SINGULARITIES
IN TWIST-3 GPDS & PDFS

Fatma Aslan, Matthias Burkardt
OUTLINE

➢ What is and why study twist-3 GPDs
➢ Discontinuities in twist-3 GPDs
➢ Discontinuities and DVCS factorization
➢ Singularities in Twist-3 PDFs and quasi-PDFs
➢ Conclusions
➢ Outlook
What is TWIST-3?

Twist → The order in $Q^2$ at which a matrix element contributes to the physical amplitude.

Leading order → Twist 2
Next to leading order → Twist 3

2-particle correlations → Twist 2
3-particle correlations (such as quark-gluon-quark) → Twist 3

Twist → Behavior under longitudinal momentum boost in the IMF

| Behavior of twist-2 and twist-3 distributions under longitudinal momentum boost, $P^+$. |
|----------------------------------|----------------|
| Twist-2                         | Twist-3       |
| Independent of $P^+$            | $1/P^+$       |

$P^+$ (Longitudinal nucleon momentum)
Why study TWIST-3 GPDs?

➢ Twist-3 effects may not be negligible in the measurement of DVCS amplitude at 12 GeV.

➢ Quark-gluon-quark correlations leading to (average) transverse force acting on a quark in a polarized nucleon:

\[ \int dx \, x^2 g_2(x) , \int dx \, x^2 e(x) \rightarrow \perp \text{force} \]

M. Burkardt, Transverse Force on Quarks in DIS (2008).

➢ There is a relation between one particular twist-3 GPD and the orbital angular momentum of quarks:

\[ L^q_{\text{kin}} = -\int dx \, x G^q_2(x, \xi = 0, t = 0) \]

Penttinen, Polyakov, Shuvaev and Strikman, DVCS amplitude in the parton model (2000).
**$G_2$ in quark target model**

\[ \frac{1}{2} \int \frac{dz^-}{2\pi} e^{i z^+ p^+} \langle P', S' | \bar{u}(-\frac{z^-}{2}) \gamma^j q(\frac{z^-}{2}) | P, S \rangle \]

\[ = \frac{1}{2p^+} u(P', S') \left[ \frac{\Delta_1^j}{2M} G_1 + \gamma^j (H + E + G_2) + \frac{\Delta_1^j}{p^+} \gamma^+ G_3 + \frac{i e^{jk} \Delta_i^k}{p^+} \gamma^+ \gamma_5 G_4 \right] (P, S) \]

D. V. Kiptily, M. V. Polyakov, Genuine twist-3 contributions to the generalized parton distributions from instantons (2003)

$G_2$ has discontinuities

\[ \xi = \frac{\Delta_1^+}{2p^+} \]
There are discontinuities in $G_2$.

Factorization?

\[
\int_{-1}^{1} dx \frac{GPD}{x \pm \xi + i \varepsilon}
\]

The relevant DVCS amplitude involves $G_2 \pm \frac{\tilde{G}_2}{\xi}$

\[
\int_{-1}^{1} dx \frac{G_2 + \frac{1}{\xi} \tilde{G}_2}{x + \xi + i \varepsilon}
\]

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GPDs & PDFs
$\tilde{G}_2$ in quark target model

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ip^+z^-} \langle P', S' | \bar{q}(-\frac{z^-}{2}) \gamma^j \gamma_5 q(\frac{z^-}{2}) | P, S \rangle$$

$$= \frac{1}{2p^+} \bar{u}(P', S') \left[ \frac{\Delta_j}{2M} \gamma_5 \tilde{E} + \tilde{G}_1 \right] + \gamma^j \gamma_5 (\tilde{H} + \tilde{G}_2) + \frac{\Delta_j}{p^+} \gamma^+ \gamma_5 \tilde{G}_3 + \frac{i c^{jk} \Delta_k}{p^+} \gamma^+ \tilde{G}_4 \right] \bar{u}(P, S).$$

Quark target model in a symmetric frame

$\tilde{G}_2$ too has discontinuities
Twist-3 DVCS factorization is safe.

\[
\int_{-1}^{1} dx \frac{G_2 + \frac{1}{\xi} \tilde{G}_2}{x + \xi + i\epsilon}
\]

\[
\int_{-1}^{1} dx \frac{G_2 - \frac{1}{\xi} \tilde{G}_2}{x - \xi + i\epsilon}
\]

- \(G_2 + \frac{1}{\xi} \tilde{G}_2\) continuous at \(x = -\xi\)
- \(G_2 - \frac{1}{\xi} \tilde{G}_2\) continuous at \(x = \xi\)

\(\xi = 0.5\)

Fatma Aslan, Matthias Burkardt, Cédric Lorcé, Andreas Metz, Barbara Pasquini, Twist-3 GPDs in Deeply Virtual Compton Scattering (2018)
Factorization is fine, but what about the discontinuities?

How do they behave?
What do they represent?
What happens in different models?
What about the forward limit?
How do the discontinuities behave as $\xi \to 0$?

$G_2$ and $\tilde{G}_2$ in Quark Target Model

<table>
<thead>
<tr>
<th>Twist-3 GPD</th>
<th>Discontinuities as $\xi \to 0$</th>
</tr>
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<tbody>
<tr>
<td>$G_2$</td>
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<tr>
<td>$\tilde{G}_2$</td>
<td>Finite</td>
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What happens in different models?

$G_2$ and $\bar{G}_2$ in Scalar Diquark Model

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The behavior of the discontinuities of the twist-3 GPDs, \( \tilde{G}_2 \) and \( G_2 \) as \( \xi \to 0 \) in quark target model (QTM) and scalar diquark model (SDM).

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<tr>
<th>Twist-3 GPD</th>
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<th>SDM</th>
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<td>( G_2 )</td>
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<td>( \tilde{G}_2 )</td>
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<td>Divergent</td>
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The ERBL region of \( \tilde{G}_2 \) behaves like a \( \delta(x) \) in SDM.

Now let’s check the forward limit \( \tilde{G}_2 \to g_2 \)
Twist -3 pdf $g_T$ & Twist -3 quasi-pdf $g_T^{\text{quasi}}$ in scalar diquark model

Twist -3 pdf $g_T$ is calculated using LF coordinates, $(+, -, \perp)$

$$2g_T S^\perp = \int \frac{d^2k_\perp}{(2\pi)^2} \frac{dk^-}{2\pi} \bar{u}(p) \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)} \gamma^\perp \gamma^5 \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)} u(p) \frac{i}{[(p - k)^2 - \lambda^2 + i\epsilon]}$$

Twist -3 quasi-pdf $g_T^{\text{quasi}}$ is calculated using normal coordinates, $(0, \perp, z)$

$$2g_T^{\text{quasi}} S^\perp = \int \frac{d^2k_\perp}{(2\pi)^2} \frac{dk^0}{2\pi} \bar{u}(p) \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)} \gamma^\perp \gamma^5 \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)} u(p) \frac{i}{[(p - k)^2 - \lambda^2 + i\epsilon]}$$

$g_T^{\text{quasi}} \overset{P^z \to \infty}{\longrightarrow} g_T$

Behavior of twist-2 and twist-3 distributions under longitudinal momentum boost, $P^+$. 

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$p^z = p^+ = 5$

$g_T(k^+), \ g_T^{\text{quasi}}(k^z)$
$p^z = p^+ = 10$

$g_T(k^+), g_T^{\text{quasi}}(k^z)$

Aslan, Burkardt- Singularities in Twist-3
GPDs & PDFs
\[ p^z = p^+ = 15 \]

\[ g_T(k^+), \ g_T^{\text{quasi}}(k^z) \]
$p^z = p^+ = 20$

$g_T(k^+), \ g_T^{\text{quasi}}(k^z)$
\[ p^z = p^+ = 25 \]

\[ g_T(k^+), \; g_T^{\text{quasi}}(k^z) \]
$p^z = p^+ = 30$  \hspace{1cm} g_T(k^+), \quad g_T^{\text{quasi}}(k^z)$
There is a momentum component in the nucleon state which does not scale as the nucleon is boosted to the infinite momentum frame.
$p^z = p^+ = 5$

$g_T(x), \ g_T^{\text{quasi}}(x)$
$p^{z} = p^{+} = 10$

$g_{T}(x), \ g_{T}^{\text{quasi}}(x)$

Aslan, Burkardt- Singularities in Twist-3
GPDs & PDFs
$p^z = p^+ = 15$

$g_T(x), \ g_T^{\text{quasi}}(x)$

![Graph showing $g_T(x)$ and $g_T^{\text{quasi}}(x)$]
$p^z = p^+ = 20$

$g_T(x), \, g_T^{\text{quasi}}(x)$
$p^z = p^+ = 25$

$g_T(x),\; g_T^{\text{quasi}}(x)$
$p^z = p^+ = 30$

$g_T(x), \ g_T^{\text{quasi}}(x)$
The origin of the singularities

Whenever $k^-$ appears in the numerator the propagator is cancelled

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S | q(0) \gamma^\mu \gamma_5 q(\lambda n) | P, S \rangle$$

$$= 2 \{ g_1(x) p^\mu (S \cdot \hat{n}) + g_T(x) S_\perp^\mu + M^2 g_3(x) \hat{n}^\mu (S \cdot \hat{n}) \}$$

$$g_T(x) = g_1(x) + g_2(x)$$

$$2g_T S_\perp = \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{dk^-}{2\pi} \bar{u}(p) \frac{i(k + m)}{k^2 - m^2 + i\epsilon} \gamma^\perp \gamma_5 \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)u(p)} \frac{i}{[(p - k)^2 - \lambda^2 + i\epsilon]}$$

$$k^- = \frac{M^2}{2p^+} - \frac{[(p - k)^2 - \lambda^2]}{2(p^+ - k^+)} - \frac{(k^2 + \lambda^2)}{2(p^+ - k^+)}$$

for $k^+ \neq 0$, $\int \frac{dk^-}{(k^2 - m^2 + i\epsilon)^2} = \int \frac{dk^-}{2k^+} \left[ k^- \left( \frac{k^2 + m^2}{k^2 + m^2 + i\epsilon} \right) \right] = 0$

$$\int \frac{dk^+ dk^-}{(k^2 - m^2 + i\epsilon)^2} = \int \frac{dk^+ dk^-}{(2k^+ k^- - k^2 + m^2 + i\epsilon)^2}$$

$$= \int \frac{d^2 k_L}{k_L^2 - k^2 + m^2 + i\epsilon} = \frac{i\pi}{k_L^2 + m^2} \delta(k^+).$$
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<tbody>
<tr>
<td>e</td>
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</tr>
<tr>
<td>h_L</td>
<td>✓</td>
</tr>
<tr>
<td>g_T (g_2)</td>
<td>✓</td>
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✓: There is $\delta(x)$
✗: There is no $\delta(x)$
**Twist-2 pdf $g_1$ & Twist-2 quasi-pdf $g_1^{\text{quasi}}$ in scalar diquark model**

$$\int \frac{d\lambda}{2\pi} e^{i \lambda x} \langle P, S | \bar{q}(0) \gamma^\mu \gamma_5 q(\lambda n) | P, S \rangle$$

$$= 2 \{ g_1(x) \hat{p}^\mu (S \cdot \hat{n}) + g_T(x) S^\mu_\perp + M^2 g_3(x) \hat{n}^\mu (S \cdot \hat{n}) \}$$

**Twist-2 pdf $g_1$** is calculated using LF coordinates, $(+,−,\perp)$

$$2g_1 S^+ = \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{dk^-}{2\pi} \bar{u}(p) \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)} \gamma^+ \gamma_5 \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)} u(p) \frac{i}{[(p - k)^2 - \lambda^2 + i\epsilon]}$$

**Twist-2 quasi-pdf $g_1^{\text{quasi}}$** is calculated using normal coordinates, $(0, \perp, z)$

$$\left( \frac{P_z}{P_0} - 1 \right) g_1^{\text{quasi}} S^z = \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{dk^0}{2\pi} \bar{u}(p) \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)} \gamma^z \gamma_5 \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)} u(p) \frac{i}{[(p - k)^2 - \lambda^2 + i\epsilon]}$$

$$g_1^{\text{quasi}} P_z \to \infty \quad g_1$$

Behavior of twist-2 and twist-3 distributions under longitudinal momentum boost, $P^+$.

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GPDs & PDFs

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✓: There is $\delta(x)$  
✗: There is no $\delta(x)$
AT TWIST-3 THERE IS SOMETHING THAT DOES NOT EXIST IN TWIST-2: THERE ARE DELTA FUNCTIONS

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✓: There is $\delta(x)$
✗: There is no $\delta(x)$
There is no $\delta(x)$.

We identify these delta functions with momentum components in the nucleon state that do not scale as the nucleon is boosted to the infinite momentum.
**PAULI VILLARS (PV) REGULARIZATION**

Without PV Regularization

\[
2g_T S^\perp = \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{dk^-}{2\pi} \overline{u}(p) \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)} \gamma^\perp \gamma_5 \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)} u(p) \frac{i}{[(p - k)^2 - \lambda^2 + i\epsilon]}
\]

\[
2g_T(x) = \frac{1}{4\pi p^+}(x + \frac{m}{M}) \left\{ M(m + M)(1 - x) - \lambda^2 \right\} \frac{1}{k_\perp^2 + \omega(k)} |K_0^2| + \frac{1}{4\pi p^+} (2x - 1 + \frac{m}{M}) \left\{ \frac{\omega}{k_\perp^2 + \omega(\lambda)} + \ln(k_\perp^2 + \omega(\lambda)) \right\} |K_0^2|
\]

\[
\omega = -x(1 - x)M^2 + (1 - x)m^2 + x\lambda^2
\]

With PV Regularization

\[
2g_T S^\perp = \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{dk^-}{2\pi} \overline{u}(p) \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)} \gamma^\perp \gamma_5 \frac{i(k + m)}{(k^2 - m^2 + i\epsilon)} u(p) \left\{ \frac{i}{[(p - k)^2 - \lambda^2 + i\epsilon]} - \frac{i}{[(p - k)^2 - \Lambda^2 + i\epsilon]} \right\}
\]

\[
2g_T(x) = \frac{1}{4\pi p^+}(x + \frac{m}{M}) \left\{ M(m + M)(1 - x) - \lambda^2 \right\} \frac{1}{k_\perp^2 + \omega(\lambda)} |K_0^2| + \frac{1}{4\pi p^+} (2x - 1 + \frac{m}{M}) \left\{ \frac{\omega(\lambda)}{k_\perp^2 + \omega(\lambda)} + \ln(k_\perp^2 + \omega(\lambda)) \right\} |K_0^2|
\]

\[
- \frac{1}{4\pi p^+}(x + \frac{m}{M}) \left\{ M(m + M)(1 - x) - \Lambda^2 \right\} \frac{1}{k_\perp^2 + \omega(\Lambda)} |K_0^2| - \frac{1}{4\pi p^+} (2x - 1 + \frac{m}{M}) \left\{ \frac{\omega(\Lambda)}{k_\perp^2 + \omega(\Lambda)} + \ln(k_\perp^2 + \omega(\Lambda)) \right\} |K_0^2|
\]
\[ 2g_T(x) = \frac{1}{4\pi p^+} (x + 1)(1 - 2x) \frac{1}{k_{\perp}^2 + \omega(\lambda = 1)} + \frac{1}{4\pi p^+} (2x) \left\{ \frac{\omega(\lambda = 1)}{k_{\perp}^2 + \omega(\lambda = 1)} + \ln(k_{\perp}^2 + \omega(\lambda = 1)) \right\} \mid_{K_0^2}^{K^2} \]

\[ - \frac{1}{4\pi p^+} (x + 1) \left\{ 2(1 - x) - \Lambda^2 \right\} \frac{1}{k_{\perp}^2 + \omega(\Lambda)} \mid_{K_0^2}^{K^2} - \frac{1}{4\pi p^+} (2x) \left\{ \frac{\omega(\Lambda)}{k_{\perp}^2 + \omega(\Lambda)} + \ln(k_{\perp}^2 + \omega(\Lambda)) \right\} \mid_{K_0^2}^{K^2} \]
PAULI VILLARS (PV) REGULARIZATION

\[ 2g_T(x) = \frac{1}{4\pi p^+} (x + 1)(1 - 2x) \frac{1}{k_1^2 + \omega(\lambda = 1)} |K_0^2 + \frac{1}{4\pi p^+} (2x) \left\{ \frac{\omega(\lambda = 1)}{k_1^2 + \omega(\lambda = 1)} + \ln(k_1^2 + \omega(\lambda = 1)) \right\} |K_0^2 \\
- \frac{1}{4\pi p^+} (x + 1) \left\{ 2(1 - x) - \Lambda^2 \right\} \frac{1}{k_1^2 + \omega(\Lambda)} |K_0^2 - \frac{1}{4\pi p^+} (2x) \left\{ \frac{\omega(\Lambda)}{k_1^2 + \omega(\Lambda)} + \ln(k_1^2 + \omega(\Lambda)) \right\} |K_0^2 \]
$$2g_T(x) = \frac{1}{4\pi p^+} (x + 1)(1 - 2x) \left( \frac{1}{k_{\perp}^2 + \omega(\lambda = 1)} - \frac{1}{k_{\perp}^2 + \omega(\lambda = 1)} \right) \ln(k_{\perp}^2 + \omega(\lambda = 1)) \right) \right) \right)^{\frac{\omega(\Lambda)}{k_{\perp}^2 + \omega(\Lambda)} - \frac{\omega(\Lambda)}{k_{\perp}^2 + \omega(\Lambda)}}$$
PAULI VILLARS (PV) REGULARIZATION

\[ 2g_T(x) = \frac{1}{4\pi p^+} (x+1)(1-2x) \frac{1}{k_\perp^2 + \omega(\lambda = 1)} K_0^2 + \frac{1}{4\pi p^+} (2x) \left\{ \frac{\omega(\lambda = 1)}{k_\perp^2 + \omega(\lambda = 1)} + \ln(k_\perp^2 + \omega(\lambda = 1)) \right\} K_0^2 \]

\[ - \frac{1}{4\pi p^+} (x+1) \left\{ 2(1-x) - \Lambda^2 \right\} \frac{1}{k_\perp^2 + \omega(\Lambda)} K_0^2 - \frac{1}{4\pi p^+} (2x) \left\{ \frac{\omega(\Lambda)}{k_\perp^2 + \omega(\Lambda)} + \ln(k_\perp^2 + \omega(\Lambda)) \right\} K_0^2 \]
\[ 2g_T(x) = \frac{1}{4\pi p^+}(x + 1)(1 - 2x)\left(\frac{1}{k_\perp^2 + \omega(\lambda = 1)}\right)^{K^2_0} + \frac{1}{4\pi p^+}(2x)\left(\frac{\omega(\lambda = 1)}{k_\perp^2 + \omega(\lambda = 1)} + \ln(k_\perp^2 + \omega(\lambda = 1))\right)^{K^2_0} \]

\[ - \frac{1}{4\pi p^+}(x + 1)\left\{2(1 - x) - \Lambda^2\right\}\left(\frac{1}{k_\perp^2 + \omega(\Lambda)}\right)^{K^2_0} - \frac{1}{4\pi p^+}(2x)\left(\frac{\omega(\Lambda)}{k_\perp^2 + \omega(\Lambda)} + \ln(k_\perp^2 + \omega(\Lambda))\right)^{K^2_0} \]
Sum rules involving twist-3 distributions are violated if we do not take the $\delta(x)$ into account.

Lorentz invariance of twist-3 GPDs

$$\int_{-1}^{1} dx G_i(x, \xi, \Delta) = 0$$

$$\lim_{\epsilon \to 0} \int_{-1}^{\epsilon} dx G_i(x, \xi = 0, \Delta) + \lim_{\epsilon \to 0} \int_{\epsilon}^{1} dx G_i(x, \xi = 0, \Delta) \neq 0.$$
CONCLUSIONS

➢ There is a $\delta(x)$ in twist -3 PDFs.
CONCLUSIONS

 Coroutineing twist -3 GPDs have discontinuities at $x = \pm \xi$. 

CONCLUSIONS

➢ No issues with DVCS factorization for twist – 3.
CONCLUSIONS

- \( \delta(x) \) functions are related to the wave function components that do not scale in when the nucleon is boosted to the infinite momentum frame.
CONCLUSIONS

➢ **Not taking the** $\delta(x)$ **functions into account imply apparent violations of sum rules.**

Lorentz invariance of twist-3 GPDs

If there is a $\delta(x)$ and if it is not included:

\[
\int_{-1}^{1} dx G_i(x, \xi, \Delta) = 0 \quad \Rightarrow \quad \lim_{\epsilon \to 0} \int_{-1}^{\epsilon} dx G_i(x, \xi = 0, \Delta) + \lim_{\epsilon \to 0} \int_{\epsilon}^{1} dx G_i(x, \xi = 0, \Delta) \neq 0.
\]

\[
\int_{-1}^{1} dx \tilde{G}_i(x, \xi, \Delta) = 0 \quad \Rightarrow \quad \lim_{\epsilon \to 0} \int_{-1}^{\epsilon} dx \tilde{G}_i(x, \xi = 0, \Delta) + \lim_{\epsilon \to 0} \int_{\epsilon}^{1} dx \tilde{G}_i(x, \xi = 0, \Delta) \neq 0
\]
OUTLOOK

➢ Calculation of twist-3 GPDs in models with dynamical chiral symmetry breaking.
   Current quark $\rightarrow \delta(x)$
   Dressed quark $\rightarrow ?$

➢ Decomposition of twist 3.

\[ g_2(x) = g_{2WW}(x) + g_2^m(x) + g_2^3(x) \]

\[ g_{2WW}(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) \]

Quark mass term potentially contains a $\delta(x)$ function
Genuine twist-3 term potentially contains a $\delta(x)$ function

Previously

\[ h_L(x) = h_{WW}^L(x) + h_{L}^m(x) + h_{L}^3(x) \]

We remark that the above calculation indicates that the $\delta(x)$ term appears not only in $h_{L}^m$ but also in $h_{L}^3$. Furthermore they do not cancel but add up to give rise to $-\delta(x)$ in $h_L(x, Q^2)$ itself.

Burkardt & Koike, Violation of Sum Rules for Twist-3 Parton Distributions in QCD, 2001

➢ Using the $x^2$ moments of genuine twist-3 GPDs we can map out the transverse force acting on a quark in a polarized nucleon. (Aslan, Burkardt, Schlegel)
Upon boosting the system to infinite momentum the partons would all become very far from $\eta = 0$, where $\eta$ is the fraction of the particle’s longitudinal momentum carried by the parton. Since all the vacuum activity takes place at $\eta = 0$, it seems very curious how these partons (at finite $\eta$) could “feel” what is going on at $\eta = 0$.

The right way to think about spontaneous breaking of chiral symmetry on the LC is that it somehow manifests itself through interactions between partons at finite $\eta$ and $\eta = 0$ (the vacuum). The problem or puzzle with this is that matrixelements connecting states which are separated by a large distance in rapidity\textsuperscript{1} are suppressed. So how could the valence quarks possibly feel what is going on at $\eta = 0$?

Leonard Susskind, Matthias Burkardt

*A Model of Mesons based on $\chi$SB in the Light-Front Frame* (1994)

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There is a momentum component in the nucleon state which does not scale as the nucleon is boosted to the infinite momentum frame.

*Thank you for listening*