Neutron-antineutron in nuclei

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History

- $n - \bar{n}$ oscillation, see Mohapatra and others
- Meeting Maurice Goldhaber: 1982, before and after, e.g. 2006

Controversy
- Hot seminars on the subject
- Nazaruk and others

Our results supported by Alberico et al., Kopeliovich et al.,
S-D mixing in the deuteron

Rarita-Schwinger equations

\[
\psi = \frac{u(r)}{r} |^3 S_1 \rangle + \frac{w(r)}{r} |^3 D_1 \rangle
\]

\[-u''(r) + m V_{00} u(r) + m V_{02} w(r) = m E u(r) ,
\]

\[-w''(r) + \frac{6}{r^2} w(r) + m V_{22} w(r) + m V_{02} u(r) = m E w(r) ,
\]

\[V_{00} = V_c , \quad V_{22} = V_c - 2 V_T - 3 V_{LS} - 3 V_{LL} , \quad V_{02} = \sqrt{8} V_T .\]

Only about 5%, but crucial for the deuteron and many other nuclear states. See Ericson & Rosa-Clot and Blatt & Weisskopf
**S-D mixing in charmonium**

- At first, \( J/\psi = 1S, \psi' = 2S, \psi'' = 1D \), etc.
- Leptonic coupling of \( \psi'' \) requires some \( S \)-wave admixture.
- Usually
  \[
  |S(3D_1)\rangle \simeq \frac{\langle 2^3S_1 | \sqrt{8} V_T | 1^3D_1 \rangle}{E_0(2S) - E_0(1D)} |2^3S_1\rangle.
  \]
- Solving RS eqs. in specific models indicate some important \( 1S \) admixture: states with same node structure mix better.
- Also
  \[
  \psi(n) \leftrightarrow D^*(\bar{D}^*) \leftrightarrow \psi(m)
  \]
  e.g., Cornell model

\[ n - \bar{n} \in \text{nuclei} \]
S-D mixing in muonium or hydrogen

- Quadrupole deformation of an atom such as $(\mu^+, e^-)$
- Small effect in principle measurable in a gradient of electric field
- More delicate than in a $(Q\bar{Q})$ potential model, as the sum on intermediate states (if performed!), extends over the continuum

$$|D(\text{g.s.})\rangle = \sum_n \frac{\langle n^3 D_1 | \sqrt{8} V_T | 1^3 S_1 \rangle}{E_0(1S) - E_0(nD)} |n^3 D_1\rangle.$$ 

- Sternheimer (Dalgarno & Lewis) equation

$$-w''(r) + \frac{6}{r^2} w(r) + m V_{22} w(r) + m V_{02} u_0(r) = m E_0 w(r),$$

- Good surprise: can be solved analytically, leading to a compact expression for the quadrupole moment of the ground state.
Deuteron lifetime

- Simplest nucleus. We restrict to $S$-wave, but including $D$-wave is straightforward
- Hulten wave function
  \[ u(r) = N \left[ \exp(-a r) - \exp(-b r) \right], \]
  with $a = 0.04570$ and $b = 0.2732 \text{ GeV}^{-1}$, and the proper behavior at $r \to 0$ and $r \to \infty$.
- Antineutron component given by the Sternheimer equation
  \[-w''(r) + m W w(r) - m E_0 w(r) = -m \gamma u(r),\]
  with $E_0 = -0.0022 \text{ GeV}$ deuteron energy, $\gamma = 1/\tau(n\bar{n})$ strength of transition, and $W$ complex potential of the $NN$ interaction.
- Width given by
  \[ -\frac{\Gamma}{2} = \int_0^\infty \text{Im} W |w(r)|^2 \, dr = -\gamma \int_0^\infty u(r) \text{Im} w(r) \, dr, \]
Deuteron lifetime

- One gets (valid for other nuclei)

\[ \Gamma \propto \gamma^2, \quad T = T_r \tau(n\bar{n})^2, \]

- where \( T_r \) is the reduced lifetime (in \( s^{-1} \)).

- \( N\bar{N} \) potential by Dover-Richard-Sainio (Khono-Weise, for instance, give similar results)

\[
W(r) = -\frac{V_0 + i W_0}{1 + \exp[(r - R)/a]},
\]

\[ V_0 = W_0 = 0.5 \text{ GeV}, \quad a = 0.2 \text{ fm}, \quad R = 0.8 \text{ fm}, \]

\[ T_r \simeq 3 \times 10^{22} \text{ s}^{-1}. \]

Thus \( T \gtrsim 10^{33} \text{ yr} \) for the deuteron \( \Rightarrow \tau(n\bar{n}) \sim 10^9 \text{ s} \).
Lifetime of the deuteron -3

- Spatial extension of $n$, $\bar{n}$ and annihilation density
  $\propto \gamma u(r) \operatorname{Im} w(r)$.

\begin{align*}
\text{u(x)}^2 \cdot \text{w(x)}^2 \\
\text{r (GeV}^{-1}\text{)}
\end{align*}

\begin{align*}
\text{u(x)}^2 \\
\text{u(x) Im w(x)} \\
\text{r (GeV}^{-1}\text{)}
\end{align*}
Lifetime of the deuteron

- Alternative formula
  \[ T_R \approx \frac{\langle V_n - \text{Re} V_\bar{n} \rangle^2 + \langle \text{Im} V_\bar{n} \rangle^2}{-2 \langle \text{Im} V_\bar{n} \rangle}, \]

- is not too bad, but not too good either
- does not distinguish inner from outer neutrons
- works in the limit of deep binding!
- underestimates the rate of decay, especially in case of weakly-bound external neutrons
Lifetime of $^{16}$O

- As an example of medium-size nucleus proton-decay exp.
- See Dover, Gal, R., and Friedman & Gal, ...
- Shell-model with individual wave function for $S_{1/2}$, $P_{1/2}$, ... to reproduce the observed properties (mainly r.m.s.)
- Summarized as an effective neutron potential for each shell, $V_n = V(n - ^{15}O)$
- While $V_{\bar{n}}$ taken from $\bar{p}$-nucleus phenomenology (exotic atoms, low-energy scattering)
- Same inhomogeneous eqn. as for deuteron, for each shell
Results for \( ^{16}\text{O} \) and \( ^{56}\text{Fe} \)

### TABLE I. Reduced lifetime \( T_R \) (in units of \( 10^{23} \text{ sec}^{-1} \)) for the neutrons in \( ^{16}\text{O} \).

<table>
<thead>
<tr>
<th>Orbit ( lj )</th>
<th>( s_{1/2} )</th>
<th>( p_{3/2} )</th>
<th>( p_{1/2} )</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I (Ref. 18)</td>
<td>1.63</td>
<td>1.11</td>
<td>0.94</td>
<td>1.2</td>
</tr>
<tr>
<td>Model II (Ref. 19)</td>
<td>1.21</td>
<td>0.85</td>
<td>0.75</td>
<td>0.8</td>
</tr>
</tbody>
</table>

### TABLE II. Reduced lifetime \( T_R \) (units \( 10^{23} \text{ sec}^{-1} \)) for the neutrons in \( ^{56}\text{Fe} \).

<table>
<thead>
<tr>
<th>Orbital ( lj )</th>
<th>( T_R ) (Model II)</th>
<th>( T_R ) (Model I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_{1/2} )</td>
<td>1.68</td>
<td>3.32</td>
</tr>
<tr>
<td>( p_{3/2} )</td>
<td>1.50</td>
<td>2.75</td>
</tr>
<tr>
<td>( p_{1/2} )</td>
<td>1.54</td>
<td>2.92</td>
</tr>
<tr>
<td>( d_{5/2} )</td>
<td>1.26</td>
<td>2.04</td>
</tr>
<tr>
<td>( 2s_{1/2} )</td>
<td>1.09</td>
<td>1.60</td>
</tr>
<tr>
<td>( d_{3/2} )</td>
<td>1.33</td>
<td>2.29</td>
</tr>
<tr>
<td>( f_{7/2} )</td>
<td>0.98</td>
<td>1.34</td>
</tr>
<tr>
<td>( 2p_{3/2} )</td>
<td>0.57</td>
<td>0.64</td>
</tr>
<tr>
<td>Average</td>
<td>1.13</td>
<td>1.69</td>
</tr>
</tbody>
</table>
Results for $^{16}\text{O}$ and $^{56}\text{Fe}$

![Graphs showing neutron and antineutron distributions](image)

- **neutron**
- **antineutron** (arb. sca.)
- Neutron density
- Annihilation density

**JMR** \( n - \bar{n} \in \text{nuclei} \)
Results for $^{16}\text{O}$ and $^{56}\text{Fe}$

- Neutron density
- Annihilation density

$u(x)^2$ $w(x)^2$

$20$ $40$ $r (\text{GeV}^{-1})$

$u(r)^2$ $\text{Im } w(r)$

$10$ $20$ $30$ $r (\text{GeV}^{-1})$

JMR $n - \bar{n} \in \text{nuclei}$
Conclusions

- Oscillations mainly outside
- Subsequent annihilation mainly at the surface
- So minimal risk of dramatic medium renormalization of the basic process
- Good knowledge of the antinucleon-nucleus interaction in this region
- Nuclei with neutron skin or neutron halo favored

\[ T_R \sim 10^{23} \text{ s} \text{ in } T(\text{nucleus}) = T_r \tau_{n\bar{n}}^2 \]