Nucleon Polarisabilities and $\chi$EFT: Bridging Between QCD and Data

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1. Two-Photon Response Explores System Dynamics
2. Polarisabilities, Compton Data, $\chi$EFT and Lattice-QCD
3. Spin Polarisabilities and Nucleon Spin Structure
4. Concluding Questions at the Intensity & Precision Frontier

How do constituents of the nucleon react to external fields?
How to reliably extract proton, neutron, spin polarisabilities?
How to bridge between QCD and Nuclear Physics?

Comprehensive Theory Effort:
1. Two-Photon Response Explores System Dynamics

(a) Polarisabilities: Stiffness of Charged Constituents in El.- Mag. Fields

Example: induced electric dipole radiation from harmonically bound charge, damping $\Gamma$ Lorentz/Drude 1900/1905

$$\vec{d}_{\text{ind}}(\omega) = \frac{q^2}{m} \frac{1}{\omega_0^2 - \omega^2 - i\Gamma\omega} \vec{E}_{\text{in}}(\omega)$$

$$=: 4\pi \alpha_{E1}(\omega) \text{"displaced volume" } [10^{-4} \text{ fm}^3]$$

electric scalar dipole polarisability

Dis-entangle interaction scales, symmetries & mechanisms with & among constituents.

Fundamental hadron properties, like charge, mass, mag. moment, $\langle r_N^2 \rangle$…PDG
Polarisabilities: Energy-dependent Multipoles of real Compton scattering.

\[
T = \begin{cases} 
\text{Powell: point spin-} \frac{1}{2} \text{ with} \\
\text{anomalous mag. moment} 
\end{cases} + 2\pi \omega^2 \left[ \frac{\alpha_{E1}(\omega)}{2} \left( \vec{e}' \cdot \vec{e} \right) + \frac{\beta_{M1}(\omega)}{2} \left( \hat{k}' \times \vec{e}' \right) \cdot \left( \hat{k} \times \vec{e} \right) \right] + \ldots
\]

Neither more nor less information about two-photon response of constituents, but more readily accessible.

For \( \omega \ll m_\pi \) more than “static + slope”! \( \Rightarrow \) Need to understand dynamics to reliably extrapolate from data to “the (static) polarisabilities” \( \alpha_{E1}(\omega = 0) \) etc.

\( \Rightarrow \) Compresses rich dynamics into few numbers.
(c) Example: Why the Magnetic Polarisability $\beta_{M1}$ Matters

$2\gamma$ in Lamb shift: proton radius

$M_p^\gamma - M_n^\gamma \approx [1.1 \pm 0.5]$ MeV

$T_1(v, Q^2) = -v^2 \int_{v^2_{th}}^{\infty} \frac{dv'}{v'\sqrt{v'^2 - v^2}} W_1(v', Q^2) + 4\pi \beta Q^2 + O(Q^4)$

Dispersion Relations: Cottingham Sum Rule and VVCS
The special status of pions and kaons in QCD and their marked impact on the long-distance structure of hadrons can be systematically encoded in an effective theory, applicable to processes at low energy. This effective theory, as well as emerging LQCD calculations, can provide benchmark predictions for so-called polarizabilities that parameterize the deformation of hadrons due to electromagnetic fields, spin fields, or even internal color fields. Great progress has been made in determining the electric and magnetic polarizabilities. Within the next few years, data are expected from the High Intensity Gamma-ray Source (HiγS) facility that will allow accurate extraction of proton-neutron differences and spin polarizabilities. JLab also explores aspects

**HiγS (DOE):** a central goal; > 3000 hrs committed at 60 – 100 MeV
- proton doubly & beam pol. (E-06-09/10)
- ³He unpol & doubly pol. (E-07-10, E-08-16)
- ⁴He unpol
- ⁶Li unpol. (E-15-11, first!)

**A2 @ MAMI (DFG: 5-year SFB):** running, data cooking and planned
- proton 100 – 400 MeV: beam & target pol. deuteron, ³He, ⁴He unpol., beam & target pol.

**MAXlab:** data cooking
- deuteron 100 – 160 MeV: unpol.
2. Polarisabilities, Compton Data, $\chi$ EFT and Lattice-QCD

(a) The Low-Energy Method: Chiral Effective Field Theory

Degrees of freedom $\pi, N, \Delta(1232)$ + all interactions allowed by symmetries: Chiral SSB, gauge, iso-spin, ...

>>> Chiral Effective Field Theory $\chi$ EFT $\equiv$ low-energy QCD

$$\mathcal{L}_{\chi\text{EFT}} = (D_\mu \pi^a)(D^\mu \pi^a) - m_\pi^2 \pi^a \pi^a + \cdots + N^\dagger [i D_0 + \frac{\bar{D}^2}{2M} + \frac{g_A}{2f_\pi} \bar{\sigma} \cdot \bar{D} \pi + \cdots] N + C_0 \left(N^\dagger N\right)^2 + \cdots$$

Controlled approximation $\implies$ Model-independent, error-estimate.

Two Low-Energy Régimes:

Low régime: $\omega \lesssim m_\pi$: $\Delta(1232)$ suppressed

High régime: $\omega \approx M_\Delta - M_N \approx 300$ MeV: $\Delta(1232)$ dominates $\implies$ propagator:

$$\frac{1}{E - (M_\Delta - M_N) - \Box} + \text{relativity}$$

Expand in $\frac{\omega}{\Lambda_\chi}$ and $\delta = \frac{M_\Delta - M_N}{\Lambda_\chi} \approx 1$ GeV $\approx \sqrt{\frac{m_\pi}{\Lambda_\chi}} = \frac{p_{\text{typ}}}{\Lambda_\chi} \ll 1$ (numerical fact) Pascalutsa/Phillips 2002.
Unified Amplitude: gauge & RG invariant set of all contributions which are
in low régime \( \omega \lesssim m_\pi \) at least \( N^4 \)LO \( (e^2 \delta^4) \): accuracy \( \delta^5 \lesssim 2\% \);
or in high régime \( \omega \sim M_\Delta - M_N \) at least NLO \( (e^2 \delta^0) \): accuracy \( \delta^2 \lesssim 20\% \).

\[ \omega \lesssim m_\pi \quad \sim M_\Delta - M_N \approx 300 \text{ MeV} \]

\[ e^2 \delta^0 \text{ LO} \quad e^2 \delta^0 \text{ NLO} \]

\[ e^2 \delta^2 \text{ N}^2 \text{LO} \quad e^2 \delta^1 \text{ N}^2 \text{LO} \]

\[ e^2 \delta^3 \text{ N}^3 \text{LO} \quad e^2 \delta^{-1} \text{ NLO} \]

\[ e^2 \delta^4 \text{ N}^4 \text{LO} \quad e^2 \delta^2 \text{ N}^3 \text{LO} \]

Unknowns: short-distance \( \delta \alpha, \delta \beta \Leftrightarrow \) Fit static \( \alpha_{E1}, \beta_{M1} \) (offset). \( \Rightarrow \) Predict \( \omega \)-dependence.
(c) Nucleon Polarisabilities from a Consistent Database

McGovern/Phillips/hg 2013
database: Feldman PPNP 2012

Noisy database, partially conflicting → reproducible trimming necessary.

Fit focuses on different Physics in different regions:
> 200 MeV: $\Delta(1232)$ fit $b_1 = 3.61 \pm 0.02$ ↔ < 170 MeV: polarisabilities
(d) Neutron Polarisabilities & Nuclear Binding

χEFT: consistency between wave functions, potentials, currents, meson-exchange, 1-N and few-N.

- **Nucleon Structure**: average of neutron & proton polarisabilities:
  
  \[ \chi \text{EFT, Disp. Rel.: } p-n \text{ difference is small } \]
  
  \[ \chi \text{EFT, Pasquini/... 2005} \]

- **Parameter-free coherent Rescattering Contributions**:
  
  \[ \frac{i}{B_d \pm \omega - \frac{q^2}{M}} : 2N \]

  coherent for \( \omega \sim 20 \text{ MeV} \)

  incoherent for \( \omega \sim m_\pi \)

  \[ \Rightarrow \text{less relevant for } \omega \gtrsim 40 \text{ MeV} \]

- **Parameter-free charged Meson-Exchange Currents are large**, dictated by gauge & chiral symmetry:

Model-independently subtract binding

\[ \Rightarrow \chi \text{EFT: quantify reliable uncertainties.} \]

Test charged-pion component of \( NN \) force.
Proton (Baldin, N^2LO) McGovern/Phillips/hg EPJA 2013

Neutron, with data from Compton@MAXlab Compton@MAX-lab PRL 2014

\[ \alpha_{E1} [10^{-4} \text{ fm}^3] = 10.65 \pm 0.35_{\text{stat}} \pm 0.2_{\Sigma} \pm 0.3_{\text{theory}} \]
\[ \beta_{M1} [10^{-4} \text{ fm}^3] = 3.15 \pm 0.35_{\text{stat}} \pm 0.2_{\Sigma} \pm 0.3_{\text{theory}} \]
\[ \chi^2 / \text{d.o.f.} = 113.2 / 135 \]

\[ \alpha_{E1}^{p-n} = -0.9 \pm 1.6_{\text{tot}} \text{ ; exp. error dominates.} \]

\[ \text{Cottingham} \Sigma R \text{ explains } M_{\gamma}^p - M_{\gamma}^n \text{ with } \alpha_{E1}^{p-n} = -1.7 \pm 0.4_{\text{tot}}. \]
**Consistency of Fit Error:**

Example $1\sigma$-contours for proton

Consistent with Baldin $\Sigma$ Rule

$$\alpha_{E1} + \beta_{M1} = \frac{1}{2\pi^2} \int_{v_0}^{\infty} dv \frac{\sigma(\gamma p \rightarrow X)}{v^2}$$

$$= 13.8 \pm 0.4$$

Olmos de Leon 2001

**Fit Stability:** floating norms within exp. sys. errors; vary dataset, $b_1$, vertex dressing,…

**need more forward data** to constrain.
**(g) What Does “Conservative” Theory Uncertainty Mean?**

\[
\chi^{\text{EFT}} \alpha^{(p)}_{E1} - \beta^{(p)}_{M1} \left[10^{-4} \text{ fm}^3\right] = \begin{align*}
7.5 & \pm 0.7_{\text{stat}} \pm 0.6_{\text{th}}^\text{LO} - 3.6_{\text{NLO}} - 0.1_{\text{N}^2\text{LO}} \pm 0.6_{\text{th}}
\end{align*}
\]

*Observable as series:*

\[
O = c_0 + c_1 \delta^1 + c_2 \delta^2 + \text{unknown} \times \delta^3
\]

Assuming \(\delta \approx 0.4\):

\[
11.2 - 9.1 \delta^1 - 0.6 \delta^2 \pm (11.2 \times \delta^3 \approx 0.7??)
\]

\[\implies \text{Estimate next term “most conservatively” as } |\text{unknown } c_3| \lesssim \max \{|c_0|; |c_1|; |c_2|\}.\]

**No infinite sampling pool; data fixed; more data changes confidence.**

\[\implies \text{Call upon the Reverend Bayes!}\]

**New information (new order) increases level of confidence.**

\[\implies \text{Smaller corrections, more reliable uncertainties.}\]

**Bayes makes you specify your premises/assumptions about series.**

**Priors:** leading-omitted term dominates (\(\delta \ll 1\)); putative distributions of all \(c_k\)'s and of largest value \(\bar{c}\) in series.

**“Least informed/informative”:** All values \(c_k\) equally likely, given upper bound \(\bar{c}\) of series.

**“Any upper bound”:** ln-uniform prior sets no bias on scale of \(\bar{c}\).
Quantifying One’s Beliefs in $O = \delta^n (c_0 + c_1 \delta^1 + c_2 \delta^2 + \ldots) = 11.2 - 9.1 \delta^1 - 0.6 \delta^2 \pm 0.6$ th

**Input:** Expansion parameter $\delta \simeq 0.4$, number of orders $k = 1$ (LO) and probable “largest number” $R = \delta^{k=1} \times \max\{|c_0| = 11.2|\} \approx 4.5$.

**Result:** Posterior $\equiv$ Degree of Belief (DoB) that next term $c_k \delta^k$ differs from order-$k$ central value by $\Delta$.

\[
\text{pdf of } c_k/\max\{c_0..c_{k-1}\} \text{ after } k \text{ tests}
\]

<table>
<thead>
<tr>
<th>order</th>
<th>DOB in $\pm R$</th>
<th>$\sigma$: 68%</th>
<th>$\Delta$(95%)</th>
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<td>2.0 $\sigma$</td>
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Quantifying One’s Beliefs in $\mathcal{O} = \delta^n (c_0 + c_1 \delta + c_2 \delta^2 + \ldots) = 11.2 - 9.1 \delta - 0.6 \delta^2 \pm 0.6_{th}$

**Input:** Expansion parameter $\delta \simeq 0.4$, number of orders $k = 2$ (NLO) and probable “largest number” $R = \delta^{k=2} \times \max\{|c_0 = 11.2|; |c_1 = -9.1|\}$

**Result:** Posterior $\equiv$ Degree of Belief (DoB) that next term $c_k \delta^k$ differs from order-$k$ central value by $\Delta$.

\[
\text{pr}(\Delta|\text{max. } R, \text{order } k) \propto \int_0^\infty d\bar{c} \text{ pr}(\bar{c}) \text{ pr}(c_k = \frac{\Delta}{\delta^k} | \bar{c}) \prod_{n} \text{ pr}(c_n | \bar{c}) \rightarrow \frac{k}{k+1} \frac{1}{2R} \left\{ \begin{array}{ll}
1 & |\Delta| \leq R \\
\left(\frac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R
\end{array} \right. 
\]

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Quantifying One’s Beliefs in \( \mathcal{O} = \delta^n(c_0 + c_1 \delta + c_2 \delta^2 + \ldots) = 11.2 - 9.1 \delta - 0.6 \delta^2 \pm 0.6 \)th

**Input:** Expansion parameter \( \delta \simeq 0.4 \), number of orders \( k = 3 \) \((N^{k-1} \text{LO})\) and probable “largest number” \( R = \delta^k \times \max\{|c_0| = 11.2; |c_1| = -9.1; |c_2| = -0.6; \ldots; |c_{k-1}|\} = 0.7 \).

**Result:** Posterior \( \equiv \text{Degree of Belief (DoB)} \) that next term \( c_k \delta^k \) differs from order-\( k \) central value by \( \Delta \).

**BUQYE 1506.01343 eq. (22)**

\[
\begin{align*}
\text{pr}(\Delta|\text{max. } R, \text{order } k) &\propto \int_0^\infty d\bar{c} \text{ pr}(\bar{c}) \text{ pr}(c_k = \frac{\Delta}{\delta^k|\bar{c}|}) \prod_{n} \text{pr}(c_n|\bar{c}) \rightarrow \frac{k}{k+1} \frac{1}{2R} \left( \frac{R}{|\Delta|} \right)^{k+1} \quad |\Delta| \leq R \\
&\left\{ \begin{array}{ll}
1 & |\Delta| > R
\end{array} \right.
\end{align*}
\]

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<td>1.6 ( R )</td>
<td>11( R ) = 7( \sigma )</td>
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<tr>
<td>NLO</td>
<td>( \frac{2}{3} = 66.7% )</td>
<td>1.0 ( R )</td>
<td>2.7( R ) = 2.6( \sigma )</td>
</tr>
<tr>
<td>N(^2)LO</td>
<td>( \frac{3}{4} = 75% )</td>
<td>0.9 ( R )</td>
<td>1.8( R ) = 1.9( \sigma )</td>
</tr>
<tr>
<td>N(^{k-1})LO ( k ) terms</td>
<td>( \frac{k}{k+1} )</td>
<td>0.68( \frac{k+1}{k} ) ( R(k \geq 2) )</td>
<td></td>
</tr>
<tr>
<td>Gauß</td>
<td>68.27%</td>
<td>1.0 ( R )</td>
<td>2.0( \sigma )</td>
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</table>

For “high enough” order, largest number \( R \) limits \( \gtrsim 68\% \) degree-of-belief interval.

**Varying priors:** When \( k \geq 2 \) orders known, DoBs with different assumptions about \( \bar{c}, c_n \) vary by \( \lesssim \pm 20\% \).

**Posterior pdf not Gauß’ian:** Plateau & power-law tail.– Do not add in quadrature in convolution!

\[ \Longrightarrow \text{Interpretation of all theory uncertainties, with these priors;} \quad \text{“A} \pm \sigma \text{” 68\% DoB interval} \quad [A - \sigma; A + \sigma]. \]
Prior Choice: What is “Natural Size”? (SCOTUS: I Know It When I see It.)

Observable/ Series \( \mathcal{O} = c_0 + c_1 \delta^1 + c_2 \delta^2 + \text{unknown} \times \delta^3 \) with “naturally-sized coefficients” \( c_i \).

“Least informative/informed”: characterised by 1 number: \( \bar{c} \).

More informed choices: more complicated structures, more thought, more parameters: \( \bar{c} \), typ. size, spread, . . .

**BUQYE** (Wesolowski/Klco/ . . .): When \( k \geq 2 \) orders known, DoBs with different assumptions about \( \bar{c}, c_n \) vary by \( \lesssim \pm 20\% \) for some “reasonable priors”.

\[ \text{Gaußian} \]
(i) Chiral Corridors of Uncertainties: $m_\pi$-Dependence Reveals Fine-Tuning

Observable $\mathcal{O} = c_0(m_\pi) + c_1(m_\pi)\delta^1 + c_2(m_\pi)\delta^2 + \text{unknown} \times \delta^3$.

χEFT: explicit $m_\pi$-dependence, parameters fixed at $m_\pi^{\text{phys}}$.

Propagating Uncertainties: Bayesian order-by-order as before, now at each $m_\pi$.

Some new terms linear in $m_\pi$. $\implies$ Conservatively expand in $\delta(m_\pi) = 0.4 \times \frac{m_\pi}{m_\pi^{\text{phys}}}$, fade as $m_\pi \uparrow \frac{m_\pi^{\text{phys}}}{0.4}$.

At physical $m_\pi = 140$ MeV: paramagnetic $\Delta(1232)$ fine-tuned against diamagnetic NLO $\pi N$ loops.

Only physical point has no substantial isospin splitting: stat. significant only for $m_\pi \lesssim 120$ MeV.
**Isovector Contributions and the Anthropic Principle**

\[ \Rightarrow \text{SPECULATION – NO ERROR BARS} \]

\[ \sum \propto \begin{array}{c} \text{Subtracted DR} \\ 2 \end{array} + 4\pi\beta Q^2 \]

**Cottingham \( \Sigma \) Rule:**

\[ \beta_{p-n}^{M_1} \iff \text{proton-neutron self-energy difference: } M_{p-n} = M_{p-n}^{\text{strong}} + M_{p-n}^{\text{em, elastic}} - A\beta_{p-n}^{M_1} \]

- If \( -A\beta_{p-n}^{M_1} \approx 0.5 \text{ MeV} \) and if dispersive \( A \propto \int_0^\Lambda dQ^2 \left( \frac{m_p^2 Q}{m_p^2 + Q^2} \right)^2 \) weakly \( m_\pi \)-dependent

Then

\[ \left. \frac{dM_{p-n}(m_\pi)}{d\ln m_q} \right|_{m_\pi^{\text{phys}}} = -0.65 \text{ MeV: Might not be negligible} \text{ vs. } \left. \frac{dM_{p-n}^{\text{strong}}}{d\ln m_q} \right|_{m_\pi^{\text{phys}}} \approx -2.1 \text{ MeV} \]

**Impact on p-n mass difference?:**

\( -A\beta_{p-n}^{M_1} \approx 0.5 \text{ MeV} \) wants more stable \( n \) as \( m_q \searrow \), competes with \( M_{p-n}^{\text{strong}} \).

\( \rightarrow \text{Neutron lifetime} \rightarrow \text{Big Bang Nucleosynthesis} \rightarrow \text{Anthropic Principle?} \)
It's A Bit More Complicated...

Both **magnitude** and **relative** importance of contributions change with $m_\pi$:

<table>
<thead>
<tr>
<th></th>
<th>$\sim m_\pi^{\text{phys}}$</th>
<th>$\sim M_\Delta - M_N \approx 300$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>charged pion cloud</td>
<td>$e^2\delta^2$ LO</td>
<td>$e^2\epsilon^1$ LO isoscalar only</td>
</tr>
<tr>
<td>infinite in chiral limit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta(1232)$ + its $\pi$ cloud</td>
<td>$e^2\delta^3$ NLO</td>
<td></td>
</tr>
<tr>
<td>covariant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>chiral corr.</td>
<td>$e^2\delta^4$ N$^2$LO</td>
<td>$e^2\epsilon^2$ NLO incomplete: no $\chi$ correction to $\Delta$ &amp; $\Delta\pi$; isovector incomplete</td>
</tr>
<tr>
<td>fit</td>
<td>etc.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>etc.</td>
<td></td>
</tr>
</tbody>
</table>

(i) Close to $m_\pi^{\text{phys}}$ 

$$\Longrightarrow \sqrt{\frac{m_\pi}{\Lambda_\chi \approx 800 \text{ MeV}}} \approx \frac{M_\Delta - M_N}{\Lambda_\chi} =: \delta\text{-counting}$$

Pascalutsa/Phillips 2002

(ii) Close to 300 MeV

$$\Longrightarrow \frac{m_\pi \sim (M_\Delta - M_N)}{\Lambda_\chi} =: \epsilon\text{-counting}$$

Manohar/Jenkins 1994, ...

(iii) Beyond $\Lambda_\chi \approx 800$ MeV 

$$\Longrightarrow$$ no small parameter, no convergence $\Longrightarrow$ **at best qualitatively useful!**

Use unified amplitude: 

$$\Longrightarrow$$ **Accuracy $N^2$LO ($\sim 6\%$) for** $m_\pi \sim 140$ MeV, **LO ($\sim 40\%$) for** $m_\pi \sim 300$ MeV.

**Gradual loss of accuracy, isovector incomplete, more sensitive to Bayesian prior when only LO.**

$$\Longrightarrow$$ **Fade corridors out beyond $\sim 250$ MeV.**

At this order, $g_A, f_\pi, M_N, (M_\Delta - M_N), \ldots$ independent of $m_\pi$. 
Towards comparable uncertainties in experiment, $\chi$EFT and lattice QCD.

$\chi$EFT: reliable error estimate for $\frac{m_\pi}{\Lambda_\chi}$ extrapolation. $\implies$ Fading corridors beyond $\sim 250$ MeV.

Example: static electric polarisability $\alpha_{E1}$

Not A Fit to Lattice Computations!

Active lattice groups:
Alexandru/Lee/...2005-;
Engelhardt/LHPC 2006-;
EMC/NPLQCD 2006-, 2015-;
Leinweber/Primer/Hall/... 2013-
Magnetic Polarisabilities: Surprises and Numerology

Why $\pi$-independent offset?

Why isoscalar off by "exactly" $\pi \times 10^{-4}$ fm$^3$?

Why isovector "exactly" matched?

Principle of Chiral Persistence?

PECULATION

Polarisabilities, INT Elweak, 45+X', 20.06.2018

Grießhammer, INS@GWU 18-1
(m) Magnetic Polarisabilities: Surprises and Numerology

Why $m_\pi$-independent offset? Why isoscalar off by “exactly” $\pi \times 10^{-4}$ fm$^3$? Why isovector “exactly” matched? *Principle of Chiral Persistence*?
3. Spin Polarisabilities and Nucleon Spin Structure

(a) Spin Polarisabilities: Nucleonic Bi-Refringence and Faraday Effect

**Optical Activity:** Response of spin-degrees of freedom, complements JLab spin programme.

\[ \mathcal{L}_{\text{pol}} = 4\pi N^\dagger \times \left\{ \frac{1}{2} \left[ \alpha_{E1} \vec{E}^2 + \beta_{M1} \vec{B}^2 \right] \right. \]

\[ + \frac{1}{2} \left[ \gamma_{E1E1} \vec{\sigma} \cdot (\vec{E} \times \dot{\vec{E}}) + \gamma_{M1M1} \vec{\sigma} \cdot (\vec{B} \times \dot{\vec{B}}) \right] \]

"pure" spin-dependent dipole

\[ -2 \gamma_{M1E2} \sigma_i B_j E_{ij} + 2 \gamma_{E1M2} \sigma_i E_j B_{ij} \]

"mixed" spin-dependent dipole

+ quadrupole etc.

\[ E_{ij} := \frac{1}{2} (\partial_i E_j + \partial_j E_i) \text{ etc.} \]

\[ \pi N \gamma : -\frac{g_A}{2f_\pi} \vec{\sigma} \cdot (\vec{q} + e\vec{e}) + \ldots \]

\[ \Rightarrow \pi \text{ emission/absorption depends on } N \text{ spin.} \]

\[ \Rightarrow \text{Test } \chi^\text{iral Symmetry!} \]
(b) Spin Polarisabilities: Theory Speaks

$\chi$EFT: Parameter-free predictions; lattice-QCD: Ramping up.

$\chi_{E1E1} [10^{-4}fm^4]$

$\chi_{M1M1} [10^{-4}fm^4]$

$\chi_{E1M2} [10^{-4}fm^4]$

$\chi_{M1E2} [10^{-4}fm^4]$

$\Delta n$: Engelhardt 2011
Unpolarised/linear/circular beam on scalar/vector/tensor target/recoil:

**Proton** $^3$He (spin-$\frac{1}{2}$): 7 Asymmetries: 1 beam, 1 target, 2 circpol. double, 3 linpol. double

5 Polarisation Transfers: 2 circpol. beam on pol. recoil, 3 linpol. beam on pol. recoil

**Deuteron** (spin-1): 17 Asymmetries: 1 beam, 4 target, 4 circpol. double, 8 linpol. double

12 Polarisation Transfers: 4 circpol. beam on pol. recoil, 8 linpol. beam on pol. recoil

$$\frac{d\sigma}{d\Omega}_\text{unpol} \times \left[ 1 + \sum_{\omega, \theta} P^{(\gamma)}_{\text{lin}}(\phi) \cos 2\phi_{\text{lin}} \right.$$ 

$$+ \sum_{J=1,2} \sum_{0\leq M \leq J} T_{JM}(\omega, \theta) P_{M0}^{(d)}(\theta) \cos\left( M\phi - \frac{\pi}{2} \delta_{J1} \right)$$

$$+ \sum_{J=1,2} \sum_{0\leq M \leq J} T_{JM}(\omega, \theta) P_{M0}^{(\text{circ})}(\theta) \sin\left( M\phi + \frac{\pi}{2} \delta_{J1} \right)$$

$$+ \sum_{J=1,2} \sum_{-J\leq M \leq J} T_{JM}(\omega, \theta) P_{M0}^{(\text{lin})}(\theta) \cos\left( 2\phi_{\text{lin}} - \frac{\pi}{2} \delta_{J1} \right) \left. \right]$$

**6 p & n polarisabilities + constraints on** $\alpha_{E1} + \beta_{M1}, \gamma_0, \ldots$; **experiment:** detector settings, feasibilities,...

No single measurement to provide definitive answers: multi-parameter extractions, systematics, validation.

⇒ Experiment & Theory collaborate to identify **observables with biggest impact.**
Fading Colours for $\omega \gtrsim 250$ MeV indicate breakdown of $\chi$EFT expansion.
Spin Polarisabilities from Polarised Photons

**Proton** best: Incoming $\gamma$ circularly polarised, sum over final states. $N$-spin in $(\vec{k}, \vec{k}')$-plane, perpendicular to $\vec{k}$:

$$\Sigma_{2x} : \begin{array}{c}
\includegraphics[width=0.2\textwidth]{fig1a.png} \\
\includegraphics[width=0.2\textwidth]{fig1b.png}
\end{array}$$

$\chi$ EFT predicts $\gamma_{E1E1} = -1.1 \pm 1.9$; Martel (MAMI) fit: $-3.5 \pm 1.2$

$\mathcal{O}(e^2\delta^4)$ $\chi$ EFT prediction $hg/McGovern/Phillips$ 2014 vs. MAMI extraction Martel/...2014

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_{E1E1}$</th>
<th>$\gamma_{M1M1}$</th>
<th>$\gamma_{E1M2}$</th>
<th>$\gamma_{M1E2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAMI 2014 proton</td>
<td>$-3.5 \pm 1.2$</td>
<td>$3.2 \pm 0.9$</td>
<td>$-0.7 \pm 1.2$</td>
<td>$2.0 \pm 0.3$</td>
</tr>
<tr>
<td>$\chi$ EFT proton <strong>predicted</strong></td>
<td>$-1.1 \pm 1.9_{th}$</td>
<td>$2.2 \pm 0.5_{stat} \pm 0.6_{th}$ <strong>fit to unpol.</strong></td>
<td>$-0.4 \pm 0.6_{th}$</td>
<td>$1.9 \pm 0.5_{th}$</td>
</tr>
<tr>
<td>$\chi$ EFT neutron <strong>predicted</strong></td>
<td>$-4.0 \pm 1.9_{th}$</td>
<td>$1.3 \pm 0.5_{stat} \pm 0.6_{th}$</td>
<td>$-0.1 \pm 0.6_{th}$</td>
<td>$2.4 \pm 0.5_{th}$</td>
</tr>
</tbody>
</table>
Fading Colours for $\omega \gtrsim 250$ MeV indicate breakdown of $\chi$ EFT expansion.

$d\Sigma_{2\times}/d\xi$ [inverse canonical units]
Fading colours for $\omega \gtrsim 250$ MeV indicate breakdown of $\chi$ EFT expansion.

$d\Sigma_3/d\xi \text{ [inverse canonical units]}$
**Neutron Polarisabilities & Nuclear Binding**

One nucleon + few-nucleon

- **N structure**
  - \( \alpha_{E1}, \beta_{M1}, \gamma_i \)

- **N Born**
  - charge & mag. moment

- **\( \pi^\pm \) exchange**
  - largest part

- Coherent rescattering
  - relevant for \( \omega \rightarrow 0 \)

**Parameter-free, except \( N \) polarisabilities.**

**Experiment:**
- More charge & MECs \( \rightarrow \) more counts \( \rightarrow \) heavier nuclei

**Theory:**
- Reliable only if nuclear binding & levels accurate \( \rightarrow \) lighter nuclei

**Find sweet-spot between competing forces:**
- deuteron, \(^3\text{He},^4\text{He}.

**Use Complementing Targets of Opportunity.**

**Deuteron, \(^4\text{He}:**
- sensitive to average p+n polarisabilities \( \rightarrow \) neutron pols

**\(^3\text{He}:**
- sensitive to \( 2\alpha_{E1}^p + \alpha_{E1}^n \) & \( 2\beta_{M1}^p + \beta_{M1}^n \) \( \rightarrow \) neutron pols.

Model-independently subtract binding effects.

\( \rightarrow \chi \text{EFT: quantify reliable uncertainties.} \)

- Chirally consistent 1N & few-N: potentials, wave functions, currents, \( \pi \)-exchange.

Test charged-pion component of \( NN \) force.

---

**Graphs:**
- \( \omega_{lab}=60 \text{ MeV} \)

- Deuteron, \(^4\text{He} HI \gamma \text{S data at } 61\text{MeV} \)

- \(^3\text{He} \) deuteron, proton, neutron

---

**Math Symbols:**
- \( N \), \( \alpha_{E1}, \beta_{M1}, \gamma_i \)
- \( \pi^\pm \)
- \( \omega_{lab}=60 \text{ MeV} \)
- \( \theta_{lab} \) [deg]
(i) Improve on the Neutron: Target $^3$He

**Correction of Code’s Isospinology**

of $\mathcal{O}(e^2 \delta^2)$ (no $\Delta(1232)$)

increases rates found by Margaryan

erratum to Shukla/...2009 in press

---

**Example unpolarised $^3$He**: Sensitivity on $\Delta(1232)$ and $\alpha_{E1}^n$ at $\omega_{lab} = 120$ MeV

\[ \omega_{lab}=120 \text{ MeV, neutron } \delta\alpha_{E1}=\pm 2 \]

\[ \omega_{lab}=\{60;120\} \text{ MeV} \]

---

Beyond $\omega \in [80;120]$ MeV: rescattering (Thomson, $T_{NN}$); explicit $\Delta(1232)$ also in MECs.
3He as “effective” spin target: sensitivity to neutron spin, not to proton spin.

Sensitivity to $\gamma$'s at $\omega_{\text{lab}} = 120$ MeV enhanced by interference with charged Born+MEC.

Polarisabilities, INT Elweak, 45+X', 20.06.2018
Polarisabilities, INT Elkevir, 45+X, 20.06.2018

(k) Experiment and Theory in Sync at the Precision and Intensity Frontier

“At present, single and double polarised data is sorely missing.” Theory letter [arXiv:1409.1512]

No single measurement will provide definitive answer: multi-parameter extraction, systematics, validation. Experiment & Theory collaborate to identify observables with biggest impact.
4. Concluding Questions at the Intensity & Precision Frontier

**Polarisabilities:** $\omega$-dependence maps out scales, symmetries & mechanisms of interactions:
- Chiral symmetry of pion-cloud, $\Delta(1232)$ properties, nucleon spin-constituents.

**Spin Polarisabilities:** Stiffness of Spin Constituents; Nuclear Faraday Effect.
- $\chi$EFT: parameter-free predictions, lattice QCD catching up.

<table>
<thead>
<tr>
<th>Target</th>
<th>Opportunities</th>
<th>Theory Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>proton</td>
<td>$p$ spin pols.</td>
<td>“done” well ahead of exp.</td>
</tr>
<tr>
<td>deuteron</td>
<td>sensitive to $p+n$ average polarised, d-wave interference: mixed spin pols $\gamma_{E1M2}, \gamma_{M1E2}$</td>
<td>$\omega \lesssim 120$ MeV done</td>
</tr>
<tr>
<td>$^3$He: increased rates</td>
<td>unpolarised: sensitive to $2p+n$ polarised: “$n$-spin” $\implies$ sensitive to $\gamma_i^n$</td>
<td>$\omega \in [50;120]$ MeV done</td>
</tr>
<tr>
<td>$^4$He: increased rates</td>
<td>sensitive to $p+n$ average</td>
<td>starting</td>
</tr>
<tr>
<td>$\gamma X \rightarrow NY\gamma$ quasifree</td>
<td>tag $n$ or $p$ directly – both in one go?</td>
<td>$\gamma d \rightarrow np\gamma$ done</td>
</tr>
</tbody>
</table>

**We Need Data:** elastic & inelastic cross-sections & asymmetries – reliable systematics!

Only combination of dedicated experiments meaningful!

(Not “one datum for one answer”.)

$\implies$ Synergy of Experiment, Low-Energy Theory & Lattice QCD, competitive uncertainties!

$\implies$ Compton Community programme outlined in White Paper for a Next Generation Laser Compton Gamma-ray Beam Facility, sent to DoE.
The efficient person gets the job done right. The effective person gets the right job done.
Proton-neutron difference $\alpha_{E1}^v := \alpha_{E1}^p - \alpha_{E1}^n$ etc. probes details:

Explicit $\chi$iral-symmetry-breaking in pion-cloud, . . . , elmag. p-n self-energy difference $[0 \pm 1]$ MeV $\propto \beta_{M1}^p - \beta_{M1}^n$

$O(e^2 \delta^4)$ (N$^2$LO) in $\chi$EFT; compatible with $\approx \frac{\text{iso-scalar}}{10}$ in Dispersion Relations. $\text{hg/Pasquini/...2005}$

No free neutron targets $\rightarrow \chi$EFT for model-independent subtraction of nuclear binding.
The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities.

It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication whenever practicable, and physical effects not included in the calculation from the beginning, such as electron correlation and relativistic corrections. It is all too often the case that the numerical results are presented without uncertainty estimates. Authors sometimes say that it is difficult to arrive at error estimates. Should this be considered an adequate reason for omitting them? In order to answer this question, we need to consider the goals and objectives of the theoretical (or computational) work being done. Theoretical papers are expected to include uncertainty estimates for comparisons with experimental data.

The aim is to estimate the uncertainty, not to state the exact amount of the error or provide a rigorous bound.

---

**Theoretical uncertainty: Truncation of Physics**

**EFT claim: systematic in** \( Q = \frac{\text{typ. low scale} \, p_{\text{typ}}}{\text{typ. high scale} \, \Lambda_{\text{EFT}}} \)

**Does Nuclear Structure emerge from QCD?**

**Beyond-Standard-Model Physics from Supernovae?**

**Religion**

**Science: Degree of Belief**

Thou Shalt Believe!

Conjecture

Evidence

---

**Scientific Method: Quantitative results with corridor of theoretical uncertainties for falsifiable predictions.**

Need procedure which is established, economical, reproducible: room to argue about “error on the error”.

“Double-Blind” Theory Errors: Assess with pretense of no/very limited data.
(b) Statistical Interpretation of the Max-Criterion: A Simple Example

I take this table of $\pi N$ scattering parameters in $\chi$EFT with effective $\Delta (1232)$ degrees of freedom from the talk by Jacobo Ruiz de Elvira. Here, I am not interested in the Physics, but use it as series $c_i = c_{i0} + c_{i1} \epsilon^1 + c_{i2} \epsilon^2$ in a small expansion parameter.

<table>
<thead>
<tr>
<th>parameter</th>
<th>LO total</th>
<th>NLO total</th>
<th>N²LO total</th>
<th>expansion</th>
<th>perturbative expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>-0.69</td>
<td>-1.24</td>
<td>-1.11</td>
<td>$c_{i0} + c_{i1} \epsilon^1 + c_{i2} \epsilon^2$</td>
<td>$\epsilon \approx 0.4$ (guess)</td>
</tr>
<tr>
<td>$c_2$</td>
<td>+0.81</td>
<td>+1.13</td>
<td>+1.28</td>
<td>$c_{i0} + c_{i1} \epsilon^1 + c_{i2} \epsilon^2$</td>
<td>$\epsilon \approx 0.4 + 5.75 \epsilon^1 - 4.44 \epsilon^2$</td>
</tr>
<tr>
<td>$c_3$</td>
<td>-0.45</td>
<td>-2.75</td>
<td>-2.04</td>
<td>$c_{i0} + c_{i1} \epsilon^1 + c_{i2} \epsilon^2$</td>
<td>$\epsilon \approx 0.4 + 5.75 \epsilon^1 - 4.44 \epsilon^2$</td>
</tr>
<tr>
<td>$c_4$</td>
<td>+0.64</td>
<td>+1.58</td>
<td>+2.07</td>
<td>$c_{i0} + c_{i1} \epsilon^1 + c_{i2} \epsilon^2$</td>
<td>$\epsilon \approx 0.4 + 5.75 \epsilon^1 - 4.44 \epsilon^2$</td>
</tr>
</tbody>
</table>

Now pick the largest absolute coefficient to estimate typical size of next-order correction $c_{i(n+1)} = c_{i3}$ in our case:

Max-Criterion: $c_{i(n+1)} \lesssim \max_{n \in \{0;1;2\}} \{ |c_{in}| \} : R$ is labelled as red in the table.

Multiply that number with $\epsilon^3$ to finally get a corridor of uncertainty/typical size of the $\epsilon^3$ contribution.

For $c_1$: $\max_{n \in \{0;1;2\}} \{ | -0.69 |; |1.38 |; | -0.81 | \} = 1.38 \implies \text{error} \pm 1.38 \times (\epsilon = 0.4)^3 \approx 0.09 \implies c_1 = -0.69 \pm 0.09$.

Similar: $c_2 = 1.28 \pm 0.06$, $c_3 = -2.04 \pm 0.37$, $c_4 = 2.07 \pm 0.20$ (round significant figures conservatively).

Notes: (1) Provide a theoretical error estimate that is reproducible. You can then discuss with others who have different opinions. No estimate, no discussion possible. – (2) Sometimes, one discards the LO→NLO correction if it's anomalously large. That is a “prior information” you need to disclose as “bias” of your estimate. – (3) Coefficients $c_{in}$ appear “more natural” for $c_1$ and $c_2$ than for $c_4 - c_4$ not that well-converging? – (4) The uncertainty estimate is agnostic about the Physics details. Somebody just handed me a table. – (5) If you are not happy with the input “$\epsilon \approx 0.4$”, pick another number. BUQYE $1511.03618$ developed the Bayesian technology to extract degrees of belief on what value of the expansion parameter the series suggests. – (6) The $c_i$ are not observables, but they are renormalised couplings which – according to Renormalisation – should follow a perturbative expansion.
Statistical Interpretation of the Max-Criterion: A Simple Example

The Bayesian interpretation of the max-criterion on the next slide will provide probability distribution (pdf)/degree-of-belief functions using a “reasonable” set of assumptions (“priors”) which give nice, analytic expressions. That’s one choice of assumptions, but other reasonable assumptions provide very similar pdf’s see BUQEYE: 1506.01343, 1511.03618, . . .

But before that, let’s do something intuitive which gives the same statistical likeliness interpretation of the max-criterion as the Bayesian one. The Bayesian analysis formalises the example and provides actual pdf’s.

Estimating a Largest Number: Given a finite set of (finite, positive) numbers in an urn. You get to draw one number at a time. Your mission, should you choose to accept it: Guess the largest number in the urn from a limited number of drawings.

For \( c_1 \), we first draw \( c_{10} = 0.69 \). I would say it’s “natural” to guess that there is a 1-in-2 = 50% chance that the next number is lower. But there is also a pretty good chance that if it is higher, then its distribution up there is not Gauß’ian but with a stronger tail.

Next, we draw \( c_{11} = 1.38 \) which is larger. So I revise my largest-number projection to \( R = 1.38 \), but I also get more confident that this may be pretty high (if not he highest already). After all, I already found one number which is lower, namely \( c_{10} = 0.69 \). With 2 pieces of information (0.69 and 1.38), it’s “natural” that the 3rd drawing has a 2-in-3 or 2/3 chance to be lower.

Next, we draw \( c_{12} = 0.81 < R \). Looking at my set of 3 numbers, I am even more confident that \( R = c_{11} = 1.38 \) is the largest number, with 3-in-4 or 75% confidence. For \( c_1 \), evil forces interfere and we have no more drawings to draw information from.

But if we could reach into the urn \( k \) times and look at the collected \( k \) results, every time revising our max-estimate, it’s “natural” to assign a 100% × \( k/(k + 1) \) confidence that I have actually gotten the largest number \( R \).

The Bayesian procedure on the next slide provides the same result. Read the BUQEYE papers for details and formulae!

In our example, we had \( k = 3 \) terms (drawings) for \( c_1 \). So the confidence that \( R = 1.38 \) is indeed the highest number is 3/4 = 75%, which is larger than \( p(1\sigma) \approx 68% \). For a 1\( \sigma \) corridor, I reasonably assume that the numbers are equi-distributed between 0 and the maximum \( R \). Then, the 68%-error corridor is set by \( \pm 68\% \times (k + 1)/k \times R \) amongst the known numbers.

Now, I multiply that number with 3 powers of the expansion parameter \( \epsilon \approx 0.4 \) (estimate N\(^3\)LO terms!) (but see Note (5) on the previous slide): \( \pm 1.38 \times (68%/75\%) \times 0.4^3 = \pm 0.08 \) is a good uncertainty estimate for a traditional 68% confidence region.

I also get a feeling that the probabilities outside the interval \([0; R]\) may not be Gauß’ian-distributed. Bayes will confirm that.
Isovector polarisabilities \( \xi^v := \frac{1}{2} (\xi^p - \xi^n) \) at \( N^2 \text{LO}; \) parameter-free. \( \Rightarrow \sim 20\% \) of LO?

**Fits:**

\[
\begin{align*}
\alpha_{E1}^{p-n} & = -0.9 \pm 1.3_{\text{tot}} \\
\beta_{M1}^{p-n} & = -0.5 \pm 1.3_{\text{tot}}
\end{align*}
\]

\( \Rightarrow \) Consider \( m_q \)-dependence!

\[
\frac{d\beta_{M1}^v}{d\ln m_q} \bigg|_{m^\text{phys}} = 0.65 \pm 0.4_{\text{th}}
\]

\[
\frac{d\alpha_{E1}^v}{d\ln m_q} \bigg|_{m^\text{phys}} = 0.7 \pm 0.4_{\text{th}}
\]

HW: Get \( \sigma \)! Know isovector only at LO: \( k = 1 \)

solution: \( \sigma = 1.6R = 1.6 \times \text{LO} \times (\delta = 0.4) \)

Possible fine-tuning at \( m^\text{phys}_\pi \) (statistically weak signal).
**Low-Energy Theorem:** Thomson limit \( A(\omega = 0) = -\frac{e^2}{M_d} \vec{c} \cdot \vec{c}' \).

Thirring 1950, Friar 1975, Arenhövel 1980: Thomson limit \( \iff \) current conservation \( \iff \) gauge invariance.

**Exact Theorem** \( \implies \) At each \( \chi \)EFT order \( \implies \) Checks numerics.

Amplitudes at \( \omega = 0 \):
\[
\langle \Psi_d | -\frac{e^2}{M_N} \vec{c} \cdot \vec{c}' | \Psi_d \rangle = 2A_{\text{Thomson}}
\]

\( S_{NN} = -A_{\text{Thomson}} \)

Significantly reduces cross section for \( \omega \lesssim 50 \text{ MeV} \), but **less important** at \( \omega \gtrsim 50 \text{ MeV} \). Urbana, Lund data

Wave function & potential dependence significantly reduced even as \( \omega \to 150 \text{ MeV} \implies \text{gauge invariance.} \)
Polarisabilities, INT Elweak, 45+X’, 20.06.2018

Pruning: World data statistically consistent after 2 points with \(\chi^2 > 9\) removed.

Number of points per \(\chi^2\) compared to analytic \(\chi^2\) distribution

\(\omega_{\text{lab}}=49\text{MeV}\)

\(\omega_{\text{lab}}=55\text{MeV}\)

\(\omega_{\text{lab}}=66\text{MeV}\)

\(\omega_{\text{lab}}=78\text{MeV}\)

\(\omega_{\text{lab}}=86.2\text{MeV}\)

\(\omega_{\text{lab}}=94.5\text{MeV}\)

\(\omega_{\text{lab}}=104\text{MeV}\)

\(\omega_{\text{lab}}=112.5\text{MeV}\)