Role of neutrino cross sections and nuclear models on oscillation experiments;

Connection electron scattering and neutrino scattering.

Camillo Mariani
Center for Neutrino Physics, Virginia Tech
Motivation and Contents

• Determination of neutrino oscillation parameters requires knowledge of neutrino energy

• Modern experiments use complicated nuclear targets: from Carbon to Argon

• Nuclear effects affect everything:
  – event identification
  – final state particles
  – reconstructed neutrino energy
  – event cross section measurements
  – neutrino oscillation parameters
Neutrino Oscillations

• 2-Flavor Oscillation:

\[ P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right) \]

Know: \( L \), need \( E_\nu \) to determine \( \Delta m^2, \theta \)

• 3-Flavor Oscillation: allows for CP violation
Observable Oscillation Parameters

\[ P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right) \]
Oscillation probability

Long-Baseline Accelerator Appearance Experiments

• Oscillation probability complicated and dependent not only on \( \theta_{13} \) but also:
  1. CP violation parameter (\( \delta \))
  2. Mass hierarchy (sign of \( \Delta m_{31}^2 \))
  3. Size of \( \sin^2 \theta_{23} \)

\[
P(\nu_\mu \rightarrow \nu_e) = 4C_{13}^2 S_{13}^2 S_{23}^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E} \times \left( 1 + \frac{2a}{\Delta m_{31}^2} \left( 1 - 2S_{13}^2 \right) \right) \\
+ 8C_{12}^2 S_{12}^2 S_{23} (C_{12} C_{23} \cos \delta - S_{12} S_{13} S_{23}) \cos \frac{\Delta m_{32}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{21}^2 L}{4E} \\
- 8C_{13} C_{12} C_{23} S_{12} S_{13} S_{23} \sin \delta \sin \frac{\Delta m_{32}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{21}^2 L}{4E} \\
+ 4S_{12}^2 C_{12} \{ C_{12}^2 C_{23}^2 + S_{12}^2 S_{23}^2 S_{13}^2 - 2C_{12} C_{23} S_{12} S_{23} S_{13} \cos \delta \} \sin^2 \frac{\Delta m_{21}^2 L}{4E} \\
- 8C_{13}^2 S_{13}^2 S_{23}^2 \cos \frac{\Delta m_{32}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \frac{aL}{4E} \left( 1 - 2S_{13}^2 \right)
\]

\( \Rightarrow \) These extra dependencies are both a “curse” and a “blessing”

Reactor Disappearance Experiments

\[
P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_{13}^2 L}{4E} + \text{small terms}
\]
Current Knowledge:

<table>
<thead>
<tr>
<th>Normal Ordering</th>
<th>$\theta_{12}$</th>
<th>$\theta_{13}$</th>
<th>$\theta_{23}$</th>
<th>$\Delta m_{21}^2 / 10^{-5}$</th>
<th>$\Delta m_{3j}^2 / 10^{-3}$</th>
<th>$\delta_{CP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$33.56_{-0.75}^{+0.77}$</td>
<td>$8.46_{-0.15}^{+0.15}$</td>
<td>$41.6_{-1.2}^{+1.5}$</td>
<td>$7.50_{-0.17}^{+0.19}$</td>
<td>$2.524_{-0.040}^{+0.039}$</td>
<td>$261_{-59}^{+51}$</td>
</tr>
<tr>
<td>Inverted Ordering</td>
<td>$33.56_{-0.75}^{+0.77}$</td>
<td>$8.49_{-0.15}^{+0.15}$</td>
<td>$50.0_{-1.4}^{+1.1}$</td>
<td>$7.50_{-0.17}^{+0.19}$</td>
<td>$-2.514_{-0.041}^{+0.038}$</td>
<td>$277_{-46}^{+40}$</td>
</tr>
</tbody>
</table>

Current and Future Goals:

- Establish whether there is CP violation in the lepton sector and, if so, measure $\delta_{CP}$
- Improve the accuracy on $\theta_{23}$
- Determine the neutrino mass ordering: $m_1 < m_2 < m_3$ or $m_3 < m_1 < m_2$

Current and Future Experiments:

- **MiniBooNE** (concluded, re-running), **NOvA** (running), **T2K** (running), **T2HK** (under construction), etc.
- **SBN Program: MicroBooNE** (running), **ICARUS** (under construction), **SBND** (under construction)
- **DUNE** (under construction)
Current Knowledge:

<table>
<thead>
<tr>
<th></th>
<th>$\theta_{12}$</th>
<th>$\theta_{13}$</th>
<th>$\theta_{23}$</th>
<th>$\Delta m_{21}^2/10^{-5}$</th>
<th>$\Delta m_{32}^2/10^{-3}$</th>
<th>$\delta_{CP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Ordering</td>
<td>33.56$^{+0.77}_{-0.75}$</td>
<td>8.46$^{+0.15}_{-0.15}$</td>
<td>41.6$^{+1.5}_{-1.2}$</td>
<td>7.50$^{+0.19}_{-0.17}$</td>
<td>2.524$^{+0.039}_{-0.040}$</td>
<td>261$^{+51}_{-59}$</td>
</tr>
<tr>
<td>Inverted Ordering</td>
<td>33.56$^{+0.77}_{-0.75}$</td>
<td>8.49$^{+0.15}_{-0.15}$</td>
<td>50.0$^{+1.1}_{-1.4}$</td>
<td>7.50$^{+0.19}_{-0.17}$</td>
<td>-2.514$^{+0.038}_{-0.041}$</td>
<td>277$^{+40}_{-46}$</td>
</tr>
</tbody>
</table>

- **SBN Program**: MicroBooNE (running), ICARUS (under construction), SBND (under construction)
- **DUNE** (under construction)
Accelerator-based neutrino-oscillation experiments

Experiments measure event rates which, for a given observable topology, can be naively computed as:

**Event Rate at near detector:**

\[ N_{\text{ND}}^\alpha(p_{\text{reco}}) = \sum_i \phi_\alpha(E_{\text{true}}) \times \sigma_i^\alpha(p_{\text{true}}) \times \epsilon_\alpha(p_{\text{true}}) \times R_i(p_{\text{true}}; p_{\text{reco}}) \]

**Event Rate at far detector:**

\[ N_{\text{FD}}^{\alpha\rightarrow\beta}(p_{\text{reco}}) = \sum_i \phi_\alpha(E_{\text{true}}) \times P_{\alpha\beta}(E_{\text{true}}) \times \sigma_i^\beta(p_{\text{true}}) \times \epsilon_\beta(p_{\text{true}}) \times R_i(p_{\text{true}}; p_{\text{reco}}) \]

\[ P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \]

\[ \text{two neutrino flavors, for simplicity} \]
Event Rate at far detector:

\[
N_{PD}^{\alpha \rightarrow \beta}(p_{reco}) = \sum_i \phi_{\alpha}(E_{true}) \times P_{\alpha \beta}(E_{true}) \times \sigma^i_{\beta}(p_{true}) \times \epsilon_{\beta}(p_{true}) \times R_i(p_{true}; p_{reco})
\]

\[
P(\nu_\alpha \rightarrow \nu_\beta) \simeq \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E}\right)
\]

Neutrino Energy: Reconstruction

For CCQE process (assuming single nucleon knock out), the reconstructed neutrino energy is

\[
E_\nu = \frac{m_p^2 - m_\mu^2 - E_n^2 + 2E_\mu E_n - 2k_\mu \cdot p_n + |p_n|^2}{2(E_n - E_\mu + |k_\mu| \cos \theta_\mu - |p_n| \cos \theta_\nu)}
\]

where \(|k_\mu|\) and \(\theta_\mu\) are measured, while \(p_n\) and \(E_n\) are the unknown momentum and energy of the interacting neutron.

Existing simulation codes routinely use \(|p_n| = 0\), \(E_n = m_n - \epsilon\), with \(\epsilon \sim 20\) MeV for carbon and oxygen, or the Fermi gas (FG) model.
For CCQE process (assuming single nucleon knock out), the reconstructed neutrino energy is

\[ E_\nu = \frac{m_p^2 - m_\mu^2 - E_n^2 + 2E_\mu E_n - 2k_\mu \cdot p_n + |p_n^2|}{2(E_n - E_\mu + |k_\mu| \cos \theta_\mu - |p_n| \cos \theta_n)} \]

- Neutrino energy reconstructed using \(2 \times 10^4\) pairs of \(|p|, E\) values sampled from realistic (SF) and FG oxygen spectral functions.
- The average value \(\langle E_\nu \rangle\) obtained from the realistic spectral function turns out to be shifted towards larger energy by \(~70\) MeV.
Neutrino-nucleus cross section

**Event Rate at far detector:**

\[
N_{\text{PD}}^{\alpha \rightarrow \beta}(p_{\text{reco}}) = \sum_i \phi_{\alpha}(E_{\text{true}}) \times P_{\alpha \beta}(E_{\text{true}}) \times \sigma_{\beta}^i(p_{\text{true}}) \times \epsilon_{\beta}(p_{\text{true}}) \times R_i(p_{\text{true}}; p_{\text{reco}})
\]

- Need realistic nuclear model (in Monte-Carlo simulations) that can describe neutrino-nucleus cross sections over a wide range of energies.
Appearance Probability as function of neutrino energy

Need energy to distinguish between different $\delta_{CP}$
Effect of an underestimation of the missing energy in the calorimetric energy reconstruction on the coincidence regions in the $\theta_{13}, \delta$ plane.
Expected sensitivity of DUNE to CP violation as a function of exposure in kt·MW·year for a range of values for the $\nu_e$ and $\bar{\nu}_e$ signal normalization uncertainties from $5\% \pm 3\%$ to $5\% \pm 1\%$. 

75% CP Violation Sensitivity

DUNE Sensitivity
Normal Hierarchy
$\sin^2 2\theta_{13} = 0.085$
$\sin^2 \theta_{23} = 0.45$

$\sigma = \sqrt{\Delta \chi^2}$

Exposure (kt-MW-years)
Oscillation Signal: Dependence on Hierarchy and Mixing Angle

Energy has to be known better than 50 MeV

Shape sensitive to hierarchy and sign of mixing angle

C. Mariani, CNP Virginia Tech

INT, Seattle Mar. 5
Appearance experiment

• Near detector:
  – Neutrino Flux
  – Background
  – Intrinsic $\nu_e$
  – Neutrino energy

• Far detector:
  – Extrapolate Flux
  – Background
  – Neutrino energy

\[
P(\nu_\mu \rightarrow \nu_e) = 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{1.27 \Delta m^2 L}{E_\nu} \right) + \text{other}
\]
Neutrino Beams

- Neutrinos do not have fixed energy nor just one reaction mechanism

Have to reconstruct energy from final state of reaction
Different processes are entangled

C. Mariani, CNP Virginia Tech
INT, Seattle Mar. 5
Neutrino Interactions

- **for $\nu_e$ appearance**
  - beam $\nu_e$
  - NC$\pi^0$ events

- **for $\nu_\mu$ disappearance (muon energy measurement)**
  - inelastic processes

$\nu_\mu + n \rightarrow \mu + p + \pi$

$\nu_\mu + n \rightarrow \nu + p + \pi^*$

 NC$\pi^0$ MC event in SK
2 rings reconstructed

Quasi-Elastic process
Energy reconstruction

\[ \nu_\mu + n \rightarrow \mu^- + p \]
\[ E_\nu = E_\nu(E_\mu, \theta_\mu) \]

Kinematic:
- Rely on underlying interaction to use relate outgoing lepton kinematics to neutrino energy
- Advantage:
  - Don’t need hadron reconstruction
- Disadvantages
  - Energy is wrong if underlying interaction is wrong (i.e. not CCQE)
  - Nuclear effects smear resolution

\[ \nu_\mu + N \rightarrow \mu^- + X \]
\[ E_\nu = E_\mu + E_X \]

Calorimetric
- Add up the energy from the leptonic and hadronic components
- Advantages
  - No a priori assumption about underlying interaction
- Disadvantages
  - Relies on hadron reconstruction
Background: Nuclear re-interactions

Modeling $\nu$ interactions in nucleus

- Underlying $\nu$-nucleon/quark interaction
  - Mode (CCQE, resonance, etc.)
  - Determine “final” state of interaction

- Initial state nucleon/quark
  - Fermi motion, binding energy
- Final state effects
  - Pauli blocking
  - Propagate hadrons within nucleus
    - Absorption, scattering, CEX, etc.

- Lepton kinematics shifted/smeared
- Outgoing hadronic final state (“topology”) may differ from expectation from “underlying” $\nu$-nucleon interaction
- FSI effects may appear degenerate with hadronic interactions outside of the target nucleus.
How to quantify effects on oscillation

- Ideal, perfect near detector ($^{12}\text{C}$), 1 km, 1kton
- Far detector at 295 km, 22.5 kton, Carbon (SF)
- Use flux that peak at 0.6 GeV, 750kW, 5 years running
- Use a second flux that peaks at 1.5 GeV, 750kW, 5 years running
- Use Super Kamiokande (water cherenkov detector) reconstruction efficiency as function of energy
- Use migration matrices to take into account how neutrino energy reconstruction is affected by the what kind of interaction the neutrino undergo in the detector and how well we can identify them
- Muon neutrino disappearance only -> fit to atmospheric parameters
How to read the plots in the following slides

reconstructed from naive QE dynamics

1, 2 and 3σ allowed regions

Simulation of long baseline neutrino oscillation
Dependence from target material (C vs O)

PRD D89, 073015 (2014) - arxiv:1311.4506
Dependence from nuclear model (1p1h)
Dependence from nuclear model (2p2h)

PRD D93, 113004 (2016) - arxiv:1603.01072
Two ways to reconstruct the neutrino energy

- Kinematic: use only info on the outgoing lepton kinematic
- Calorimetric: sum all energy in final state
Simulating a non perfect detector

• Detection thresholds
  – 20 MeV for mesons,
  – 40 MeV for protons

• Efficiencies
  – 60% for $\pi^0$,
  – 80% for other mesons,
  – 50% for protons,
  – neutrons undetected

$$\sigma(|k_\mu|) = 0.02|k_\mu| \quad \text{and} \quad \sigma(\theta) = 0.7^\circ$$

$$\frac{\sigma(E_{\pi^0})}{E_{\pi^0}} = \max \left\{ \frac{0.107}{\sqrt{E_{\pi^0}}}, \frac{0.02}{E_{\pi^0}} \right\} \quad \text{and} \quad \frac{\sigma(E_h)}{E_h} = \max \left\{ \frac{0.145}{\sqrt{E_h}}, 0.067 \right\}$$
Detector effects on kinematic energy reconstruction

$L = 295$ km, Kinematic Reconstruction

- Real setup
- $\alpha = 0.1$
- $\alpha = 0.3$

$E_\gamma$ [GeV]

$\Delta m^2_{31} [x 10^{-3} \text{ eV}^2]$

Contours for $\Delta \chi^2 = 2.3$

- Real setup
- $\alpha = 0.1, \chi^2_{\text{res-fit}}/\text{dof} = 0.02/15$
- $\alpha = 0.2, \chi^2_{\text{res-fit}}/\text{dof} = 0.04/15$
- $\alpha = 0.3, \chi^2_{\text{res-fit}}/\text{dof} = 0.1/15$

PRD D92, 091301 (2015) - arxiv:1507.08561
Detector effects on calorimetric energy reconstruction

\[ L = 295 \text{ km}, \text{Calorimetric Reconstruction} \]

- Real. setup
- \( \alpha = 0.1 \)
- \( \alpha = 0.3 \)

\[ E_\nu [\text{GeV}] \]

\[ \Delta m_{31}^2 [\times 10^{-3} \text{ eV}^2] \]

\( \theta_{23}[^\circ] \)

Contours for \( \Delta \chi^2 = 2.3 \)

\[ \chi^2_{\text{best-fit}}/dof = 1.6/15 \]

\[ \chi^2_{\text{best-fit}}/dof = 6.1/15 \]

\[ \chi^2_{\text{best-fit}}/dof = 13.1/15 \]
Electron vs neutrino scattering
QE e-A scattering

\[ \left( \frac{d^2\sigma}{d\omega_e d\Omega} \right)_e = \frac{\alpha^2}{Q^4} \left( \frac{2}{2J_i + 1} \right) \frac{1}{k_f E_i} \times \zeta^2 (Z', E_f, q_e) \left[ \sum_{J=0}^{\infty} \sigma_{L,e}^J + \sum_{J=1}^{\infty} \sigma_{T,e}^J \right] \]

\[ \sigma_{L,e} = v_e^L R_e^L \]

\[ \sigma_{T,e} = v_e^T R_e^T \]

ν’s → Leptonic coefficients → Purely kinematical → Easy to calculate
\[
\frac{d^2\sigma}{d\omega_e d\omega} = \frac{\alpha^2}{Q^4} \left( \frac{2}{2J_i + 1} \right) \frac{1}{k_f E_i} \times \zeta^2(Z', E_f, q_e) \left[ \sum_{J=0}^{\infty} \sigma^J_{L,e} + \sum_{J=1}^{\infty} \sigma^J_{T,e} \right]
\]

\[
\frac{d^2\sigma}{d\omega_{\nu} d\omega} = \frac{G_F^2 \cos^2 \theta_c}{(4\pi)^2} \left( \frac{2}{2J_i + 1} \right) \varepsilon_f k_f \times \zeta^2(Z', \varepsilon_f, q_{\nu}) \left[ \sum_{J=0}^{\infty} \sigma^J_{CL,\nu} + \sum_{J=1}^{\infty} \sigma^J_{T,\nu} \right]
\]

\[
\sigma_{L,e} = v_e^L R_e^L
\]

\[
\sigma_{T,e} = v_e^T R_e^T
\]

\[
\sigma^J_{CL,\nu} = [v_{\nu}^MR_{\nu}^M + v_{\nu}^LR_{\nu}^L + 2 v_{\nu}^{ML} R_{\nu}^{ML}]
\]

\[
\sigma^J_{T,\nu} = [v_{\nu}^TR_{\nu}^T \pm 2 v_{\nu}^{TT} R_{\nu}^{TT}]
\]

\(\nu's \rightarrow \text{Leptonic coefficients} \rightarrow \text{Purely kinematical} \rightarrow \text{Easy to calculate}\)

\(R's \rightarrow \text{Response functions} \rightarrow \text{Nuclear dynamics} \rightarrow \text{Need nuclear models to calculate!}\)
Electron scattering data as a validation

Longitudinal (left) and transverse (right) electromagnetic responses of $^{12}\text{C}$ at $|q| = 570$ MeV, as function of energy transfer.

Theoretical results obtained using the Green’s Function Monte Carlo (GFMC) technique, a realistic nuclear Hamiltonian and consistent one- and two-nucleon currents.

Note that, even at moderate momentum transfer, the non relativistic approach fails to describe the transverse response in the region of large energy transfer, where the contribution of inelastic processes is large.
Electron scattering data as a validation

$e + ^{12}\text{C} \rightarrow e' + X$ quasi elastic cross section computed within the IA including FSI. The predictions of the Relativistic Fermi Gas Model (RFGM) are also shown for comparison.

[$\omega$ (MeV) vs $d\sigma/d\omega d\Omega$ for different energies and angles]
$e^+ {^{12}C} \rightarrow e^\prime + X$

(A) $E_e = 0.68$ GeV
$\theta_e = 36$ deg

(B) $E_e = 1.3$ GeV
$\theta_e = 37.5$ deg
Little advertisement ... (Vishvas’s talk later today)
Conclusions

• Energy reconstruction essential for precision determination of neutrino oscillation parameters and neutrino-hadron cross sections

• Impact on neutrino oscillation experiments due to nuclear models, what they are and how they are implemented is not negligible (order 10%)
  – comparing systematically generators is important
  – neutrino event generators use almost same data set so there are correlations that are non-negligible
  – using wrong models affect neutrino oscillation parameters determination
• In future extend the case to CP violation:
  • neutrino vs anti-neutrino cross-section,: do we have reliable event generators for anti-neutrino?

• Energy reconstruction requires reliable event generators, of same quality as experimental equipment

• Precision era of neutrino physics requires much more sophisticated generators and a dedicated effort in theory

• Theorists-phenomenologists and experimentalists need to work together.
Generators are a crucial part of any experiment! Must be of same quality as the experimental equipment itself! Needed resources are relatively small, but still not available

"What we especially like about these theoretical types is that they don't tie up thousands of dollars worth of equipment."