Transport phenomena in strong magnetic fields

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Seminar in INT Program INT-18-1b Week 1
Multi-Scale Problems Using Effective Field Theories
Strong magnetic fields induced by relativistic heavy-ion collisions

Z \sim 80, \quad v > 0.99999 c, \quad \text{Length scale} \sim 1/\Lambda_{\text{QCD}}

One can study the interplay btw QCD and QED.
Besides,

- Weyl & Dirac semimetals
- Neutron stars/magnetars
- High intensity laser fields
- Strong B field by lattice QCD simulations
- Cosmology
“Harmonic oscillator” in the transverse plane

Relativistic: \[ \epsilon_n = \sqrt{p_z^2 + (2n + 1)eB} + m^2 \]

Nonrelativistic: \[ \epsilon_n = \frac{p_z^2}{2m^2} + \left( n + \frac{1}{2} \right) \frac{eB}{m^2} \]  
Cyclotron frequency

The Zeeman effect for spin-1/2 cancels the “zero-point energy”.
Dynamics in the lowest Landau level (LLL)

(1+1)-D dispersion relations
“Effective dimensional reduction”

\[ \varepsilon = \pm p_z \]

Macroscopic consequences

Anomalous transport phenomena
→ Chiral magnetohydrodynamics (MHD)
KH, Y.Hirono, H.-U.Yee, Y.Yin

Any consequence in usual (dissipative) transport phenomena?

SSB and Kondo effect in analogy to dense systems
Chiral magnetohydrodynamics

KH, Yuji Hirono, Ho-Ung Yee, and Yi Yin, \texttt{arXiv:1711.08450 [hep-th]}
Anomaly-induced transports in a magnetic and vortex field

\[
\begin{pmatrix}
j^\mu_V \\
j^\mu_A
\end{pmatrix} = C_A \begin{pmatrix}
q_f \mu_A & \mu_V \mu_A \\
q_f \mu_V & (\mu^2_V + \mu^2_A)/2 + C_A^{-1} T^2/12
\end{pmatrix} \begin{pmatrix}
B^\mu \\
\omega^\mu
\end{pmatrix}
\]

\[
B^\mu = \tilde{F}^{\mu \nu} u_\nu, \quad \omega^\mu = \frac{1}{2} \epsilon^{\mu \alpha \beta \gamma} u_\alpha \partial_\beta u_\gamma
\]

Non-dissipative transport phenomena with time-reversal even and nonrenormalizable coefficients.

Anomaly relation:

\[
\partial_\mu j^\mu_A = q_f^2 C_A E \cdot B
\]

\[
C_A = \frac{1}{2\pi^2}
\]

Cf., An interplay between the B and \( \omega \) leads to a new nonrenormalizable transport coefficient for the magneto-vorticity coupling.

Anomalous hydrodynamics in STRONG & DYNAMICAL magnetic fields

-- Anomalous hydrodynamics \( \mu_A \neq 0, \ B \sim \mathcal{O}(\partial A) \) and external
Son & Surowka

-- Anomalous magnetohydrodynamics (MHD) \( \mu_A \neq 0, \ B \sim \mathcal{O}(1) \) and dynamical
This work.

Slow variables in chiral MHD: \( \{\epsilon, u^\mu, B^\mu, \text{ and } n_A\} \)
\( n_A \): # density of axial charge
Neutral plasma \( (n_V = 0) \)
No E-field in the global equilibrium

\[
\text{EoM: } \partial_\mu T_{\text{fluid+EM}}^{\mu\nu} = 0, \ \partial_\mu \tilde{F}^{\mu\nu} = 0, \ \partial_\mu j_A^\mu = -C_A E^\mu B_\mu.
\]
Constitutive eqs. in the ideal order determined by the entropy conservation

\[ T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - X \Delta^{\mu\nu} - Y B^\mu B^\nu \]
\[ \widetilde{F}^{\mu\nu}_{(0)} = B^\mu u^\nu - B^\nu u^\mu \]
\[ \dot{j}^{\mu}_{A(0)} = n_A u^\mu \]

\[ \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \quad (u_\mu \Delta^{\mu\nu} = 0) \]

E-field is first order.

\[ B^{(\mu} u^{\nu)} \] is absent in \( T^{\mu\nu} \) when \( n_V = 0 \).

From EoM + thermodynamic relation \( ds = \frac{1}{T}(d\epsilon - \mu_A d\ln A - H_\mu dB^\mu) \)

\[ \partial_\mu (su^\mu) = u \cdot \partial s + s \partial \cdot u \]
\[ = (p - X) \partial \cdot u + (H^\mu - Y B^\mu) B \cdot \partial u_\mu \]
\[ = 0 \quad \text{for the ideal part.} \]

Therefore,

\[ T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - p \Delta^{\mu\nu} - \mu^{-1} B^\mu B^\nu \]

\( \epsilon \) and \( p \) are the total (fluid+magnetic) energy and pressure.
Constitutive eqs. and entropy current in the first order

\[ T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} \quad \text{Note that } \partial_\mu j^\mu_A = -C_A E^{\mu}_{(1)} B_\mu. \]

\[ \tilde{F}^{\mu\nu} = \tilde{F}^{\mu\nu}_{(0)} - \epsilon^{\mu\nu\alpha\beta} u_\alpha E^{(1)\beta} \]

\[ j^\mu_A = j^\mu_A_{(0)} + j^\mu_A_{(1)} \]

How much can we constrain \( T^{\mu\nu}_{(1)}, E^{\mu}_{(1)}, j^\mu_A_{(1)} \sim \mathcal{O}(\partial^1) \) from \( \partial_\mu s^\mu \geq 0 \)?

Again, computing the entropy current,

\[ \partial_\mu (su^\mu) = \partial_\mu [\cdots] + T^{\mu\nu}_{(1)} \partial_\mu (\beta u_\nu) - j^\mu_A_{(1)} \partial_\mu (\beta \mu_A) \]

\[ + E^{\mu}_{(1)} \{ \mu_A C_A B_\mu - \epsilon_{\mu\nu\alpha\beta} u^\nu \partial^\alpha (\beta H^\beta) \} \]

The total derivative term \( \partial_\mu [\cdots] \) is identified as the first order correction to the entropy current, \( \partial_\mu S^{\mu}_{(1)}. \)
Insuring the semi-positivity with bilinear forms

The second law of the thermodynamics $\partial_\mu (s u^\mu) \geq 0$ constrains the tensor structures of the first order corrections.

Each term should be semi-positive definite (see previous slide). For example,

$$E^\mu_{(1)} \left\{ \mu_A C_A B_\mu - \epsilon_{\mu \nu \alpha \beta} u^\nu \partial^\alpha (\beta H^\beta) \right\} \geq 0$$

Should have a bilinear form

$$E^\mu_{(1)} X_{\mu \nu} E^\nu_{(1)} \geq 0$$

$$X_{\mu \nu} = \sigma_\parallel b_\mu b_\nu - \sigma_\perp (\Delta_{\mu \nu} + b_\mu b_\nu) - \sigma_{\text{Hall}} \epsilon_{\mu \nu \alpha \beta} u^\alpha b^\beta$$

$$\sigma_\parallel, \sigma_\perp \geq 0, \text{ but } \sigma_{\text{Hall}} \propto \mu \nu. \quad b^\mu = B^\mu / (\sqrt{-B_\mu B_\mu})$$

Therefore, we get a “constitutive eq.” of the E-field:

$$E^\mu_{(1)} = X^{-1\mu \rho} \left\{ \mu_A C_A B_\rho - \epsilon_{\rho \nu \alpha \beta} u^\nu \partial^\alpha (\beta H^\beta) \right\}$$
Transport coefficients
--- CME and other dissipative terms

\[ X_{\mu\nu} E^\nu_{(1)} = \mu_A C_A B_\mu - \epsilon_{\mu\nu\alpha\beta} u^\nu \partial^\alpha (\beta H^\beta) \]

with \( X \) given in the previous slide.

Eliminating \( \partial^\alpha H_\beta \) by the Ampère’s law \( J^\nu = \partial_\mu F^{\mu\nu} \),

\[ J^\mu_V = C_A \mu_A B^\mu + \left[ \sigma_\parallel E_\parallel^\mu + \sigma_\perp E_\perp^\mu + \sigma_{\text{Hall}} \epsilon^{\mu\nu\alpha\beta} u_\nu b_\alpha E_\beta \right] + \cdots \]

The CME current is given by \( C_A \) without any other unknown coefficient, and is necessary for insuring the semi-positive entropy production.

There appear 3 conductivities (see later slides).

Similarly,

\[ T^{\mu\nu}_{(1)} \partial_\mu (\beta u_\nu) \geq 0 \]

provides 5 shear and 2 bulk viscous coefficients (see later slides)

Landau & Lifshitz; Huang, Sedrakian, & Rischke; Tuchin; Hernandez & Kovtun; ...
Phases of collective excitations and new instabilities from the CME
Collective excitations in MHD without anomaly

2 transverse waves (Alfven waves)  
4 longitudinal waves (fast and slow magneto-sonic waves)

*T Magnetic lines move together with the fluid volume.

Tension of B-field = Restoring force  
Fluid energy density = Inertia

Transverse Alfven wave

$$v_{Alf}^2 = \frac{B_0^2}{\epsilon + p + B_0^2}$$
Alfven wave from a linear analysis

0. Stationary solutions
\[ \nu^\mu = (1, 0), \quad B^\mu = (0, B_0), \quad j^\mu = (0, 0) \]

1. Transverse perturbations
\[ \nu \rightarrow \nu + \delta \nu \]
\[ B_0 \rightarrow B_0 + \delta B \]

Linearize the set of hydrodynamic eqs. with respect to the perturbation.

2. Wave equation
\[ \partial_t^2 \delta B(t, z) = \frac{B_0^2}{\epsilon + \rho} \partial_z^2 \delta B(t, z) \]
Alfven wave velocity

Same wave equation for \( \delta \nu \)

\( \rightarrow \) Fluctuations of B and \( \nu \) propagate together.

\[ B_0 \parallel k \]
How does the CME change the hydrodynamic waves in chiral fluid?

--- Drastic changes by only one term in the current

\[ j^\mu = \sigma_{\text{CME}} B^\mu \]
Excitations in anomalous MHD

Linearized EoM \( (\mathbf{v} \rightarrow \mathbf{v} + \delta \mathbf{v}, \ B \rightarrow \mathbf{B} + \delta \mathbf{B}) \)

\( \rightarrow \) Secular eq. as a cubic eq. of \( \omega^2 \)

\( \rightarrow \) 3 modes propagating in the opposite directions (6 solutions in total)

\[
(\omega^2 - x_1)(\omega^4 + b\omega^2 + c) = 0
\]

\( x_1 \): Real solution

Stability of the waves from classification of solutions

\[ \mu_A \neq 0, \ \mu_V = 0 \]
New hydrodynamic instability in a chiral fluid

Signs of the imaginary parts (Damping/growing modes in the hydrodynamic time evolution)

Helicity decomposition (Circular R/L polarizations)
\[ \nabla \times \mathbf{e}_{R/L} = \pm \mathbf{e}_{R/L} \]

A helicity selection, depending on the sign of \( \mu_A \).
Helicity conversions as the topological origin of the instability

Chiral imbalance btw R and L fermions

Chiral Plasma Instability (CPI)

Magnetic helicity

Fluid helicity (structures of vortex strings)

Dynamical & beyond-linear analysis demanded.
Short summary 1:
Formulation of Chiral MHD

The second law of thermodynamics determines the zeroth order derivative expansion, and constrains the tensor structures in the first order.

In the MHD regime, the CME current is completely fixed by the anomaly coefficient without any ambiguity.

The other dissipative parts are characterized by 3 conductivities, and 5 shear and 2 bulk viscous coefficients. (Will be discussed shortly.)
Short summary 2:
Collective excitations and instabilities of Chiral MHD

The CME drastically changes the time evolution of the fluid with the axial charge and B-field.

Not stable against a small perturbation on $v$ and $B$.
$\rightarrow$ New hydrodynamic instability!

Helicities of the unstable modes are selected by $\mu_A$. 
Dissipative transport phenomena in the lowest Landau Levels

In the LLL for the strong B limit, charged fermions transport the charge and momentum only along the B.

Longitudinal, transverse, and Hall currents; 5 shear and 2 bulk viscous coefficients.

Landau & Lifshitz; Huang, Sedrakian, & Rischke; Tuchin; Hernandez & Kovtun

In the LLL for the strong B limit, charged fermions transport the charge and momentum only along the B.

Longitudinal conductivity
Longitudinal bulk viscosity

KH, S.Li, D.Satow, H.-U. Yee
KH, X.-G.Huang, D.Satow, D.Rischke
1. Electrical conductivity in strong magnetic field

KH, Shiyong Li, Daisuke Satow, and Ho-Ung Yee, arXiv:1610.06839 [hep-ph].

\[ j = \sigma \text{Ohm} E \]

\[ T \neq 0, \mu = 0 \]

“Mismatched dimensions”
Quarks live in (1+1) D
Gluons live in (3+1) D

Strong B
Linear response in kinetic theory

\[ \dot{j}_z = \sigma_{zz} E_z \]

Acceleration by the electric field

\[ \dot{p}_z = \pm q_f E_z \quad f_\pm \rightarrow f^{eq} \pm \delta f \]

Total current integrated over \( p_z \) from the off-equilibrium components

\[ j_z = \frac{|q_f B|}{2\pi} \cdot q_f \int \frac{dp_z}{2\pi} v_z (f_+ - f_-) \]

Density of states

“Landau degeneracy factor”

\( (1+1) \) Dim.

2\( \delta f \)

Boltzmann eq. in stationary and homogeneous limit

\[ \frac{\partial f_\pm}{\partial t} + \dot{z} \frac{\partial f_\pm}{\partial z} + \dot{p}_z \frac{\partial f_\pm}{\partial p_z} = C[f_\pm] \]

External driving force v.s. Relaxation
Quark-damping mechanism in strong magnetic fields

Finite B opens 1-2 processes

\[ |\mathcal{M}|^2 \sim \mathcal{O}(\alpha_s) \]
This work with B
(Cf., Cyclotron radiation)

\[ |\mathcal{M}|^2 \sim \mathcal{O}(\alpha_s^2) \]
AMY without B

\[ \epsilon_{\text{quark}}^2 = p_z^2 + m_f^2 \]
\[ \epsilon_{\text{gluon}}^2 = k_z^2 + |\mathbf{k}_\perp|^2 \]

\[ |\mathbf{k}_\perp| \text{ works as a gluon mass for 2D kinematics.} \]
Analogue of a massive weak boson production from \( q\bar{q} \) annihilation in 4D.
Chirality selection in the massless limit

Chirality conservation at the vertex

The scattering in the (1+1) D is prohibited by the chirality conservation in the massless limit (cf., CME).

Therefore, the scattering rate \( \to 0 \) as \( m \to 0 \).

\[ \epsilon_p = \pm p_z \]

\[ \gamma \propto g^2 m_f^2 \]

\[ A_\mu \propto (0, 1, \pm i, 0) \]

unless \( |k_\perp| \) is finite.
\[ \sigma^{zz} = q_f^2 \left| q_f B \right| \frac{4T}{g^2 m_f^2 \ln(T/M)} \]

When \( eB = 0 \),

Arnold, Moore, and Yaffe

\[ \sigma_{B=0} = q_f^2 \frac{T}{g^4 \ln[T/(gT)]} \]

See KH and Satow for a consistent result from diagrammatic calculation, and Fukushima and Hidaka for effects of the higher Landau levels.

Transition from strong B (LLL) to weak B (hLL).

\[ eB = 10m_{\pi}^2 \]
Lifetime of the B-field in HIC

A longer lifetime due to the Lenz’s law? Tuchin

\(B(t) \rightarrow \text{Induced } J \text{ sustains } B.\)

\(\partial_t B(t) < 0\)

Time dependent B induces E.

E induces J if QGP is conducting.

\(\rightarrow\) Induced J sustains B.

Important to know the conductivity of QGP.

There is no transverse current in the LLL, because the quarks are confined in the longitudinal direction.

\(\rightarrow\) No backreaction effect in the “very” strong magnetic field.
2. Bulk viscosity in a strong magnetic field

Bulk viscosity of the QGP

\[ \zeta = 0 \]  
Adiabatic expansion in an equilibrium

\[ \zeta \neq 0 \]  
Rapid expansion in (slightly) off equilibrium

Scale inv. in the massless & classical limits:
\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}i\not{\partial}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \]

A finite bulk viscosity demands conformal-symmetry breakings.
\[ T_\mu = m^2 \bar{\psi}\psi + \frac{\beta(g)}{g^3}F^{\mu\nu}F_{\mu\nu} \]
\[ \zeta \sim m^4 \#_1 + \beta^2 \#_2 \]

Arnold, Dogan, Moore (2006)
Pressure evolution in response to an expansion

\[ \delta P_\parallel = \frac{|eB|}{2\pi} \int \frac{dk_z}{2\pi} k_z v_z \delta f(k_z) \quad v_z = \frac{k_z}{\epsilon_k} \]

In the linear response regime, \( \delta f \propto \partial_z u_z \).

\[ \zeta_\parallel = -\frac{1}{3} \frac{\delta P_\parallel}{\partial_z u_z} \]
Boltzmann eq. in an expanding system

\[ \frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} = C[f] \]

Perturbation by the expansion of the system

\[ f(t, z; k_z) = \frac{1}{\exp[\beta(t)(\epsilon - k_z u_z(z))] + 1} \]

Solve the linearized Boltzmann eq. for \( \delta f = f - f_{eq} \)

Competition between the conformal symmetry and the chirality conservation in the massless limit.

\[ \zeta_{||} = \frac{|eB|}{2\pi} \cdot \frac{1}{T} \int \frac{dk_z}{2\pi \epsilon_k^2} \left[ \epsilon^2 - k_z^2 \right]^2 [\ldots] \gamma_{damp} \]

Remember \( \gamma_{damp} \propto m^2 g^2 \) in B !!
Results

\[ M^2 = \alpha_s eB \]

\[ \zeta_{\parallel} \sim \frac{|eB|}{2\pi} \frac{m^2}{g^2 T \ln(T/M)} \]

Consistent result can be obtained from diagrammatic calculation with Kubo formula.

When \( eB = 0 \),

\[ \zeta_{B=0} \sim (\text{typical momentum})^4 \frac{(\text{conformal breaking factor})^2}{(\text{mean free path})^{-1}} \sim T^4 \left( \frac{m_f^2}{T^2} \right)^2 \frac{1}{g^4 T \ln(1/g)}. \]

Arnold, Dogan, Moore (2006)
Short summary

The chirality selection plays crucial roles in the non-anomalous transports.

Consequences of the chirality selection rule and the competition with the conformal symmetry.

Electrical conductivity: \( \sigma_{zz} \propto eB \frac{1}{g^2 m_f^2} \)

Bulk viscosity: \( \zeta_{||} \propto eB \frac{m_f^2}{g^2} \)
Analogy between the systems at high density and in strong B --- Consequences of dimensional reductions

Condensed matter in finite density

<table>
<thead>
<tr>
<th>BCS instability</th>
<th>Magnetic catalysis of $\chi_{SB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kondo effect</td>
<td>Kondo effect in B-field</td>
</tr>
</tbody>
</table>

They are all understood from the dimensional reduction.

KH, K. Itakura, S. Ozaki, A review paper to appear in PPNP
Analogy btw the dimensional reduction in a large B and μ

(1+1)-D dispersion relations

\[ \varepsilon = \pm p_z \quad \leftrightarrow \quad \varepsilon = \pm \ell_\parallel \quad (\ell_\parallel \ll \mu) \]

\[ \rho = \frac{N_{\text{state}}}{S} = \frac{eB}{2\pi} \sim \frac{1}{r_{\text{cyclotron}}^2} \]

\[ \rho_F \sim \mu^2 \]

Strong B

2-dimensional density of states

Large Fermi sphere
IR scaling dimensions

When $\epsilon \to s\epsilon$, $l_\parallel \to sl_\parallel$. ($s < 1$)

Kinetic term

$$S^{\text{kin}} = \int dt \sum_{\nu_F} \int \frac{d^2 \ell_\perp d\ell_\parallel}{(2\pi)^3} \bar{\psi}_+(i\partial_t - \ell_\parallel)\gamma^0 \psi_+ + O(1/\mu)$$

$$0 = \frac{\bar{\psi} \cdot \psi}{dt} + \frac{1}{d\ell_\parallel} + \frac{1}{\partial_t}$$

$$d_\psi = -\frac{1}{2}$$

Four-Fermi operators for superconductivity

Polchinski (1992)

$$S^{\text{int}} = \int dt \left[ \int \frac{d^2 \ell_\perp d\ell_\parallel}{(2\pi)^3} \right]^4 G[\bar{\psi}^{(4)}(\gamma^{\mu}\psi^{(2)})][\bar{\psi}^{(2)}(\gamma^{\mu})\psi^{(1)}] \delta^{(3)}(p^{(1)} + p^{(2)} - p^{(3)} - p^{(4)})$$

In general momentum config.

$$p^{(1)} + p^{(2)} \sim \mu$$

$$d_{4-\text{Fermi}} = (-1) + 4(1 - \frac{1}{2}) = +1$$

In the BCS config.

$$p^{(1)} + p^{(2)} \sim \ell_\parallel \ll \mu$$

$$d_{4-\text{Fermi}} = (-1) + 4(1 - \frac{1}{2}) - 1 = 0$$
SSB by the dimensional reduction

(1+1)-D dispersion relation $\Rightarrow d_\psi = -1/2$

Again, the 4-Fermi interaction is a marginal operator in the (1+1) dimensions in a strong B!

Wilsonian RG flow driven by the logarithmic quantum corrections

$\Lambda \frac{dG(\Lambda)}{d\Lambda} = -G^2(\Lambda) \rho_B$  \hspace{1cm} Negative beta function!!

The solution diverges at $\Lambda_{dy} = \Lambda_{UV} \exp \left( -\frac{1}{\rho_B G(\Lambda_{UV})} \right)$

The IR regime of the LLL is strong coupling.

$\Rightarrow \chi_{SB}$ occurs even in QED!!!

(Cf., SC occurs with any weak attraction.)
Chiral symmetry and its spontaneous breaking

No sign problem in B-field!

Interplay with the strong coupling QCD

Lattice QCD results inspired so many authors, Including KH, T.Kojo, N.Su; KH, K.Itakura, S.Ozaki.
Kondo effect

--- Another consequence of the dimensional reduction
Impurity scatterings near a Fermi surface

+ Electron-impurity scattering in conde. matt.
+ Light-Heavy quark scattering in quark matter

$G(\Lambda)(\bar{\psi}\psi)(\bar{\Psi}\Psi)$

4-Fermi operator for the heavy-light scattering is marginal when the light particle lives in (1+1) dimensions.
RG analysis for “QCD Kondo effect”

\[ G(\Lambda - d\Lambda) = G(\Lambda) + G(\Lambda) \]

From above to \( \varepsilon_F \)

\[ \rho_F \int_{\varepsilon_F}^{d\varepsilon} \frac{d\varepsilon}{\varepsilon} \sim \log \frac{\Lambda}{\Lambda - d\Lambda} \]

From below to \( \varepsilon_F \)

\[ \rho_F \int_{\varepsilon_F}^{\varepsilon_F} \frac{d\varepsilon}{\varepsilon} \sim -\log \frac{\Lambda}{\Lambda - d\Lambda} \]

\( \rightarrow \) Opposite signs in the log corrections. Nevertheless, do not cancel because of the non-Abelian matrices.
**RG equation**

\[ \Lambda \frac{dG(\Lambda)}{d\Lambda} = -\frac{k_{F}^{2}}{8\pi^{2}} g^{2} N_{c} G^{2}(\Lambda) \]

**Negative beta function**
→ **Asymptotic-free solution**

**Consequence of the Kondo effect**

Anomalous increase of the resistance

\[ \Lambda_{K} \sim k_{F} \exp \left( -\frac{8\pi^{2}}{N_{c} g^{2}} \right) \]

Landau pole ("Kondo scale")

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**Graphical representation**

*Resistance/Resistance (T=0 Celsius) x 10000*

(from W.J. de Haas and G.J. van den Berg, Physica vol. 3, page 440, 1936)

Low temperature resistivity of Au
Kondo effect at high density and in a strong $B$

--- Analogy from the dimensional reduction

Kondo Effect at high density

$$\Lambda_K \sim k_F \exp \left( -\frac{8\pi^2}{N_c g^2} \right)$$

Kondo Effect in $B$-field

$$\Lambda_K \sim \sqrt{q_{\text{em}}B} \exp \left( -\frac{8\pi^2}{N_c g^2} \right)$$

$$k_F \leftrightarrow \sqrt{eB}$$

Correspondence btw the density of states
Possible application to the heavy-quark diffusion dynamics in QGP

The drag force on the heavy quarks may be enhanced by the Kondo effect in the strong magnetic field.

Interactions btw the heavy and light particles are strongly coupled in the low-energy.

The conductivity is enhanced because the carriers are “trapped” near the impurity.

There are reactions on the heavy particles, which may enhance the drag force.
Anisotropic diffusion constant at LO
---Generation of additional $v_2$ of heavy flavors

Magnetic anisotropy gives rise to $v_2$ of HQs even without the $v_2$ of medium.
→ Possible to generate $v_2$ of HQs in the early QGP stage.

Kondo effect in B-field may occur in the NLO!

Summary

The effective dimensional reduction in the LLL gives rise to rich macroscopic consequences.

The low energy dynamics of the chiral fluid in a dynamical magnetic field is captured by the chiral MHD and contains novel collective excitations and instabilities.

Analogy between the systems at high density and in strong magnetic fields can be understood in terms of the dimensional reduction.
Backup slides
Kubo formulas

\[ \kappa_{||} = \frac{\partial}{\partial \omega} \text{Im} G_{j^3 j^3}^{R} \bigg|_{p=0, \omega \to 0}, \]

\[ \zeta_{||} = \frac{1}{3} \frac{\partial}{\partial \omega} \left[ 2 \text{Im} G^R_{\tilde{P} \perp \tilde{P}_{||}} + \text{Im} G^R_{\tilde{P}_{||} \tilde{P}_{||}} \right] \bigg|_{p=0, \omega \to 0}, \]

\[ \zeta_{\perp} = \frac{1}{3} \frac{\partial}{\partial \omega} \left[ 2 \text{Im} G^R_{\tilde{P}_{\perp} \tilde{P}_{\perp}} + \text{Im} G^R_{\tilde{P}_{||} \tilde{P}_{\perp}} \right] \bigg|_{p=0, \omega \to 0}, \]

\[ \eta_0 = \frac{\partial}{\partial \omega} \text{Im} G_{T_{12} T_{12}}^{R} \bigg|_{p=0, \omega \to 0}, \]

\[ \eta_1 = -\frac{4}{3} \eta_0 - 2 \frac{\partial}{\partial \omega} \text{Im} G^R_{\tilde{P}_{||} \tilde{P}_{\perp}} \bigg|_{p=0, \omega \to 0}, \]

\[ \eta_2 = -\eta_0 + \frac{\partial}{\partial \omega} \text{Im} G_{T_{13} T_{13}}^{R} \bigg|_{p=0, \omega \to 0}, \]

\[ \eta_3 = \frac{1}{2} \frac{\partial}{\partial \omega} \text{Im} G^R_{\tilde{P}_{\perp} T_{12}} \bigg|_{p=0, \omega \to 0}, \]

\[ \eta_4 = \frac{\partial}{\partial \omega} \text{Im} G_{T_{13} T_{23}}^{R} \bigg|_{p=0, \omega \to 0}, \]

\[ \tilde{P}_{||} \equiv P_{||} - \Theta_{\beta} \epsilon, \]

\[ \tilde{P}_{\perp} \equiv P_{\perp} - (\Theta_{\beta} + \Phi_{\beta}) \epsilon, \]

with \( \Theta_{\beta} \equiv (\partial P_{||}/\partial \epsilon)_B \)

and \( \Phi_{\beta} \equiv -B(\partial M/\partial \epsilon)_B. \)
Consistent result from diagrammatic calculation

Response function

\[ j^\mu = \Pi^{\mu\nu}_R A_\nu(q) \]

(\(\omega, q\) \(\to\) 0)

Kubo formula

\[ E = i\omega A \]

\[ j^i = \frac{\Pi^{ij}_R}{i\omega} E^j \]

Regularized by quark damping rate

\[ \Pi^{33}_R \sim \int \frac{dp_z}{2\pi} \frac{1}{i\epsilon} \]

Divergence in free theory

\[ i\epsilon \to i\gamma_{\text{damp}} \propto g^2 m_f^2 \ln(T/m_f) \]

No need to resum the pinch singularities in the present case
Energy-momentum tensor in the LLL

\[ T^{\mu \nu}(x) = \frac{i}{2} \mathcal{S} \sum_{f} \left[ \overline{\psi} D^{\mu} \gamma^{\nu} \psi + \overline{\psi} D^{\mu} \gamma^{\nu} \psi \right] \]

In the Landau gauge,

\[ \psi(x) = \int_{p_L, p^2} e^{-i(p_L \cdot x_L - p^2 x^2)} \mathcal{H}(x^1 - \frac{p^2}{eB}) \mathcal{P}_+ \chi(p_L) + \text{(Contributions from } n \geq 1) \]

\[ p_L^\mu \equiv (p^0, 0, 0, p^3), \quad \bar{p}^\mu \equiv (p^0, 0, p^2, p^3) \]

\[ \mathcal{P}_\pm = (1 + \text{sgn}(eB)i\gamma^1\gamma^2)/2 \]

When \( q \to 0 \),

\[ T^{\mu \nu}(q_L) = \int_{\bar{p}} \bar{\chi}(p_L + q_L) \gamma_L^\mu p_L^\nu \mathcal{P}_+ \chi(p_L) \]
Subtraction of the equilibrium component

\[ \delta P_{\parallel} \rightarrow \delta \tilde{P}_{\parallel} = \delta P_{\parallel} - \frac{\partial P_{\parallel}}{\partial e} e \quad e: \text{Energy density} \]

\[ \delta \tilde{P}_{\parallel} = \frac{|eB|}{2\pi} \int \frac{dk_z}{2\pi} \frac{k_z^2 - \Theta \epsilon_k^2}{\epsilon_k} \delta f(k_z) \quad \Theta = \frac{\partial P_{\parallel}}{\partial e} \]

Note that \( \Theta = \frac{1}{d_{\text{space}}} \) in the massless limit.

\[ \epsilon_k^2 = k_z^2 + m^2 \]

\( \delta \tilde{P}_{\parallel} \propto m^2 \) due to a small deviation from the conformal limit.
Possible Phenomenological Implications

2. Soft Dilepton Production


\[ \frac{d\Gamma}{d^4 p} = \frac{\alpha}{12\pi^4 \omega^2} T \sigma^{33} \]

\[ \Rightarrow (\text{virtual photon emission rate}) \sim n_B(\omega) \text{Im} \Pi^{\mu \nu} \sim T \sigma^{33} \]

\[ \sigma^{33} \text{ is large} \]

\[ e\sqrt{eB} \ll \omega \ll \frac{g^2 m^2}{T} \ln \left( \frac{T}{M} \right) \]

\[ \Rightarrow \text{Soft dilepton production is enhanced by } B? \]
Heavy quark (HQ) dynamics in the QPG -- In soft regime

Langevin equation
\[ \frac{dP}{dt} = \xi(t) - \eta_D P \]

Random kick (white noise)
\[ \langle \xi_i(t)\xi_j(t') \rangle = \kappa \delta_{ij} \delta(t - t') \]

Drag force coefficient: \( \eta_D \)
Diffusion constant: \( \kappa \)

Einstein relation
\[ \eta_D = \frac{\kappa}{2MT} \]

Perturbative calculation by finite-T field theory (Hard Thermal Loop resummation) LO and NLO without B are known (Moore & Teaney, Caron-Huot & Moore).
Perturbative computation of momentum diffusion constant

\[ \kappa_i = \int d^3q q_i^2 \frac{d\Gamma}{d^3q} \]

Momentum transfer rate in the LO Coulomb scatterings

\[ \frac{d\Gamma}{d^3q} = \text{HQ} \quad \text{Thermal quarks} + \text{HQ} \quad \text{Thermal gluons} \]

c.f.) LO and NLO without B (Moore & Teaney, Caron-Huot & Moore)

Effects of a strong magnetic field (\(eB >> T^2\))

1. Modification of the dispersion relation of thermal quarks
2. Modification of the Debye screening mass
1. Prohibition of the longitudinal momentum transfer

Linear dispersion relation \( k^0 = \pm k_z \)

From the chirality conservation

\[
q^0 = \pm (k'_z - k_z) = \pm q_z
\]

In the static limit (or HQ limit) \( q^0 \to 0 \)

\[
q_z \to 0.
\]

\( \kappa_\parallel = 0 \) in massless limit, while \( \kappa_\perp \neq 0 \).
2. Screening effect in a strong $B$

\[ \rho = \frac{N_{\text{state}}}{S} = \frac{eB}{2\pi} \]

Gluon self-energy
\[ \Pi^{\mu\nu}(q) = \frac{eB}{2\pi} \Pi^{\mu\nu}_{1+1} \]

Schwinger model
\[ \Pi^{\mu\nu}_{1+1} = \text{tr}[t^a t^a] \frac{g^2}{\pi} (q_\parallel^2 g^{\mu\nu}_{\parallel} - q_\parallel^{\mu} q_\parallel^{\nu}) \]

\[ m_D^2 \sim \frac{eB}{2\pi} \cdot \frac{g^2}{\pi} \gg (gT)^2 \]
### Anisotropic momentum diffusion constant

<table>
<thead>
<tr>
<th></th>
<th>Longitudinal</th>
<th>Perpendicular</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarks</td>
<td>$\kappa_{\parallel}^{\text{quark}} = 0$</td>
<td>$\kappa_{\perp}^{\text{quark}} \sim \alpha_s^2 T \times eB \times \log \frac{T^2}{\alpha_s eB}$</td>
</tr>
<tr>
<td>Gluons</td>
<td>$\kappa_{\parallel}^{\text{gluon}} \sim \alpha_s^2 T \times T^2 \times \log \frac{T^2}{\alpha_s eB}$</td>
<td>$\kappa_{\perp}^{\text{gluon}} \sim \alpha_s^2 T \times T^2 \times \log \frac{T^2}{\alpha_s eB}$</td>
</tr>
</tbody>
</table>

Remember the density of states in B-field,

$$ \rho = \frac{N_{\text{state}}}{S} = \frac{eB}{2\pi} $$

In the strong B limit,

$$ \frac{\kappa_{\parallel}}{\kappa_{\perp}} \sim \frac{\kappa_{\text{gluon}}}{\kappa_{\text{quark}} + \kappa_{\text{gluon}}} \sim \frac{T^2}{eB} < 1 $$
Transverse diffusion constant in massless limit

\[ \kappa_\perp = \alpha_s \lim_{q^0 \to 0} \frac{T}{q^0} \int d^3q \, q^2 \, q^0 \, \frac{\text{Im}\Pi(q)}{[q^2 + m_D^2]^2} \]

Distribution function
\[ n(q^0) \sim \frac{T}{q^0} \]

Screened Coulomb scattering amplitude (squared)
\[ m_D^2 \sim \alpha_s eB \]

Spectral density
\[ 2\text{Im}\Pi(q) = \rho(q) \sim m_D^2 \, q^0 \, \delta(q_z) \]

\[ \kappa_\perp \sim \alpha_s T \int d^2q \, q^2 \, \frac{m_D^2}{[q^2 + m_D^2]^2} \sim \alpha_s T m_D^2 \log 1/\alpha_s \]
3. Anomalous transports 

from magneto-vorticity coupling

Strong magnetic field & vorticity/angular momentum induced by heavy-ion collisions

\[ \mathbf{\omega} = \nabla \times \mathbf{v}_{\text{fluid}} \]

**Vorticity in HIC**

- Pang, Petersen, Wang, Wang (2016)
- Becattini et al., Csernai et al., Huang, Huovinen, Wang
- Jiang, Lin, Liao (2016)
- Deng, Huang (2016)

**Super-strong \( B \)**

- Deng & Huang (2012), KH & Huang (2016)
- Skokov et al. (2009), Voronyuk et al. (2011), Bzdak, Skokov (2012)
- McLerran, Skokov (2014)
Spin polarizations from spin-rotation coupling

\[ f^\pm(\epsilon, \omega) = f_0^\pm(\epsilon - S \cdot \omega) \quad f_0^\pm(\epsilon) = \frac{1}{e^{\pm\beta(\epsilon-\mu)} + 1} \]

\( \Lambda \) polarization

See talks by Becattini, Niida, Konyushikhin, Li

Becattini et al., Glastad & Csernai, Gyulassy & Torrieri, Xie, etc.
An interplay $B \otimes \omega$

Could the magneto-vorticity coupling be important??

For dimensional reason, one would get

$$j \sim \# B \cdot \omega$$

Cf., In CVE, it was

$$j^\mu_{R/L} = C_A \mu^{2}_{R/L} \omega^\mu$$

Could the magneto-vorticity coupling be important??

Q1. Is the coefficient related to any quantum anomaly?
Q2. How is $T$ and/or $\mu$ dependence?

**QFT with Kubo formula**

**Anomalous hydrodynamics due to Son and Surowka**

$$j^\mu = \sigma_{\text{Ohm}} E^\mu + \xi_B B^\mu + \xi_\omega \omega^\mu$$

The first-order derivative expansion $[A^\mu \sim O(\partial^0), \nu^\mu \sim O(\partial^0)]$

$$E^\mu \sim B^\mu \sim \omega^\mu \sim O(\partial^1)$$

$\rightarrow$ Yes, it is important when $B$ is so strong that $B >> O(\partial^1)$.

Q1. Is the coefficient related to any quantum anomaly?
Q2. How is $T$ and/or $\mu$ dependence?
Consequences of a magneto-vorticity coupling
Shift of thermal distribution functions by the spin-vorticity coupling

Spin-vorticity coupling

\[ f^\pm(\epsilon, \omega) = f_0^\pm(\epsilon - \mathbf{S} \cdot \mathbf{\omega}) \quad f_0^\pm(\epsilon) = \frac{1}{e^{\pm \beta(\epsilon - \mu)} + 1} \]

Landau & Lifshitz, Becattini et al.

In the LLL, the spin direction is aligned along the magnetic field.

\[ \Delta \epsilon^\pm \equiv -\mathbf{S} \cdot \mathbf{\omega} = \mp \text{sgn}(q_f) \frac{1}{2} \hat{\mathbf{B}} \cdot \mathbf{\omega} \quad - \text{for particle} \]

\[ + \text{ for antiparticle} \]

Number density

\[ n_R = \frac{|q_f B|}{2\pi} \left[ \int_0^\infty \frac{dp_z}{2\pi} f^+(p) + \int_{-\infty}^0 \frac{dp_z}{2\pi} f^-(p) \right] \]

At the LO in the energy shift \( \Delta \epsilon \)

\[ \Delta n_R = \frac{|q_f B|}{2\pi} \left[ \Delta \epsilon^+ \int_0^\infty \frac{dp_z}{2\pi} \frac{\partial f_0^+(p_z)}{\partial p_z} + \Delta \epsilon^- \int_{-\infty}^0 \frac{dp_z}{2\pi} \frac{\partial f_0^-(p_z)}{\partial p_z} \right] \]
\[ \Delta n_R = q_f \frac{C_A}{4} B \cdot \omega [f_0^+(0) + f_0^-(0)] = q_f \frac{C_A}{4} B \cdot \omega \]

\[ f_0^+(0) + f_0^-(0) = 1 \text{ identically for any } T \text{ and } \mu. \]

The shift is independent of the chirality, and depends only on the spin direction.

\[ \Delta n_L = \Delta n_R \]

In the V-A basis,

\[ \Delta n_V = \Delta n_R + \Delta n_L = q_f \frac{C_A}{2} B \cdot \omega \]

\[ \Delta n_A = \Delta n_R - \Delta n_L = 0 \]

Spatial components of the current

\[ \Delta j^3_R = v_R \Delta n_R \]

\[ j^1 = j^2 = 0 \text{ for the LLL} \]

Velocity: \( v_{R/L} = \pm \text{sgn}(q_f B) \)

The shift depends on the chirality through the velocity.

\[ \Delta j^3_R = -\Delta j^3_L \]

In the V-A basis, \( \Delta j^3_V = 0 \)

\[ \Delta j^3_A = |q_f| \text{sgn}(B) \frac{C_A}{2} B \cdot \omega \]
Field-theoretical computation by Kubo formula

Perturbative $\omega$ in a strong $B$

$$\lambda = -2i \lim_{q_x \to 0} \frac{\partial}{\partial q_x} \langle n_V(x)T^{02}(x')\rangle \theta(t - t')$$

Similar to the Kubo formula used to get the $T^2$ term in CVE (Landsteiner, Megias, Pena-Benitez)

We confirm
1. the previous results obtained from the shift of distributions.
2. a relation of $\langle n_V T^{02} \rangle$ to the chiral anomaly diagram in the (1+1) dim.

$$\Pi_{AV}^{\mu\nu} = j^\nu_A j^\mu_V$$

$q_\mu \Pi_{AV}^{\mu\nu} \neq 0$ !!!

There is no $T$ or $\mu$ correction in the massless limit, since it is related to the chiral anomaly!
Field-theoretical computation by Kubo formula

\[
\lambda = -2i \lim_{q_x \to 0} \frac{\partial}{\partial q_x} \langle n_V T^{02} \rangle
\]

\[
n_V(x) = \overline{\psi}(x) \gamma^0 \psi(x)
\]

\[
T^{0i}(x) = \frac{i}{2} \overline{\psi}(x) (\gamma^0 D^i + \gamma^i D^0) \psi(x)
\]

\[
S_{LLL} = 2e \frac{|p_\perp|^2}{q_f B} \frac{i}{p_\parallel + m_f} \mathcal{P}_+ \quad \mathcal{P}_+ = (1 + i \text{sgn}(q_f B) \gamma^1 \gamma^2)/2
\]

\[
\langle n_V T^{02} \rangle \propto \frac{|q_f B|}{2\pi} q_x \Pi_{1+1}^{00}
\]

\[
\Pi_{1+1}^{\mu \nu} = \int \frac{d^2p_\parallel}{(2\pi)^2} \text{tr} [\gamma_\mu^\prime S_{1+1}(p_\parallel + q_\perp) \gamma_\nu^\prime S_{1+1}(p_\parallel)] = \frac{1}{\pi} \frac{1}{q_\parallel^2} (q_\parallel^2 g_\mu^\nu - q_\parallel^\mu q_\parallel^\nu)
\]

There is no T or \( \mu \) correction in the massless limit!

\( \Rightarrow \) Consistent with the previous observation from the shift of distributions.
Summary 2

A magneto-vorticity coupling $B \otimes \omega$ induces charge redistributions without $\mu_A$.
- Related to the chiral anomaly in the (1+1) dimensions.
- No T or $\mu$ correction.

When $B \cdot \omega \neq 0$,

$$j_{EM,V}^0 = q_f^2 \frac{C_A}{2} (B \cdot \omega)$$

$$j_{EM,A}^3 = \text{sgn}(q_f)q_f^2 \frac{C_A}{2} (B \cdot \omega) \hat{B}$$

Emerges even without $\mu_A$.

Coupling between the CME and fluid velocity induces a new instability in MHD. Take by Y. Hiron. KH, Hirono, Yee, Yin, In preparation.
Heavy quarks as a probe of QGP

Non-thermal heavy-quark production in hard scatterings

Momentum distribution of HQs in log scale

Initial distribution ($\tau = 0$) from pQCD

$\sim (1/p_T)^n$

Thermal ($\tau = \infty$)

Relaxation time is controlled by transport coefficients (Drag force, diffusion constant)
Brief Introduction to Kondo effect in cond. matt.

Measurement of the resistance of alloy (with impurities)

**T_K**: Kondo Temp. (Location of the minima)

Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

**Resistance Minimum in Dilute Magnetic Alloys**

Jun Kondo
Impurity scatterings near a Fermi surface

+ Light-Heavy quark scattering in quark matter

How does the coupling evolve with the energy scale, $\Lambda \rightarrow 0$, on the basis of Wilsonian RG?

$G(\Lambda)(\bar{\psi}\psi)(\bar{\Psi}\Psi)$

The LO does not explain the minimum of the resistance.

Logarithmic quantum corrections arise in special kinematics and circumstances. → BCS, Kondo effect, etc.
“Dimensional reduction” in dense systems -- (1+1)-dimensional low-energy effective theory

\[ \epsilon = \pm \ell_\parallel \quad (\ell_\parallel \ll \mu) \]

+ Low energy excitation along radius \[(1+1)D\]

+ Degenerated states in the tangential plane \[2D\]

Phase space volume \(\sim p^{D-1} dp\).

Enhanced IR dynamics induces nonperturbative physics, such as superconductivity and Kondo effect.

Cf., Superconductivity occurs no matter how weak the attraction is.
In the BCS config.

In the BCS config. 

\[ p^{(1)} + p^{(2)} \sim \ell_{||} \ll \mu \] 

\[ d_{4-\text{Fermi}} = (-1) + 4\left(1 - \frac{1}{2}\right) - 1 = 0 \]

In general momentum config.

\[ p^{(1)} + p^{(2)} \sim \mu \] 

\[ d_{4-\text{Fermi}} = (-1) + 4\left(1 - \frac{1}{2}\right) = +1 \]
Scaling dimensions in the LLL

When $\epsilon_{\text{LLL}} \rightarrow s\epsilon_{\text{LLL}}$, $p_z \rightarrow sp_z$. ($s < 1$; $\mathbf{p}_\perp$ does not scale.)

Kinetic term

$$S_{\text{LLL}}^{\text{kin}} = \int dt \int dp_z \bar{\psi}_{\text{LLL}}(p_z)(i\partial_t \gamma^0 - p_z \gamma^3 - m_f)\psi_{\text{LLL}}(p_z)$$

$$0 = 2d_{\psi} \bar{\psi} \cdot \psi + \left(-\frac{1}{dt}\right) + \frac{1}{dp_z} + \frac{1}{\partial_t} \rightarrow d_{\psi} = -\frac{1}{2}$$

A four-Fermi operator for the LLL

$$S^{\text{int}} = \int dt \left[ \int \frac{dp_z}{2\pi} \right]^4 G[\bar{\psi}_{\text{LLL}}^{(4)} \gamma_\mu \psi_{\text{LLL}}^{(2)}][\bar{\psi}_{\text{LLL}}^{(3)} \gamma_\mu \psi_{\text{LLL}}^{(1)}] \delta(p_z^{(1)} + p_z^{(2)} - p_z^{(3)} - p_z^{(4)})$$

$$d_4 = -1 + 4 \times (1 - 1/2) - 1 = 0$$

Always marginal irrespective of the interaction type and the coupling constant thanks to the “dimensional reduction” in the LLL.

IR scaling dimension for Kondo effect

Heavy-quark Kinetic term

\[ S_{H}^{\text{kin}} = \int dt \int \frac{d^3 k}{(2\pi)^3} \psi_+^+(k) i\partial_t \psi_+(k) + \mathcal{O}(1/m_H) \]

\[ d_{\psi} = (-1) + 1 = 0 \]

Heavy-light four-Fermi operator

\[ S_{H-L}^{\text{int}} = \int dt \left[ \int \frac{d^2 l \, dl_\parallel}{(2\pi)^3} \right]^2 \left[ \int \frac{d^3 k}{(2\pi)^3} \right]^2 G[\overline{\psi}_+^{(3)} t^a \psi_+^{(1)}][\overline{\psi}_+^{(4)} t^a \psi_+^{(2)}] \]

\[ d_{H-L} = (-1) + 2(1 + d_\psi) + 2d_\psi = 0 \]

Marginal !! Let us proceed to diagrams.
Scattering in the NLO
-- Renormalization in the low energy dynamics

\[ \mathcal{M} = \]

Wilsonian RG
If \( \mathcal{M} \sim \Lambda \), interactions become less and less important.
If \( \mathcal{M} \sim \log \Lambda \), the fate depends on the sign of \( \beta \) func.
High-Density Effective Theory (LO)

Expansion around the large Fermi momentum
\[ p^0 = \ell^0, \quad p^i = \mu v^i_F + \ell^i \]

(1+1)-dimensional dispersion relation
\[ \ell^0 = v_F \cdot \ell \equiv \ell_\parallel \]

Spin flip suppressed when the mass is small \( m \ll \mu \).
\[ \gamma^\mu A_\mu \rightarrow \gamma^0 v^\mu_F A_\mu \]
Heavy-Quark Effective Theory (LO)

HQ-momentum decomposition

\[ p^\mu = m_Q v^\mu_Q + k^\mu \]

HQ velocity

\[ v^\mu_Q = \left. \frac{1}{m_Q} P^\mu \right|_{P^2 = m_Q^2} \]

Nonrelativistic magnetic moment suppressed by \(1/m_Q\)

\[ \gamma^\mu A_\mu \rightarrow v^\mu_Q A_\mu \]

\[ \gamma^\mu A_\mu = A^0 \text{ when } \vec{v}_Q = 0 \]
Gluon propagator in dense matter

\[ D^{\mu\nu}(k) = \frac{P_L^{\mu\nu}}{k^2 - \Pi_L} + \frac{P_T^{\mu\nu}}{k^2 - \Pi_T} - \xi \frac{k^\mu k^\nu}{k^4} \]

\[ P_T^{\mu\nu} = \delta^{\mu i} \delta^{\nu j} \left( \delta^{ij} - \frac{k^i k^j}{|k|^2} \right) \]

\[ P_L^{\mu\nu} = - \left( g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) - P_T^{\mu\nu} \]

Screening of the \(<A^0A^0>\) from the HDL

\[ \Pi_L \sim m_{\text{Debye}}^2 \sim (g\mu)^2 \]

Cf., Son, Schaefer, Wilczek, Hsu, Schwetz, Pisarski, Rischke, ....., showed that unscreened magnetic gluons play a role in the cooper paring.
Important ingredients for Kondo effect

1. Quantum corrections

\[ \rho_F \int_{\epsilon_F}^{\epsilon} \frac{d\epsilon}{\epsilon} \sim \log \frac{\Lambda}{\Lambda - d\Lambda} \]

2. Log enhancements from the IR dynamics

\[ \rho_F \int_{\epsilon_F}^{\epsilon} \frac{d\epsilon}{\epsilon} \sim - \log \frac{\Lambda}{\Lambda - d\Lambda} \]
3. Incomplete cancellation due to non-Abelian interactions

**Particle contribution**

\[
[t^a t^b]_{ij} [t^a t^b]_{k\ell} = c \delta_{ij} \delta_{k\ell} - \frac{1}{n} t^a_{k\ell} t^a_{ij}
\]

**Hole contribution**

\[
[t^a t^b]_{ij} [t^b t^a]_{k\ell} = c \delta_{ij} \delta_{k\ell} - \frac{1}{n} t^a_{k\ell} t^a_{ij} + \frac{n}{2} t^a_{k\ell} t^a_{ij}
\]
RG analysis for “QCD Kondo effect”

\[
G(\Lambda - d\Lambda) = G(\Lambda) + \int_{\Lambda-d\Lambda}^{\Lambda} dp \sim \log \frac{\Lambda - d\Lambda}{\Lambda}
\]

RG equation

\[
\Lambda \frac{dG(\Lambda)}{d\Lambda} = -\frac{k_F^2}{8\pi^2} g^2 N_c G^2(\Lambda)
\]

Asymptotic-free solution

\[
G(\Lambda) = \frac{G(\Lambda_0)}{1 + g^2 N_c/(8\pi^2) G(\Lambda_0) \log \Lambda/\Lambda_0}
\]

Effective coupling: \( G(\Lambda) \)

\( E = 0 \) Fermi energy

\( \Lambda_K \sim k_F \exp \left( -\frac{8\pi^2}{N_c g^2} \right) \) Landau pole (“Kondo scale”)
Short summary for Kondo effect in quark matter

1. Non-Ablelian interaction (QCD)

2. Dimensional reduction near the Fermi surface

3. Continuous spectra near the Fermi surface, and heavy impurities (gapped spectra).
An analogy between the dimensional reductions in high-density matter and in strong magnetic field


Scaling dimensions in the LLL

When $\epsilon_{\text{LLL}} \to s\epsilon_{\text{LLL}}, p_z \to sp_z$. ($p_\perp$ does not scale.)

$(1+1)$-D dispersion relation $\Rightarrow d_\psi = -\frac{1}{2}$

Four-light-Fermi operator

$$S^{\text{int}} = \int dt \left[ \int \frac{dp_z}{2\pi} \right]^4 G[\bar{\psi}_{\text{LLL}}^{(4)} \gamma_\mu \psi_{\text{LLL}}^{(2)}][\bar{\psi}_{\text{LLL}}^{(3)} \gamma_\mu \psi_{\text{LLL}}^{(1)}] \delta(p_z^{(1)} + p_z^{(2)} - p_z^{(3)} - p_z^{(4)})$$

Always marginal thanks to the dimensional reduction in the LLL. $\Rightarrow$ Magnetic catalysis of chiral condensate.

Chiral symmetry breaking occurs even in QED.

Gusynin, Miransky, and Shovkovy. Lattice QCD data also available (Bali et al.).

Heavy-light four-Fermi operator

$$S_{\text{H-L}}^{\text{int}} = \int dt \left[ \int \frac{dp_z}{2\pi} \right]^2 \left[ \int \frac{d^3 k}{(2\pi)^3} \right]^2 G[\bar{\psi}_{\text{LLL}}^{(3)} t^a \psi_{\text{LLL}}^{(1)}][\bar{\Psi}^{(4)} t^a \Psi^{(2)}]$$

Marginal !! Just the same as in dense matter.
Important ingredients of Kondo effect -- Revisited with strong B fields

1. Quantum corrections (loop effects)
2. Log enhancement from the IR dynamics due to the dimensional reduction in the strong B.
3. Incomplete cancellation due to non-Abelian color-exchange interactions

“QCD Kondo Effect”

\[ \Lambda_K \sim k_F \exp \left( -\frac{8\pi^2}{N_c g^2} \right) \]


“Magnetically Induced QCD Kondo Effect”

\[ \Lambda_K \sim \sqrt{q_{em}B} \exp \left( -\frac{8\pi^2}{N_c g^2} \right) \]