sivers function in \( p^\uparrow + p \rightarrow W^\pm/Z @ RHIC **


\[
A^W_N = \frac{d\sigma_{p \rightarrow WX}^p - d\sigma_{p \rightarrow WX}^{-p}}{d\sigma_{p \rightarrow WX}^p + d\sigma_{p \rightarrow WX}^{-p}} = \frac{d\sigma_{\uparrow} - d\sigma_{\downarrow}}{d\sigma_{\uparrow} + d\sigma_{\downarrow}}
\]

Sivers function

\[
A^W_N = \frac{\sum_{q_1,q_2} |V_{q_1,q_2}|^2 \int d^2k_{\perp_1} d^2k_{\perp_2} \delta^2(k_{\perp_1} + k_{\perp_2} - q_T) S \cdot (\hat{p}_1 \times \hat{k}_{\perp_1}) \Delta^N f_{q_1/p \uparrow}(x_1, k_{\perp_1}) f_{q_2/p \downarrow}(x_2, k_{\perp_2})}{2 \sum_{q_1,q_2} |V_{q_1,q_2}|^2 \int d^2k_{\perp_1} d^2k_{\perp_2} \delta^2(k_{\perp_1} + k_{\perp_2} - q_T) f_{q_1/p}(x_1, k_{\perp_1}) f_{q_2/p}(x_2, k_{\perp_2})}
\]
Sivers single spin asymmetry in pion induced Drell Yan @ COMPASS


190GeV/c \( \pi^- \) beam scattered off a transversely polarized NH3 target (polarized proton)
New COMPASS Sivers data (higher statistics, higher precision, multidimensional binning) require a new phenomenological extraction of the Sivers function (more detailed estimation of uncertainties, evaluation of the bias induced by parametric form, study of $Q^2$ scale dependence)

New, comprehensive study of the Sivers effect
Extraction of Sivers functions from SIDIS data

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, JHEP 1704 (2017) 046

Unpolarized TMD PDF

\[ f_{q/p}(x, k_\perp) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle} \]

Unpolarized TMD FF

\[ D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-p_\perp^2 / \langle p_\perp^2 \rangle} \]

OLD MODEL

\[ \Delta^N f_{q/p}(x, k_\perp) = 2 N_q(x) h(k_\perp) f_{q/p}(x, k_\perp) \]

\[ h(k_\perp) = \sqrt{2e} \frac{k_\perp}{M_1} e^{-k_\perp^2 / M_1^2} \]

\[ N_q(x) = N_q x^{\alpha_q} (1 - x)^{\beta_q} \frac{(\alpha_q + \beta_q) (\alpha_q + \beta_q)}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} \]

\[ N_{\bar{q}}(x) = N_{\bar{q}} \]

Sivers function parametrized in terms of the unpolarized PDF

Sivers width parametrized starting from unpolarized width
New extraction of the Sivers function

New parametrization of the Sivers function

\[ \Delta^N f_{q/p^\uparrow} = 4N_q x^\alpha_q (1 - x)^\beta_q \frac{M_p}{\langle k_{\perp}^2 \rangle_S} k_{\perp} \frac{e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle_S}}{\pi \langle k_{\perp}^2 \rangle_S} \]

- First moment of the Sivers fn. Flavour dependent (u, d)
- \( k_{\perp} \) dependence of the Sivers fn. Flavour independent

Sivers functions not proportional to TMD PDFs

- No direct control on the positivity bound
- \( M_p \) is a fixed parameter to give the right dimensions. It is fixed to 0.938 GeV

In perspective: parametrization in terms of momentum better suited for the study of TMD evolution

It makes the expression of the actual Sivers asymmetry as simple as possible (within this model)

Sivers Asymmetry (numerator)

\[ F_{UT}^{sin(\phi_S - \phi_h)} = 2 \frac{z P_T M_p}{\langle P_T^2 \rangle_S} \frac{e^{-P_T^2 / \langle P_T^2 \rangle_S}}{\pi \langle P_T^2 \rangle_S} \sum q e_q^2 \left( N_q x^\alpha_q (1 - x)^\beta_q \right) D_{h/q}(z) \]
New extraction of the Sivers function

New parametrization of the Sivers function

\[ \Delta^N f_{q/p^\uparrow} = 4N_q x^{\alpha_q} (1 - x)^{\beta_q} \frac{M_p}{\langle k_{\perp}^2 \rangle_S} \frac{k_{\perp}}{\pi \langle k_{\perp}^2 \rangle_S} e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle_S} \]

First moment of the Sivers fn. Flavour dependent (u, d)

\[ \Delta^N f_{q/p^\uparrow}^{(1)}(x) = \int d^2 k_{\perp} \frac{k_{\perp}}{4M_p} \Delta^N f_{q/p^\uparrow}(x, k_{\perp}) \]

\[ \Delta^N f_{q/p^\uparrow}^{(1)} = N_q x^{\alpha_q} (1 - x)^{\beta_q} \]

In perspective: parametrization in terms of momentum better suited for the study of TMD evolution

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Sivers functions not proportional to TMD PDFs

No direct control on the positivity bound

M_p is a fixed parameter to give the right dimensions. It is fixed to 0.938 GeV

Before attempting any “global fitting” we have to check data for compatibility
Apparently …

some tension between COMPASS and HERMES data

However, COMPASS and HERMES span different ranges in $Q^2$ and have different $< Q^2 >$.

Kinematics effects
Possible signal of TMD evolution?
Signal of some tension between independent fit solutions for COMPASS and HERMES data

Start by using a very simple model only 5 parameters and no Q evolution

Use only u flavour and only $\pi^+$ data
New extraction of the Sivers function

Signal of some tension between independent fit solutions for COMPASS and HERMES data

Start by using a very simple model only 5 parameters and no Q evolution

Use only u flavour and only π+ data

Combined fit
Before attempting any “global fitting” we have to check data for compatibility ...

... and we have to check that the unpolarized cross sections are computed consistently and reproduce data successfully
To calculate any spin asymmetry it is crucial to use the appropriate denominator, i.e. the appropriate unpolarized cross section

\[
F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}
\]

with \( \langle P_T^2 \rangle = \langle p_T^2 \rangle + z_h^2 \langle k_T^2 \rangle \)

- It is very important to measure \( p_T \) distributions of unpolarized cross sections in SIDIS, Drell-Yan, e+e- processes

- These measurements will allow us to
  - **TEST THEORY**, and assess whether or not theory errors are under control (large \( q_T \) corrections, factorization errors, kinematics ...)
  - **HAVE BETTER MODELS** for TMDs

See talks by A. Signori and N. Sato
Relevance of unpolarized $p_T$ distributions

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005

\[ F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle} \quad \text{with} \quad \langle P_T^2 \rangle = \langle p_{\perp}^2 \rangle + z_h^2 \langle k_{\perp}^2 \rangle \]

\[ \langle k_{\perp}^2 \rangle = 0.60 \pm 0.14 \text{ GeV}^2 \]
\[ \langle p_{\perp}^2 \rangle = 0.20 \pm 0.02 \text{ GeV}^2 \]
\[ \chi^2_{\text{dof}} = 3.42 \]

Simple models seem to work well, but cannot describe both data sets simultaneously


\[ \langle k_{\perp}^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2 \]
\[ \langle p_{\perp}^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2 \]
\[ \chi^2_{\text{dof}} = 1.69 \]

HERMES $M_p^{\pi^+}$

Airapetian et al, Phys. Rev. D 87 (2013) 074029
New extraction of the Sivers function

Tension relaxes when the asymmetry is computed using the appropriate unpolarized widths for each data set

SIMULTANEOUS FIT OF
HERMES-2009 (HERMES widths - no evolution)
COMPASS-2015 (COMPASS widths – no evolution)
New extraction of the Sivers function

Sivers widths: HERMES vs. COMPASS

If we use different unpolarized widths for HERMES and COMPASS data, do we have to use different Sivers widths as well?

Allowing for different Sivers widths for each experiments, does not improve the quality of the fit, and the extracted values are very similar.

Boglione Gonzalez Flore D'Alesio, in preparation

No-evolution
Simple models seem to work well, but cannot describe both data sets simultaneously ...

However, more refined calculations seem to be presenting serious difficulties

See talks by A. Signori and N. Sato
Although the shape in transverse momentum space is well described, normalization is very problematic.

\[ \chi^2_{\text{tot}} = 1.55 \]

- Y-term is neglected
- Sum of two Gaussian \( k_T \) distributions is introduced
Normalization and K factor

How can we address the normalization problem???

- K factor depends on $p_T$
- Kinematics cuts can affect the size of K factors … up to a factor 10!

Stringent cuts on the pion production angle in H1 data suppresses LO and NLO contributions in a different way.

"The rather large size of the K-factor can be understood as a consequence of the opening of a new dominant ('leading-order') channel, and not to the 'genuine' increase in the partonic cross section [...]. The dominance of the new channel is due to the size of the gluon distribution at small $x_B$ and to the fact that the H1 selection cuts highlight the kinematical region dominated by the $\gamma + g \rightarrow g + q + \bar{q}$ partonic process. In particular, without the experimental cuts for the final state hadrons, the gg component represents less than 25% of the total NLO contribution at small $x_B$ ."

Challenges with Large Transverse Momentum in Semi-Inclusive Deeply Inelastic Scattering

J. O. Gonzalez-Hernandez, T. C. Rogers, N. Sato, B. Wang

We survey the current phenomenological status of semi-inclusive deep inelastic scattering at moderate hard scales and in the limit of very large transverse momentum. At the transverse momentum becomes comparable to or larger than the overall hard scale, the differential cross sections should be calculable with fixed order qCD methods, while small transverse momentum (TMD factorization) approximations should eventually break down. We find large disagreement between HERMES and COMPASS data and fixed order calculations done with modern parton densities, even in regions of kinematics where such calculations should be expected to be very accurate. Possible interpretations are suggested.

There are large discrepancies between data and fixed order calculations. They seem to be generated by collinear PDFs and FFs.
Now, back to Sivers ...
Main indications directly inferred from data:

- $u$ seems well constrained
- $d$ is not constrained: it can be replaced by sea contributions with equally good fits - hard to distinguish where this contribution comes from.
- Sivers sea is totally unconstrained

<table>
<thead>
<tr>
<th></th>
<th>$\chi^2_{\text{tot}}$</th>
<th>$\chi^2_{\text{dof}}$</th>
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<tbody>
<tr>
<td><strong>One flavour fits (3 parameters)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u$</td>
<td>408</td>
<td>1.88</td>
</tr>
<tr>
<td>$d$</td>
<td>914</td>
<td>4.21</td>
</tr>
<tr>
<td><strong>Two flavour fits (5 parameters)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u, \bar{u}$</td>
<td>266</td>
<td>1.24</td>
</tr>
<tr>
<td>$u, \bar{d}$</td>
<td>228</td>
<td>1.06</td>
</tr>
<tr>
<td>$u, d$</td>
<td>213</td>
<td>0.99</td>
</tr>
</tbody>
</table>

It is of vital importance to gain information on the $d$ content of the Sivers function.

We strongly rely on SIDIS measurements of the Sivers asymmetry on deuterium target @ COMPASS, as well as @ the future EIC.
Study of the uncertainties in the extraction of the Sivers function


Reference fit
(N, β, no α parameters)
No - evolution
Diff - Same configuration

Exploring the low-x region
Fit with N, β and α
(hard to find convergence)

Proxy for Collinear Twist-3 Evolution

Proxy for TMD Evolution
Study of the uncertainties in the extraction of the Sivers function


Reference fit - no evolution

<table>
<thead>
<tr>
<th></th>
<th>( \chi^2_{\text{tot}} = 212.8 )</th>
<th>( \chi^2_{\text{def}} = 0.99 )</th>
<th>( \Delta \chi^2 = 11.3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HERMES</td>
<td>( \langle k_1^2 \rangle = 0.57 \text{ GeV}^2 )</td>
<td>( \langle p_1^2 \rangle = 0.12 \text{ GeV}^2 )</td>
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<td>COMPASS</td>
<td>( \langle k_1^2 \rangle = 0.60 \text{ GeV}^2 )</td>
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<td></td>
</tr>
<tr>
<td>( N_u )</td>
<td>( 0.40 \pm 0.09 )</td>
<td>( 5.43 \pm 1.59 )</td>
<td></td>
</tr>
<tr>
<td>( N_d )</td>
<td>( -0.63 \pm 0.23 )</td>
<td>( 6.45 \pm 3.64 )</td>
<td></td>
</tr>
<tr>
<td>( \langle k_1^2 \rangle_u )</td>
<td>( 0.30 \pm 0.15 \text{ GeV}^2 )</td>
<td></td>
<td></td>
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</tbody>
</table>

The \( \chi^2 \) profile can reasonably be approximated with a quadratic, Hessian approx. works well, MINUIT errors give reliable estimates of the uncertainty on the free parameters.

Parameter correlations

\[
P = \int_0^{\chi^2} \frac{1}{21^2(M/2)} \left( \frac{\chi^2}{2} \right)^{(M/2)-1} \exp\left(-\frac{\chi^2}{2}\right) d\chi^2.
\]
Study of Low-x Uncertainties
(include $\alpha_u$ and $\alpha_d$ in the parametrization of the Sivers function)

Reference fit – no evolution
5 params.

$\chi^2$ does not improve!

$\chi^2_{\text{tot}} = 212.8$
$\chi^2_{\text{def}} = 0.99$
$\Delta \chi^2 = 11.3$

| HERMES | $(k_1^2) = 0.57$ GeV$^2$ | $(p_1^2) = 0.12$ GeV$^2$ |
| COMPASS | $(k_1^2) = 0.60$ GeV$^2$ | $(p_1^2) = 0.20$ GeV$^2$ |
| $N_u = 0.40 \pm 0.09$ | $\beta_u = 5.43 \pm 1.59$ |
| $N_d = -0.63 \pm 0.23$ | $\beta_d = 6.45 \pm 3.64$ |
| $\langle k_1^2 \rangle_S = 0.30 \pm 0.15$ GeV$^2$ |

$\alpha$ fit – no evolution
7 params.

$\chi^2_{\text{tot}} = 211.5$
$\chi^2_{\text{def}} = 0.99$
$\Delta \chi^2 = 14.3$

| HERMES | $(k_1^2) = 0.57$ GeV$^2$ | $(p_1^2) = 0.12$ GeV$^2$ |
| COMPASS | $(k_1^2) = 0.60$ GeV$^2$ | $(p_1^2) = 0.20$ GeV$^2$ |
| $N_u = 0.40 \pm 0.09$ | $\beta_u = 5.93 \pm 3.86$ | $\alpha_u = 0.073 \pm 0.46$ |
| $N_d = -0.63 \pm 0.23$ | $\beta_d = 5.71 \pm 7.43$ | $\alpha_d = -0.075 \pm 0.83$ |
| $\langle k_1^2 \rangle_S = 0.30 \pm 0.15$ GeV$^2$ |

For the alpha-fit, the $\chi^2$ profile is NOT quadratic, Hessian approx. does not work, MINUIT errors do NOT give reliable estimates of the uncertainty on the parameters, especially on the N parameters.

Attempt to minimize bias
Induced by the choice of parametric form

Alpha and N parameters are totally correlated
Study of the uncertainties in the extraction of the Sivers function

Parameter correlations

Study of Low-x Uncertainties

(include $\alpha_u$ and $\alpha_d$ in the parametrization of the Sivers function)

no-evolution

$\chi^2$ scans

220 data points
7 free-parameters
$\Delta\chi^2 = 14.34$

For the alpha-fit, the $\chi^2$ profile is NOT quadratic, Hessian approx. does not work, MINUIT errors do NOT give reliable estimates of the uncertainty on the parameters, especially on the N parameters.
Uncertainty bands – Sivers first moment

Reference Fit (no-evolution)

Study of Low-x Uncertainties (include $\alpha_u$ and $\alpha_d$ in the parametrization of the Sivers function)

Sivers First moment
u-contribution

Sivers First moment
d-contribution
Uncertainty bands – Sivers Asymmetries

Reference Fit (no-evolution)

Uncertainty bands from the reference fit (light-blue) become artificially small at small $x$.

Study of Low-x Uncertainties (include $\alpha_u$ and $\alpha_d$ in the parametrization of the Sivers function)

Alpha fit gives better estimates of uncertainties at small $x$ (gray bands).
**Impact of the precision of SIDIS deuteron data**

Boglione Gonzalez Flore D’Alesio, *in preparation*

- Light-blue bands represent the uncertainties corresponding to the reference fit.
- Red meshed bands correspond to the uncertainties estimated by using the same model, with the projected experimental errors of the future COMPASS run on deuteron target.

Signals of $Q$ scale dependence
**TMD Factorization approach and Collinear twist-three factorization approach**

**TMD factorization approach**
- Spin asymmetries are generated by spin and transverse momentum correlations between the identified hadron and the active parton.
- This correlations are embedded in the TMD parton distribution or fragmentation functions, which can be interpreted as probability densities.

**Collinear twist-three factorization approach**
- The correlation between spin and transverse momentum is included into the high twist collinear parton distributions or fragmentation functions.
- Twist-3 collinear parton distributions or fragmentation functions have no probabilistic interpretation. They are interpreted as the quantum interference between a collinear active quark state in the scattering amplitude and a collinear quark–gluon composite state in its complex conjugate amplitude.

- TMDs and quark–gluon correlation functions are closely related to each other.
- The first \( k_\perp \) moment of the Sivers function is equal to the twist-3 quark–gluon correlation functions \( T_{q_F}(x, x) \)
- Evolution kernels for \( T_{q_F}(x, x) \) are known; we can exploit them in our study (off diagonal terms are not included)

\[
\frac{\partial T_{q_F}(x, x, \mu)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left\{ P_{qq}(z) T_{q,F}(\xi, \xi, \mu) + \frac{N_c}{2} \left[ \frac{1+z^2}{1-z} (T_{q,F}(\xi, \xi, \mu) - T_{q,F}(\xi, \xi, \mu)) + z T_{q,F}(x, x, \mu) + T_{\Delta q,F}(x, \xi, \mu) \right] 
- N_c \delta(1-z) T_{q,F}(x, x, \mu) + \frac{1}{2N_c} \left[ (1-2z) T_{q,F}(x, x-\xi, \mu) + T_{\Delta q,F}(x, x-\xi, \mu) \right] \right\}
\]


8 October 2018
TMD evolution of the Sivers function

Aybat, Collins, Qiu, Rogers, Phys. Rev. D 85 (2012) 034043

\[ \tilde{F}_{1T}' f (x, b_T; \mu, \xi_F) = \tilde{F}_{1T}' f (x, b_T; \mu_0, Q_0^2) \exp \left\{ \ln \frac{\sqrt{\xi_F}}{Q_0} \tilde{K} (b_*; \mu_b) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F (g(\mu'); 1) - \ln \frac{\sqrt{\xi_F}}{\mu'} \gamma_K (g(\mu')) \right] \\
+ \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{\sqrt{\xi_F}}{Q_0} \gamma_K (g(\mu')) - g_K (b_T) \ln \frac{\sqrt{\xi_F}}{Q_0} \right\} \]

A proxy for TMD evolution:

\[ \langle k_{\perp}^2 \rangle_S = g_1 + g_2 \ln \frac{Q^2}{Q_0^2} \]
**Signals of scale dependence**  

### Collinear twist-3 evolution

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<td>$\chi^2_{\text{tot}}$</td>
<td>201.5</td>
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<tr>
<td>$\chi^2_{\text{dof}}$</td>
<td>0.94</td>
</tr>
<tr>
<td>n. of points</td>
<td>220</td>
</tr>
<tr>
<td>n. of free parameters</td>
<td>5</td>
</tr>
<tr>
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</tr>
<tr>
<td>COMPASS</td>
<td>$\langle k_T^2 \rangle = 0.60 \text{ GeV}^2$</td>
</tr>
<tr>
<td></td>
<td>$\langle p_T^2 \rangle = 0.20 \text{ GeV}^2$</td>
</tr>
<tr>
<td>$N_u$</td>
<td>$0.39 \pm 0.08$</td>
</tr>
<tr>
<td>$\beta_u$</td>
<td>$3.55 \pm 1.26$</td>
</tr>
<tr>
<td>$N_d$</td>
<td>$-0.65 \pm 0.27$</td>
</tr>
<tr>
<td>$\beta_d$</td>
<td>$4.77 \pm 3.41$</td>
</tr>
<tr>
<td>$\langle k_T^2 \rangle_s$</td>
<td>$0.33 \pm 0.14 \text{ GeV}^2$</td>
</tr>
</tbody>
</table>

![Graphs showing $\Delta N_{f_i}(x)$](image-url)
TMD evolution proxy

\begin{align*}
\langle k_{1t}^2 \rangle_s &= g_1 + g_2 \ln \frac{Q^2}{Q_0^2} \\
\end{align*}

\begin{tabular}{ll}
$\chi^2_{\text{tot}}$ & 212.8 \\
$\chi^2_{\text{dof}}$ & 0.99 \\
$\Delta \chi^2$ & 12.9 \\
\hline
HERMES & $\langle k_1^2 \rangle = 0.57$ GeV$^2$ & $\langle p_1^2 \rangle = 0.12$ GeV$^2$ \\
COMPASS & $\langle k_1^2 \rangle = 0.60$ GeV$^2$ & $\langle p_1^2 \rangle = 0.20$ GeV$^2$ \\
$N_u$ & $0.40 \pm 0.09$ & $\beta_u = 5.42 \pm 1.70$ \\
$N_d$ & $-0.63 \pm 0.26$ & $\beta_d = 6.45 \pm 3.89$ \\
$\langle k_{1t}^2 \rangle_s = g_1 + g_2 \log \left( \frac{Q^2}{Q_0^2} \right)$ & \\
$g_1 = 0.28 \pm 0.29$ GeV$^2$ & $g_2 = 0.01 \pm 0.20$ GeV$^2$ \\
\end{tabular}
**TMD evolution of the Sivers function**

- Echevarria, Idilbi, Schafer, Scimemi, arXiv:1208.1281
- Echevarria, Idilbi, Scimemi, arXiv:1208.1281
- Godbole, Misra, Mukherjee, Raswoot, Phys. Rev. D89 (2014) 074013
  
  ... ... ...

... and many others ...

- ...until the most recent studies on the Sivers function in $J/\psi$ production
  A. Mukherjee et al.

- and on the gluon contribution to the Sivers functions

**EIC will give important contribution!**

See talks by E. Aschenawer, C. Pisano, A. Mukherjee
Simultaneous extraction of transversity and the Collins function
What about $Q^2$ evolution?

Simultaneous fits of SIDIS and $e^+e^- \rightarrow h_1 h_2 X$ involve data sets at very different $Q^2$ scales.

In our computation the Collins TMD function evolves according to DGLAP evolution equations, through its $D_{h/q}(z,p_t,Q^2)$ component.

Could TMD evolution be an issue?

Could TMD evolution affect our results?
New BaBar data

BaBar measurements of $A_0$ and $A_{12}$ as a function of $z_1$ and $z_2$

BaBar multidimensional data on $A_{12}$ in bins of $(z_1, z_2, p_{t1}, p_{t2})$

BaBar measurements of $A_0$ and $A_{12}$ as a function of $p_{t0}, p_{t1}$ and $p_{t2}$
CSS/TMD evolution and Collins/Transversity

➢ TMD evolution

➢ Naive TMD


CSS/TMD evolution and Collins/Transversity

➢ TMD evolution

Naive TMD


CSS/TMD evolution and Collins/Transversity

➢ TMD evolution effects at BESIII

Naive TMD

BESIII, Ablikim et al., PRL 116 (2016) 042001

BESIII, Ablikim et al., PRL 116 (2016) 042001
Other phenomenological analyses for the extraction of the Collins function

There are other ways to study the Collins function …

… for example studying the transverse momentum distribution of single hadron pp production in jets

Kang, Liu, Ringer, Xing, JHEP (2017) 11:068

These analyses, performed on independent processes, provide evidence that the Collins function extracted in these processes is well consistent with that extracted by fitting e+e- and SIDIS data simultaneously. Moreover, they confirm that the experimental data presently available do not show signals of strong evolution effects, and cannot resolve calculations done with or without TMD evolution.

Don't miss the TMD/Jet Session on Tuesday; X. Liu, Y. Makris, F. Ringer, D. Pitoniak

EIC will give important contribution!
Other phenomenological analyses for the extraction of the Collins function

There are other ways to study the Collins function …

… Using collinear twist-three formalism

Don't miss the Collinear/ Twist-3 sessions on Wednesday and Thursday
W. Vogelsang, M. Schlegel, L. Gamberg

… Or studying closely related phenomena like the di-hadron fragmentation functions coupled to transversity

Dedicated talks on Thursday by A. Vossen, M. Radici

EIC will give important contribution!
Outlooks and perspectives

Phenomenological studies of TMDs, TMD factorization and TMD extraction have come a long way. Some issues remain open and need further investigation.

$P_T$ distributions of unpolarized SIDIS cross sections need to be measured (over the largest possible $P_T$ range) and further investigated on the phenomenological point of view.

Simultaneous fits of SIDIS, Drell-Yan and $e^+e^-$ annihilation data are highly recommended, but they should be performed within a consistent and solid framework where they can be implemented.

New data allow for
- Much more reliable extraction of the Sivers function
- Detailed study of the uncertainties
- Reduce the bias introduce by the choice of a specific parametrization on the final results

Data selection is crucial in global fitting:
- not too many
  (only data within the ranges where the TMD scheme works should be considered)
- not too few
  (too strict a selection can bias the fit results and neglect important information from experimental data)

As discussed by T. Rogers this morning