Quenching, Currents, EFT, and $\beta\beta$ Decay

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June 20, 2018
Review of $0\nu\beta\beta$ Decay

Standard operator

Forbidden in Standard Model.
New physics inside blobs
Review of $0\nu\beta\beta$ Decay

Standard operator

I’ll focus on this one.

Forbidden in Standard Model.
New physics inside blobs

Other possibilities

Forbidden in Standard Model.
Significant spread. And all the models may miss important physics. And uncertainty hard to quantify but we're making progress on *ab-initio* nuclear-structure calculations of these.

Basic ingredients, on the other hand...
Nuclear Matrix Element (Simplified)

\[ M^{0\nu} = g_A^2 M^{0\nu}_{GT} - g_V^2 M^{0\nu}_{F} + \ldots \]

with

\[ M^{0\nu}_{GT} = \langle f | \sum_{a,b} H_{GT}(r_{ab}) \vec{\sigma}_a \cdot \vec{\sigma}_b \tau_a^+ \tau_b^+ | i \rangle \]

\[ M^{0\nu}_{F} = \langle f | \sum_{a,b} H_{F}(r_{ab}) \tau_a^+ \tau_b^+ | i \rangle \]

\[ H_{GT}(r) \approx H_{F}(r) \approx \frac{R_{\text{nucl.}}}{r} \]
Nuclear Matrix Element (Simplified)

\[ M^{0\nu} = g_A^2 M_{GT}^{0\nu} - g_V^2 M_{F}^{0\nu} + \ldots \]

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\[ M_{F}^{0\nu} = \langle f | \sum_{a,b} H_{F}(r_{ab}) \tau_a^+ \tau_b^+ | i \rangle \]

\[ H_{GT}(r) \approx H_{F}(r) \approx \frac{R_{nucl.}}{r} \]

Also:

\[ M_{2\nu} = g_A^2 \sum_m \frac{\langle f | \sum_a \vec{\sigma}_a \tau_a^+ | m \rangle \cdot \langle m | \sum_b \vec{\sigma}_b \tau_b^+ | i \rangle}{E_m - \frac{E_f + E_i}{2}} \]
Gamow-Teller $\beta$ Decay

Leading order decay operator is $\bar{s}\tau_+$. 

40-Year-Old Problem: Effective $g_A$ needed in all calculations of shell-model type.

Lots of suggestion about the cause but no consensus.
Other Tests of $\bar{\nu}\tau$ Strength Also Show Suppression

From Yako et al., PRL 103, 012503 (2009)

Only about $2/3$ of theoretically expected strength observed.
And $2\nu\beta\beta$ Decay…

Anti-neutrinos come out instead of being exchanged

From F. Iachello

If quenching is this severe in $0\nu$ decay, experimentalists will not be happy.
And $2\nu\beta\beta$ Decay…

Anti-neutrinos come out instead of being exchanged

What explains all this quenching?

In current paradigm — chiral EFT + ab initio computation — answer must be combination of many-body approximations and chiral currents.

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Axial Weak Current in Chiral EFT

Simplified…

Leading order:

Three orders down:
Most of the effects are from correlations outside the valence shell. Two-body currents don’t do much.
...And in the sd-Shell

Shell model seems to include most correlations. Bulk of quenching comes from two-body current.
...And in $^{100}$Sn

Coupled-Cluster Calculation of $\beta$ Decay

Again, most of the quenching accounted for by two-body current.

And quenching increases with mass.

Spectator nucleons contribute coherently to two-body current.
Naive Inclusion in $0\nu\beta\beta$ Decay

Use closure approximation:

$$\hat{O} \propto \int \frac{J^+(\vec{q})J^+(\vec{-q})}{q(q + \bar{E})} d^3q$$

Leading diagram (electron lines omitted):
Product of Currents

In first quantization, let

\[ \sum_i \hat{O}_{i}^{1b} = 1\text{-body operator in } J^+ \]

\[ \sum_{ij} \hat{O}_{ij}^{2b} = 2\text{-body operator in } J^+ \]

\[ J^+ (\vec{q}) J^+ (\vec{-q}) = \sum_{ij} \hat{O}_{i}^{1b} \hat{O}_{j}^{1b} + \sum_{ijk} \left( \hat{O}_{ij}^{2b} \hat{O}_{k}^{1b} + \hat{O}_{i}^{1b} \hat{O}_{jk}^{2b} \right) + 4\text{-body} \]

\[ + \sum_{ij} \left( \hat{O}_{ij}^{2b} \left[ \hat{O}_{i}^{1b} + \hat{O}_{j}^{1b} \right] + \left[ \hat{O}_{i}^{1b} + \hat{O}_{j}^{1b} \right] \hat{O}_{ij}^{2b} \right) \]

\[ \text{2-body op.} \]
Two-Body Currents in $0\nu\beta\beta$ Decay

Diagrams for these contributions:

Three-body

Two-body
Normal ordered two-body current, to get effective one-body current. Corresponds to:
Prior Work on Effects in Heavy Systems

Javier, Doron, Achim: Symmetric Nuclear Matter

Normal ordered two-body current, to get effective one-body current. Corresponds to:

\[
\begin{align*}
\rho & \to g_A - g_A \frac{\rho}{F_\pi^2} \left[ \frac{c_d}{g_A \Lambda} + \frac{2c_3}{3} \frac{q^2}{q^2 + 4m_{\pi}^2} + I(\rho, P) \left( \frac{2c_4 - 3}{3} + \frac{1}{6m} \right) \right] \\
I(\rho, P) & \approx \frac{2}{3} \text{ at nuclear density, with weak dependence on } P.
\end{align*}
\]

\(0\nu\beta\beta\) decay quenched by about 30\%, somewhat less than \(2\nu\beta\beta\) decay because of \(q\) dependence of effective \(g_A\).
More Complete Nuclear Matter Calculation
With Simplest Operator: $g_A$ at one-body vertex, $c_D$ at two-body vertex

Goldstone (Time-Ordered) Diagrams

\[
\sum_{i<F} \langle F \left| p_d^\dagger n_i^\dagger n_a n_b \right| p_c^\dagger n_i \right| I \rangle
\]

Three-body operators contribute (a) and (b) plus twice (c) and (d) $\approx 0$.

\[
(c) + (d) \approx -\frac{1}{2} \left[ (a) + (b) \right]
\]

\[
(e) + (f) \approx (\Lambda - k_F) \left[ (a) + (b) \right]
\]

Need counter-term to renormalize these.
L.J. Wang has done calculation with approximate $^{76}\text{Ge}$ wave function in $fp$ shell, inert core underneath.

Three-body operators

Left is contraction only within two-body current, right includes everything.
Two-Body Operators
With Nucleon Form Factors

Right side includes regulator.

![Graph with regulator](image)

Right side includes short-range correlations.

![Graph with SRC](image)

Almost entire contribution from \(c_D\) and short-range parts of \(c_3, c_4\).
Meanwhile...

Vincenzo, Emanuele, Jordy, Bira, Saori, Wouter D., Michal G.: EFT for $\beta\beta$

At $N^2LO$:

FIG. 3. Loop diagrams contributing to an effective $nnnpe$ vertex.

FIG. 4. Diagrams in the low-energy nuclear EFT contributing to the matching at $N^2LO$. The gray circle denotes an insertion of the LO strong potential of Eq. (11). The gray box denotes an insertion of the LO $\Delta L = 2$ potential $V_{\nu,0}$. The remaining notation is as in Fig. 1.

In the literature, the dipole parameterization of the vector and axial form factors is often used

$$g_V(q^2) = \left( 1 + \frac{q^2}{\Lambda_V^2} \right) - 2,$$

$$g_A(q^2) = \left( 1 + \frac{q^2}{\Lambda_A^2} \right) - 2,$$

(16)

with vector and axial masses $\Lambda_V = 850 \text{ MeV}$ and $\Lambda_A = 1040 \text{ MeV}$. The magnetic and induced pseudoscalar form factors are then assumed to be given by

$$g_M(q^2) = (1 + \kappa_1) g_V(q^2),$$

$$g_P(q^2) = -\frac{2}{m_N} g_A(q^2) q^2 + \frac{m_N^2}{\pi},$$

(17)

where $\kappa_1 = 3.7$ is the nucleon isovector anomalous magnetic moment. Expanding Eqs. (16) and (17) for small $|q|$, one recovers the LO and, for $g_A(q^2)$, the $N^2LO$ $\chi$PT expressions of the nucleon form factors. In the case of $g_V$, $g_P$ and $g_M$, the $N^2LO$ $\chi$PT results, given for example in Ref. [50], deviate from Eqs. (16) and (17). However, any parameterization that satisfactorily describes the observed nucleon form factors can be used in the neutrino potential (14).

The potential $V_{\nu,2}$ is induced by one-loop diagrams with a virtual neutrino and pions contributing to $nnn\rightarrow ppee$, built out of the leading interactions of Eqs. (8). They can be separated into

Need contact to renormalize these; would also renormalize ours.
Furthermore...

New Leading Contribution to Neutrinoless Double-$\beta$ Decay
Vincenzo Cirigliano, Wouter Dekens, Jordy de Vries, Michael L. Graesser, Emanuele Mereghetti, Saori Pastore, and Ubirajara van Kolck
Phys. Rev. Lett. 120, 202001 – Published 16 May 2018

Physics See Synopsis: A Missing Piece in the Neutrinoless Beta-Decay Puzzle

Synopsis: A Missing Piece in the Neutrinoless Beta-Decay Puzzle
May 16, 2018
The inclusion of short-range interactions in models of neutrinoless double-beta decay could impact the interpretation of experimental searches for the elusive decay.
So, to Sum Up…

1. Nice operators (three-body and long-range part of two-body in product of currents) quench $\text{O}_{\nu\beta\beta}$ less than previously. $c_D$ contributes very little.
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3. There is a similar contact term even at leading order.
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That’s all; thanks.