Building the Dynamical Diquark Model for Exotic Hadrons

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Multi-Scale Problems Using Effective Field Theories

Institute for Nuclear Theory

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Outline

1) Introduction: The exotics zoo in 2018

2) Diquarks as hadronic components

3) The dynamical diquark picture

4) Extended hadrons in the Born-Oppenheimer approximation

5) Exotics spectroscopy using B-O potentials

6) The future: Building realistic B-O based models
The Exotics Zoo

• Our textbooks still (for the most part) tell us that hadrons only appear in two species: $q\bar{q}$ mesons and $qqq$ baryons
• But so many other types of color-singlet compound hadrons, the so-called exotics, are possible:
  • $gg, ggg, \ldots$ (glueball)
  • $q\bar{q}g, q\bar{q}gg, \ldots$ (hybrid meson)
  • $q\bar{q}q\bar{q}, q\bar{q}q\bar{q}q\bar{q}, \ldots$ (tetraquark, hexaquark, ...)
  • $qqqq\bar{q}, qqqqqqq\bar{q}, \ldots$ (pentaquark, octoquark, ...)
  • $qqqqqq, \ldots$ (dibaryon, ...)
• Some of these were already suggested by Gell-Mann and Zweig in their original 1964 quark model papers!
Signs and Portents
Where Are the Light-Quark Exotics?

- The $0^{++}$ mesons $f_0(980)$ and $a_0(980)$ are widely (not universally) believed to be $s\bar{s}q\bar{q}$ tetraquarks (or, if you like, $K\bar{K}$ molecules)

- The mesons $\pi_1(1400)$ and $\pi_1(1600)$ appear to have non-$q\bar{q}$ $J^{PC} = 1^{-+}$ quantum numbers

- The baryon resonance $\Lambda(1405)$ is suspected to have a large pentaquark (or $KN$ molecular) component

- Other more recent suspects are appearing at the $NN$ threshold, in $\phi N$ processes, etc.

- And who can forget the 2002-2005 rise and fall of the $\Theta^+(1535)$ pentaquark?
The Fundamental Problem with Light-Quark Exotics

\[ \Lambda_{QCD} \gtrsim m_s \gg m_{u,d} \]

- In other words, it is not always easy to tell whether a \( q\bar{q} \) pair (\( q = u, d \), even sometimes \( s \)) is a sea-quark or valence pair
- This ambiguity is greatly diminished for \( c\bar{c} \) or \( b\bar{b} \) pairs
- It is the ultimate reason that quark potential models (e.g., the Cornell model) work well in the heavy-quark sector
- To get ironclad evidence for the existence of exotic hadrons, the clearest path is to look for heavy-quark exotics
Modern Exotics Studies Begin in 2003

The Belle Collaboration:
Evidence for a new particle at mass 3872 MeV

**X = Unknown**

- Belle found a new **charmoniumlike** resonance appearing in:
  \[ B \to K (J/\psi \pi^+\pi^-) \]
  - In the same mass range as charmonium, and it always decays into a final state containing \( c\bar{c} \)

- **Has been confirmed at** BABAR, CDF, DØ, LHCb, CMS, COMPASS

- **\( J^{PC} = 1^{++} \)**, but not believed to be ordinary \( c\bar{c} \):
  - Mass is many **10’s of MeV** below the nearest \( c\bar{c} \) candidate with these quantum numbers, \( \chi_{c1}(2P) \)

- **Now called** \( X(3872) \) [and believed to be a \( c\bar{c}u\bar{u} \) state]
  - \( m_{X(3872)} = 3871.69 \pm 0.17 \text{ MeV} \)
  - **Note:** \( m_{X(3872)} - m_{D^*0} - m_{D^0} = -0.01 \pm 0.18 \text{ MeV} \)
    - Leads to endless speculation that \( X(3872) \) is a \( D^0\bar{D}^{*0} \) hadronic molecule
  - **Width:** \( \Gamma_{X(3872)} < 1.2 \text{ MeV} \)
What the Charmonium System Should Look Like
(as predicted from quark potential models)

Lines with labels: predicted and observed experimentally
What the Charmonium System Really Looks Like
(May 2018)
Charmonium: May 2018
Charged sector

Baryonic ones too! (Pentaquarks)
The Exotics Scorecard: May 2018

• **35** observed exotics
  – **30** in the charmonium sector
  – **4** in the (much less explored) bottomonium sector
  – **1** with a single $b$ quark (and an $s$, a $u$, and a $d$)

• **15** confirmed (& none of the other 20 disproved)
Shameless Self-Promotion
Prog. Part. Nucl. Phys. 93 (2017) 143; 1610.04528

Review

Heavy-quark QCD exotica

Richard F. Lebed a,*, Ryan E. Mitchell b, Eric S. Swanson c

...to learn in detail about the history of the discoveries and the various theoretical interpretations attempted
How are Tetraquarks Assembled?


D^0 \rightarrow \bar{D}^{*-0} “molecule”

diquark-diantiquark

q\bar{q}-gluon “hybrid”

hadrocharmonium

cusp effect:
Resonance created by rapid opening of meson-meson threshold
Diquarks as Hadronic Components

• The short-distance color attraction of combining two color-3 quarks (3 = red, blue, green) into a color-3 diquark is fully half as strong as that of combining a 3 and a 3 into a color-neutral singlet (i.e., diquark attraction is nearly as strong as the confining attraction)

• Just as one computes a $SU(2)$ spin-spin coupling,
\[
\tilde{s}_1 \cdot \tilde{s}_2 = \frac{1}{2} \left[ (\tilde{s}_1 + \tilde{s}_2)^2 - \tilde{s}_1^2 - \tilde{s}_2^2 \right],
\]
from two particles in representations 1 and 2 combined into representation 1+2:

• If $s_1, s_2 = \text{spin } \frac{1}{2}$, and $\tilde{s}_1 + \tilde{s}_2 = \text{spin } 0$, get $-\frac{3}{4}$; if spin 1, get $+\frac{1}{4}$

• The exact $SU(3)_{\text{color}}$ analogue formula for color charges gives the result stated above
Evidence for Diquarks?

• As formal entities, **diquarks** have always been with us:
• In any baryon, each quark is a color $\mathbf{3}$, meaning that the other two quarks together must be in a color $\overline{\mathbf{3}}$: technically, a diquark
• In a $\Lambda_Q$ baryon, one heavier quark $Q = s, c, b$ is singled out, and the $ud$ pair is necessarily isosinglet and spin-singlet
• **Jaffe** [Phys. Rep. 409, 1 (2005)] calls this $ud$ a “good” diquark since models predict it to be the most tightly bound combination
• The production of diquarks in fragmentation processes has long been studied [e.g., Fontannaz et al., Phys. Lett. 77B (1979) 315]
• An ideal gas of $q$ and $\bar{q}$ (even including color screening) would produce preferentially diquark attraction $O(10\%)$ of the time [RFL, Phys. Rev. D94 (2016) 034039]
Diquarks as Quasiparticles

• A diquark composed of a heavy ($c$ or $b$) quark $Q$ and a light quark $q$ has a better chance of being identified as a localized quasiparticle, because the $Q$ can be localized to a space of dimension $\lambda_c = \frac{1}{m_Q} \lesssim O(0.1 \text{ fm})$

• Since the characteristic dimension of the compound is given by its reduced mass $\mu$, the heavy-light diquark should be about half the size of a light-light diquark or meson, $\lesssim 0.5 \text{ fm}$

• For example, Albertus et al. [Nucl. Phys. A 740, 333 (2004)] compute the matter radius of $\Lambda_c$ to be $\approx 0.3 \text{ fm}$
The Dynamical Diquark Picture

Stanley J. Brodsky, Dae Sung Hwang, RFL

- CLAIM: At least some of the observed tetraquark states are bound states of diquark-antidiquark pairs
- Likewise, pentaquark states are bound states of diquark-antitriquark pairs
- BUT the pairs are not in a static configuration; they are created with a lot of relative energy, and rapidly separate from each other
- Diquarks are not color neutral! They cannot, by confinement, separate asymptotically far
- They must hadronize via large-$r$ tails of mesonic wave functions, which suppresses decay widths to make them observably narrow
Nonleptonic $\bar{B}^0$ meson decay
Nonleptonic $\bar{B}^0$ meson decay
Nonleptonic $B^0$ meson decay

B.R.~22%
(Branching Ratio = probability)
What happens next?
Option 1: Color-allowed

B.R.\(\sim 5\%\)
(& similar 2-body)

Each has \(P\)
\(~1700\) MeV
What happens next?
Option 2: Color-suppressed
What happens next?
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What happens next?
Option 2: Color-suppressed

\[\begin{align*}
&c \\
&s \\
&\bar{c} \\
&\bar{d}
\end{align*}\]
What happens next?
Option 2: Color-suppressed

B.R. $\sim 2.3\%$
What happens next?
Option 2: Color-suppressed

B.R.~2.3%

$\bar{K}^{(*)0}$

$\bar{c}$

$s$

$\bar{d}$

charmonium

$c$
What happens next?
Option 3: Diquark formation

c s c̄

\(c\) \(s\) \(c\) \(d\)
What happens next?
Option 3: Diquark formation

Diagram showing quark interactions:
What happens next?
Option 3: Diquark formation

\[ K^{(*)^-} \]

\[ \bar{u}, u, c, \bar{c}, \bar{d}, d \]
What happens next?
Option 3: Diquark formation
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\[\bar{c}d - cu\]
This state, with a quantized glue field, is the proposed nature of the tetraquark 

\[ \bar{c} \bar{d} \quad \text{Z}^{+}(4430) \quad c \, u \]
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This state, with a quantized glue field, is the proposed nature of the tetraquark charmonium $\Psi(2S)$ with $\pi^+$. 
How far apart do the diquarks actually get?

- Since this is still a $3 \leftrightarrow \bar{3}$ color interaction, just use the Cornell potential:

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + br + \frac{32\pi \alpha_s}{9m_{cq}^2} \left( \frac{\sigma}{\sqrt{\pi}} \right)^3 e^{-\sigma^2 r^2} S_{cq} \cdot S_{\bar{cq}},$$

[This variant: Barnes et al., PRD 72, 054026 (2005)]

- Use that the kinetic energy released in $B^0 \rightarrow K^- + Z^+(4430)$ converts into potential energy until the diquarks come to rest
- Decay transition most effective at this point (WKB turning point)
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$r_Z = 1.16 \text{ fm}$
Fascinating Z(4430) fact:

Belle [K. Chilikin et al., PRD 90, 112009 (2014)] says:

\[
\frac{\text{B. R. } [Z^{-}(4430) \rightarrow \psi(2S)\pi^{-}]}{\text{B. R. } [Z^{-}(4430) \rightarrow J/\psi \pi^{-}]} > 10
\]

and LHCb has not reported seeing the $J/\psi$ (1S) mode

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\[
\langle r_{\psi(2S)} \rangle = 0.80 \text{ fm}
\]

\[
\langle r_{J/\psi} \rangle = 0.39 \text{ fm}
\]

\[
r_{Z} = 1.16 \text{ fm}
\]
Nonleptonic $\Lambda_b$ baryon decay
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This is a diquark!
- Color $\bar{3}$
- Isospin 0
- Spin 0
Nonleptonic $\Lambda_b$ baryon decay

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What happens next?
Diquark *and triquark* formation
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Diquark and triquark formation

\[
\begin{align*}
\bar{u} & \quad u \\
\bar{s} & \quad \bar{c} \\
\quad c & \quad ud
\end{align*}
\]
What happens next?
Diquark *and* triquark formation

\[ \text{K}^{(*)-} \]

\( \bar{u} \)
\( u \)
\( c \)
\( \bar{c} \)
\( ud \)
What happens next?
Diquark and triquark formation

\[ K^{(*)^-} \]

\[ \bar{u} \quad cu \quad \bar{c}ud \]
What happens next?
Diquark and triquark formation

\[ \bar{c}ud \leftrightarrow cu \]
The same color-triplet mechanism, supplemented with the fact that the \( ud \) in \( \Lambda \) baryons themselves act as diquarks, predicts a rich spectrum of *pentaquarks*.
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\[ J^P = \frac{1^-}{2} \quad J^P = L^{-1}\bar{L} \quad J^P = 0^+, 1^+ \]
The same color-triplet mechanism, supplemented with the fact that the $ud$ in $\Lambda$ baryons themselves act as diquarks, predicts a rich spectrum of *pentaquarks*

\[ J/\Psi \]

\[ p \]

\[ J^p = \frac{1^-}{2} \quad J^p = L^{(-1)^L} \quad J^p = 0^+, 1^+ \]
Exotics in the Born-Oppenheimer Approximation

• When studying physical chemistry or atomic physics, as students we encountered a qualitative definition of the Born-Oppenheimer approximation [Ann. Phys. 389 (1927) 457]: “The light degrees of freedom (the electrons) in an atom or molecule adapt their state rapidly and adiabatically with respect to the much more slowly changing nuclei”

• This is a true statement, but it can also be recast rigorously into particle-physics language:
  – The dynamics exhibits a scale separation in powers of $m_e/m_N$
  – The wave functions factor into light-d.o.f. and heavy-d.o.f. parts, with the light d.o.f. acting as potentials [B-O potentials] for the heavy d.o.f.
  – One can build an effective field theory, with $m_e/m_N$ as the expansion parameter [Brambilla et al., PRD 97 (2018) 016016]
When Is the B-O Approximation Needed?

- With only a single heavy source and a single light d.o.f. (e.g., hydrogen or mesons composed of constituent quarks), then the usual trick of using a reduced mass is sufficient.
- A system with at least two heavy sources plus light d.o.f. has B-O potentials that depend upon the separation and orientation of the heavy sources.
- A simple such system is the $H_2^+$ ion: 2 protons, 1 electron [Griffiths QM, Sec. 7.3]
- Another is the $\Xi_{cc} (ccq)$ baryon.
- Another is the charmoniumlike hybrids $c\bar{c}g$, as well as charmoniumlike tetraquarks $c\bar{c}q_1\bar{q}_2$ and pentaquarks $c\bar{c}q_1q_2q_3$, ...
B-O Quantum Numbers for the “Homonuclear Diatomic” $Q \bar{Q}$ System

- Symmetry group is that of a cylinder, $D_{\infty h}$:
- Rotations about the axis $\hat{r}$ (eigenvalues $\lambda \equiv \hat{r} \cdot L$)
- Reflection ($R_{\text{light}}$) through a plane containing the axis $\hat{r}$ (eigenvalues $\epsilon = \pm 1$)
- Reflection through the origin ($P_{\text{light}}$) is not a symmetry since $Q$, $\bar{Q}$ not equivalent, but $(CP)_{\text{light}}$ is a symmetry (eigenvalues $\eta = \pm 1$, called $g$ and $u$, respectively)
B-O Quantum Numbers for the “Homonuclear Diatomic” $Q\bar{Q}$ System

- $\lambda \equiv \hat{r} \cdot L$ is a pseudoscalar: Invariant under rotations, odd under reflections
  Reflection $R_{\text{light}}$ gives physically equivalent system, but $\lambda \rightarrow -\lambda$
- Thus, the energy of the system can only depend upon $\Lambda \equiv |\lambda|$
- The B-O potentials are thus labeled by $\Lambda^\epsilon_\eta$
  - $\Lambda = 0, 1, 2, \cdots$ are labeled, respectively, by the letters $\Sigma, \Pi, \Delta, \cdots$ (analogous to $S, P, D, \cdots$)
  - Can show that the $P_{\text{light}}$ eigenvalue equals $\epsilon (-1)^\Lambda$
  - If the light d.o.f. contain explicit spins ($e^-$ for molecules), then its total $s$ is also good quantum number $\Rightarrow 2s + 1 \Lambda^\epsilon_\eta$
Notes on the $D_{\infty h}$ B-O Quantum Numbers

- Only $\Sigma$ ($\Lambda = 0$) potentials are automatically eigenstates of $R_{\text{light}}$ (definite $\epsilon$), but one can make $\Pi, \Delta, \cdots$ into eigenstates of definite $\epsilon$ by taking combination of $+\lambda$ and $-\lambda$ states (just as one does to form even/odd functions).

- The term label $\Gamma \equiv \Lambda^\epsilon_\eta$ fully specifies the $D_{\infty h}$ irreducible representations, but it is still possible to specify not only $s$, but also $L$, which satisfies the constraint $L \geq |\hat{r} \cdot L| = \Lambda$.

- If the heavy sources are not truly “homonuclear” (e.g., $b\bar{c}$), then one loses the $(CP)_{\text{light}}$ eigenvalue $\eta$.

- If the light d.o.f. carry isospin (e.g., $c\bar{c}u\bar{d}$), then $C$-parity symmetry is replaced by $G$-parity symmetry, $G \equiv C(-1)^I$. 
Exotics spectroscopy using B-O potentials
RFL, Phys. Rev. D 96 (2017), 116003 [1709.06097]

- Given quantum numbers of the light d.o.f., combine with the heavy quantum numbers to find the full spectrum of states.
- For hybrid mesons, the light d.o.f. consist of an extended gluon field plus sea $q\bar{q}$ (gluelump).
- For tetraquarks, the light valence quarks can in principle be included with the light d.o.f. [Braaten et al., PRD 90 (2014) 014044]
- Diquark model: It is more appropriate to separate out $s_{q\bar{q}}$

$$J = J_{\text{light}} + L_{Q\bar{Q}} + s_{q\bar{q}} + s_{Q\bar{Q}}$$

- $L$ and $S$

- Still have $\lambda \equiv \hat{r} \cdot L = \hat{r} \cdot J_{\text{light}}$ since $\hat{r} \cdot L_{Q\bar{Q}} = 0$
Exotics spectroscopy using B-O potentials

- **Hybrid discrete symmetry quantum numbers:**
  \[ P = \epsilon (-1)^{\Lambda+L+1}, \quad C = \eta \epsilon (-1)^{\Lambda+L+s_{Q\bar{Q}}} \]

- **Tetraquark discrete symmetry quantum numbers:**
  \[ P = \epsilon (-1)^{\Lambda+L}, \quad C = \eta \epsilon (-1)^{\Lambda+L+s_{Q\bar{Q}}+s_{Q\bar{Q}}} \]

- **Pentaquark discrete symmetry quantum numbers:**
  \[ P = \epsilon (-1)^{\Lambda+L+1}, \quad C \text{ no longer good} \]

- **Now work out the multiplets based on the B-O potentials, starting with underlying states classified according to spins**
  \( s_{q\bar{q}}, s_{Q\bar{Q}}, S \) [Maiani et al., PRD 89 (2014) 114010], *e.g.*, 
  \[ Z' \equiv \left| 0_{s_{q\bar{q}}}, 1_{s_{Q\bar{Q}}} \right\rangle_{S=1} \]
## Exotics spectroscopy using B-O potentials: Tetraquarks

**Boldface** = exotic quantum numbers for $q\bar{q}$

<table>
<thead>
<tr>
<th>BO potential</th>
<th>$\Sigma_g^+$ (1S)</th>
<th>$\Sigma_g^+$ (1P)</th>
<th>$\Sigma_g^+$ (1D)</th>
<th>$\Pi_u^+$ (1P) &amp; $\Sigma_u^-$ (1P)</th>
<th>$\Pi_u^+$ (1P)</th>
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<tr>
<td>$\Sigma_g^+$ (1S)</td>
<td>$\tilde{X}^{(0)}_{0S}^{++}$</td>
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<td>$\tilde{X}^{(2)}_{0D}^{2-}$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{Z}^{(1)}_{S}^{++}$</td>
<td>$[\tilde{Z}^{(0)}_{P}^{(1)}, (1),(2)]^{++}$</td>
<td>$[\tilde{Z}^{(1)}_{P}^{(2)}]^{++}$</td>
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<td>$\tilde{Z}^{(1)}_{S}^{++}$</td>
<td>$[\tilde{Z}^{(0)}_{D}^{(2)}, (2),(3)]^{++}$</td>
</tr>
<tr>
<td></td>
<td>$2 \times 1^{+-}$</td>
<td>$2 \times (0, 1, 2)^{-}$</td>
<td>$2 \times (1, 2, 3)^{-}$</td>
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<td>$[0, 1, 2]^{++}$</td>
<td>$[1, (0, 1, 2), (1, 2, 3)]^{-}$</td>
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</table>

### State notation

- **$\tilde{X}$** represents the exotic state.
- **$X$** represents the standard state.
- **$\tilde{Z}$** represents the exotic state.
- **$Z$** represents the standard state.
- **$\tilde{X}, X, \tilde{Z}, Z$** are quantum numbers.
- **$J^{PC}$** represents the total angular momentum and parity.
### Exotics spectroscopy using B-O potentials: Pentaquarks

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<th>BO potential</th>
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<th>State $J^P$</th>
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<td>$\Sigma^+(1S)$</td>
<td>$\tilde{P}^{(1/2)<em>+}, \tilde{P}'^{(1/2)</em>+}$</td>
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</tbody>
</table>
Exotics with Known $J^{PC}$

- Can these multiplets accommodate the states with known (or favored values of) $J^{P(C)}$?
- **No problem:**

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>States</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^{++}$</td>
<td>$X(3915), X(4500), X(4700)$</td>
</tr>
<tr>
<td>$0^{- -}$</td>
<td>$Z_c^0(4240)$</td>
</tr>
<tr>
<td>$1^{- -}$</td>
<td>$Y(4008), Y(4220), Y(4260), Y(4360), Y(4390), X(4630), Y(4660), Y_b(10888)$</td>
</tr>
<tr>
<td>$1^{++}$</td>
<td>$X(3872), Y(4140), Y(4274)$</td>
</tr>
<tr>
<td>$1^{+-}$</td>
<td>$Z_c^0(3900), Z_c^0(4200), Z_c^0(4430), Z_b^0(10610), Z_b^0(10650)$</td>
</tr>
<tr>
<td>$\frac{3}{2}^{+}, \frac{5}{2}^{+}$</td>
<td>$P_c(4380), P_c(4450)$</td>
</tr>
</tbody>
</table>

- Well, what about all the other predicted ones? Only a few production modes have been used to date, which prefer certain $J^{PC}$, such as $1^{- -}$ for initial-state $\gamma$ radiation
Ordering of the B-O Potentials

• How do we know what are lowest, next lowest, etc. B-O potentials? That’s nonperturbative QCD!

• In the case of hybrids and pure-glue configurations, that information comes from numerous lattice QCD simulations

• State-of-the-art results:

• But it has a very long history:
  Griffiths, Michael, Rakow: PLB 129B (1983) 351
  Foster et al.: PRD 59 (1999) 094509
  Marsh, Lewis: PRD 89 (2014) 014502
Ordering of the B-O Potentials

• But all pure-glue simulations agree:
  – Ground-state potential: $\Sigma^+_g$
  – 1st excited potential: $\Pi_u$; 2nd excited potential: $\Sigma^-_u$

• Additionally, in the small-size limit, some potentials become degenerate gluelumps and mix, e.g., $\Pi^+_u (1P)$ and $\Sigma^-_u (1P)$
  [\Lambda doubling: Berwein et al., PRD 92 (2015) 114019]

• Great for hybrids! What about tetra/pentaquarks?

• Here, the only relevant lattice results use flavor-nonsinglet potentials for color-adjoint mesons:
  Foster, Michael: PRD 59 (1999) 094509

• What we really need for the diquark model is simulations with heavy sources that also carry isospin
Selection Rules

• Heavy-quark spin symmetry: $s_{Q\bar{Q}}$ should be conserved in a decay of a $Q\bar{Q}q_1\bar{q}_2$ (or $Q\bar{Q}q_1q_2q_3$) to $Q\bar{Q}$ + light hadrons

• Exotics with $s_{Q\bar{Q}} = 1$ should decay to $\psi$ ($\Upsilon$) or $\chi$

• Exotics with $s_{Q\bar{Q}} = 0$ should decay to $\eta$ or $h$

• The evidence is mixed: For example,
  – The $c\bar{c}u\bar{d}$ states $Z_c^+(3900) \rightarrow J/\psi$, while $Z_c^+(4020) \rightarrow h_c$
  – The $b\bar{b}u\bar{d}$ states $Z_b^+(10610), Z_b^+(10650) \rightarrow$ both $\Upsilon, h_b$

• The latter case suggests a mixture of $s_{Q\bar{Q}}$ eigenstates
  One way for this to occur is molecular states (good $s_{Q\bar{Q}}, s_{\bar{Q}q}$)
  Or, good diquark-spin quantum numbers (good $s_{Qq}, s_{\bar{Q}\bar{q}}$)
Selection Rules

• **B-O potential quantum numbers:**
  Separate conservation of light d.o.f. quantum numbers (since they undergo more rapid transitions than heavy d.o.f.)

• **Example:** Consider $Q \bar{Q} q_1 \bar{q}_2 (\Lambda_\eta^\epsilon) \rightarrow Q \bar{Q} (\Sigma_g^+) + \rho/\omega$ ($s$-wave)

• Then $J^{PC}$ conservation forbids this decay unless:
  \[ \Lambda \leq 1 + s_{q\bar{q}}, \quad \epsilon = (-1)^{\Lambda+1}, \quad \eta = + \]

• But in comparing to the known decays, these rules only work if some $\Lambda_\eta^\epsilon$ potentials besides the ones seen for pure glue are among those of lowest energy

• Again, lattice simulations with **heavy diquark sources** would completely resolve this question
We now appear to live in an age of at least four known hadron species: mesons, baryons, tetraquarks, and pentaquarks.

This talk focused on the construction of multiquark exotics composed of colored diquark (and triquark) components.

The dynamical diquark picture says that several properties of the exotics can be explained if the colored diquark components achieve a substantial spatial separation.

The most convenient framework for describing such states is the Born-Oppenheimer approximation. We studied the relevant quantum numbers, built the particle spectrum, and examined decay selection rules.
So What Next?

- Choose particular forms for $V_{\Lambda \eta}(r)$, feed into Schrödinger equations, solve for the spectrum and decay amplitudes.
- Issue: Need high-quality lattice results including heavy diquark sources to know the correct forms of $V_{\Lambda \eta}(r)$.
- Are there isospin-dependent forces analogous to $\pi$ exchange? One lesson from dense QCD color-flavor-locking [Alford, Rajagopal, Wilczek, PLB 422 (1998) 247]: Isospin-carrying Goldstone bosons exist even inside glue fields.
- Genuine hadronic (e.g., meson-meson) thresholds mix with (e.g., diquark-antidiquark) resonances and can lead to nontrivial level-crossing behavior in the spectrum and decays.
Will It All Work?

Ask me again in a couple of years!

Thank you!