Post-Sphaleron Baryogenesis and $n - \bar{n}$ Oscillations

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Based on:
K. S. Babu, R. N. Mohapatra (2017) (to appear);
K. S. Babu, P. S. Bhupal Dev, E. C. F. S. Fortes and R. N. Mohapatra,
Outline

- Idea of post-sphaleron baryogenesis
- Explicit models based on $SU(2)_L \times SU(2)_R \times SU(4)_C$ symmetry
- Relating baryogenesis with neutron-antineutron oscillation
- Other experimental tests
- Conclusions
Generating Baryon Asymmetry of the Universe

- Observed baryon asymmetry:
  \[ Y_{\Delta B} = \frac{n_B - n_{\bar{B}}}{s} = (8.75 \pm 0.23) \times 10^{-11} \]

- Sakharov conditions must be met to dynamically generate \( Y_{\Delta B} \):
  - Baryon number (\( B \)) violation
  - \( C \) and \( CP \) violation
  - Departure from thermal equilibrium

- All ingredients are present in Grand Unified Theories. However, in simple GUTs such as \( SU(5) \), \( B - L \) is unbroken

- Electroweak sphalerons, which are in thermal equilibrium from \( T = (10^2 - 10^{12}) \) GeV, wash out any \( B - L \) preserving asymmetry generated at any \( T > 100 \) GeV

Kuzmin, Rubakov, Shaposhnikov (1985)
Generating baryon asymmetry (cont.)

- Sphaleron: Non-perturbative configuration of the electroweak theory
- Leads to effective interactions of left-handed fermions:
  \[ O_{B+L} = \prod_i (q_i q_i q_i L_i) \]
  Obeys \( \Delta B = \Delta L = 3 \)
  - Sphaleron can convert lepton asymmetry to baryon asymmetry – Leptogenesis mechanism (Fukugita, Yanagida (1986))
- Post-sphaleron baryogenesis: Baryon number is generated below 100 GeV, after sphalerons go out of equilibrium (Babu, Nasri, Mohapatra (2006))
A scalar ($S$) or a pseudoscalar ($\eta$) decays to baryons, violating $B$

$\Delta B = 1$ is strongly constrained by proton decay and cannot lead to successful post-sphaleron baryogenesis

$\Delta B = 2$ decay of $S/\eta$ can generate baryon asymmetry below $T = 100$ GeV: $S/\eta \to 6 q; \quad S/\eta \to 6 \bar{q}$

Decay violates CP, and occurs out of equilibrium

Naturally realized in quark-lepton unified models, with $S/\eta$ identified as the Higgs boson of $B - L$ breaking

$\Delta B = 2 \Rightarrow$ connection with $n - \bar{n}$ oscillation

Quantitative relationship exists in quark-lepton unified models based on $SU(2)_L \times SU(2)_R \times SU(4)_C$
At high temperature, $T$ above the masses of $S/\eta$ and the mediators, the $B$-violating interactions are in equilibrium:

$$\Gamma_{\Delta B \neq 0}(T) \gg H(T) = 1.66(g^*)^{1/2} \frac{T^2}{M_{Pl}}$$

As universe cools, $S/\eta$ freezes out from the plasma while relativistic at $T = T_*$ with $T_* \geq M_{S/\eta}$. (Number density of $S/\eta$ is then comparable to $n_\gamma$.)

For $T < T_*$, decay rate of $S/\eta$ is a constant. $S/\eta$ drifts and occasionally decays. As the universe cools, $H(T)$ slows; at some temperature $T_d$, the constant decay rate of $S/\eta$ becomes comparable to $H(T_d)$. $S/\eta$ decays at $T \sim T_d$ generating $B$.

Post-sphaleron mechanism assumes $T_d = (10^{-1} - 100)$ GeV.
Dilution of Baryon Asymmetry

- $S/\eta \rightarrow 6q$ decay occurs at $T_d \ll M_{S/\eta}$. There is no wash out effect from back reactions.

- However, $S/\eta$ decay dumps entropy into the plasma. This results in a dilution:
  \[
  d \equiv \frac{s_{\text{before}}}{s_{\text{after}}} \approx g_*^{-1/4} \frac{0.6(\Gamma_\eta M_{\text{Pl}})^{1/2}}{rM_\eta} \approx \frac{T_d}{M_\eta}
  \]

- $M_\eta$ cannot be much higher than a few TeV, or else the dilution will be too strong.

- There is a further dilution of order 0.1, owing to the change of $g_*$ from 62.75 at 200 MeV to 5.5 after recombination.
Pseudoscalar $\eta$ must have $\Delta B = 2$ decays

Such decays should have CP violation to generate $B$ asymmetry

$\eta$ should freeze-out while relativistic: $T_* \geq M_\eta$. This requires $\eta$ to be feebly interacting, and a singlet of Standard Model

$T_d$, the temperature when $\eta$ decays, should lie in the range $T_d = (100 \text{ MeV} - 100 \text{ GeV})$

$\eta$ should have a mass of order TeV, or else baryon asymmetry will suffer a dilution of $T_d/M_\eta$
Quark-Lepton Symmetric Models

- Quark-lepton symmetric models based on the gauge group $SU(2)_L \times SU(2)_R \times SU(4)_C$ have the necessary ingredients for PSB.

- There is no $\Delta B = 1$ processes since $B - L$ is broken by 2 units. Thus there is no rapid proton decay. Symmetry may be realized in the 100 TeV range.

- There is baryon number violation mediated by scalars, which are the partners of Higgs that lead to seesaw mechanism.

- Scalar fields $S/\eta$ arise naturally as Higgs bosons of $B - L$ breaking.

- Yukawa coupling that affect PSB and $n - \bar{n}$ oscillations are the same as the ones that generate neutrino masses.
Models based on $SU(2)_L \times SU(2)_R \times SU(4)_C$ (Pati-Salam)

Fermions, including $\nu_R$, belong to $(2, 1, 4) \oplus (1, 2, 4)$

Symmetry breaking and neutrino mass generation needs Higgs field $\Delta(1, 3, \overline{10})$. Under $SU(2)_L \times U(1)_Y \times SU(3)_C$:

$$\Delta(1, 3, \overline{10}) = \Delta_{uu}(1, -\frac{8}{3}, 6^*) \oplus \Delta_{ud}(1, -\frac{2}{3}, 6^*) \oplus \Delta_{dd}(1, +\frac{4}{3}, 6^*) \oplus \Delta_{ue}(1, \frac{2}{3}, 3^*)$$

$$\oplus \Delta_{uv}(1, -\frac{4}{3}, 3^*) \oplus \Delta_{de}(1, \frac{8}{3}, 3^*) \oplus \Delta_{d\nu}(1, \frac{2}{3}, 3^*) \oplus \Delta_{ee}(1, 4, 1)$$

$$\oplus \Delta_{\nu e}(1, 2, 1) \oplus \Delta_{\nu \nu}(1, 0, 1).$$

$\Delta_{uu}, \Delta_{ud}, \Delta_{dd}$ are diquarks, $\Delta_{ue}, \Delta_{uv}, \Delta_{de}, \Delta_{d\nu}$ are leptoquarks, and $\Delta_{\nu \nu}$ is a singlet that breaks the symmetry

Diquarks generate $B$ violation, leptoquarks help with CP violation, and singlet $\Delta_{\nu \nu}$ provides the field $S/\eta$ for PSB
Interactions of color sextet diquarks and $B$ violating couplings:

\[ \mathcal{L}_I = \frac{f_{ij}}{2} \Delta_{dd} d_i d_j + \frac{h_{ij}}{2} \Delta_{uu} u_i u_j + \frac{g_{ij}}{2\sqrt{2}} \Delta_{ud} (u_i d_j + u_j d_i) \]

\[ + \frac{\lambda}{2} \Delta_{\nu\nu} \Delta_{dd} \Delta_{ud} \Delta_{ud} + \lambda' \Delta_{\nu\nu} \Delta_{uu} \Delta_{dd} \Delta_{dd} + \text{h.c.} \]

$f_{ij} = g_{ij} = h_{ij}$ and $\lambda = \lambda'$ from gauge symmetry.

We also introduce a scalar $\chi(1, 2, 4)$ which couples to $\Delta(1, 3, 10)$ via

\[ (\mu \chi \chi \Delta \supset \chi_{\nu} \chi_{\nu} \Delta_{\nu\nu} + \ldots) \]

Among the phases of $\Delta_{\nu\nu}$ and $\chi_{\nu}$, one combination is eaten by $B - L$ gauge boson. The other, is a pseudoscalar $\eta$:

\[ \Delta_{\nu\nu} = \left( \frac{\rho_1 + v_R}{\sqrt{2}} \right) e^{i \eta_1 / v_R}, \chi_{\nu} = \left( \frac{\rho_2 + v_B}{\sqrt{2}} \right) e^{i \eta_2 / v_B}, \eta = \frac{2 \eta_2 v_R - \eta_1 v_B}{\sqrt{4 v_R^2 + v_B^2}} \]

Advantage of using $\eta \rightarrow 6q$ is that $\eta$ has a flat potential due to a shift symmetry.
Baryon violating decay of $\eta$

$\eta \to 6q$ and $\eta \to 6\bar{q}$ decays violate $B$:
Baryon violating decay of $\eta$ (cont.)

- $B$-violating decay rate of $\eta$:

$$\Gamma_\eta \equiv \Gamma(\eta \to 6q) + \Gamma(\eta \to 6\bar{q}) = \frac{P}{\pi^9 \cdot 2^{25} \cdot 45} \frac{12}{4} |\lambda|^2 \text{Tr}(f^\dagger f) [\text{Tr}(\hat{g}^\dagger \hat{g})]^2 \left( \frac{M_{\eta}^{13}}{M_{\Delta_{ud}}^8 M_{\Delta_{dd}}^4} \right)$$

Here $P$ is a phase space factor:

$$P = \begin{cases} 
1.13 \times 10^{-4} & (M_{\Delta_{ud}}/M_S, M_{\Delta_{dd}}/M_S \gg 1) \\
1.29 \times 10^{-4} & (M_{\Delta_{ud}}/M_S = M_{\Delta_{dd}}/M_S = 2) 
\end{cases}$$

- $T_d$ is obtained by setting this rate to Hubble rate. For $T_d = (100 \text{ MeV} - 100 \text{ GeV})$, $f \sim g \sim h \sim 1$, $M_{\Delta_{ud}} \sim M_\eta$ needed

- $\eta$ has a competing $B = 0$ four-body decay mode, which is necessary to generate CP asymmetry: $\eta \to (uu)\Delta_{u\nu}\Delta_{u\nu}$

- The six-body and four-body decays should have comparable widths, or else $B$ asymmetry will be too small
Baryon conserving decay of $\eta$

$\eta \rightarrow (uu) \Delta_{uv} \Delta_{uv}$

This generates absorptive part and CP violation in $\eta \rightarrow 6q$: 
Baryon Asymmetry

- $\eta \rightarrow uu\Delta_{u\nu}\Delta_{u\nu}$ has a width:

$$\Gamma_\eta \equiv \Gamma(\eta \rightarrow uu\Delta_{u\nu}\Delta_{u\nu}) + \Gamma(\eta \rightarrow \bar{u}\bar{u}\Delta - u\nu^*\Delta^*_{u\nu}) = \frac{P}{1024\pi^5} [\text{Tr}(\hat{f}^\dagger \hat{f})]^3 \left( \frac{M_\eta^5}{M_{\nu_R}^4} \right)$$

- Here $P$ is a phase space factor, $P \approx 2 \times 10^{-3}$

- For $\lambda = 1$, $\Gamma_6 \approx (7 \times 10^{-9}) \times \Gamma_4 \times \frac{M_{\nu_R}^4}{M_{\Delta_{dd}}^4}$

- Baryon asymmetry:

$$\epsilon_B \approx \frac{|f|^2 \text{Im}(\lambda \tilde{\lambda})}{8\pi (|\lambda|^2 + \Gamma_4/\Gamma_6)} \left( \frac{M_{\Delta_{dd}}^2}{M_{\nu_R}^2} \right)$$

- With $M_{\nu_R} \sim 100 \times M_{\Delta_{dd}}$, $\Gamma_4 \sim \Gamma_6$. This choice maximizes $\epsilon_B$:

$$\epsilon_B \sim (8 \times 10^{-5}) \times \frac{|f|^2}{8\pi} \text{Im}(\frac{\tilde{\lambda}}{\lambda})$$

- For $f \sim \lambda \sim 1$, reasonable baryon asymmetry is generated with a dilution $d \sim 10^{-3}$
Other constraints for successful baryogenesis

\( \eta \) must freeze out at \( T \geq M_\eta \). Interactions of \( \eta \) with lighter particles must be weak. The scattering processes freeze out as desired, owing to the shift symmetry in \( \eta \).

\[ \Delta_{u\nu} \]
Connection with $n - \bar{n}$ oscillation

As $\eta$ is associated with $B - L$ symmetry breaking, replacing $\eta$ by the vacuum expectation value, $n - \bar{n}$ oscillation results:

\[ M_\eta \sim 3 \text{ TeV}, \quad M_{\Delta_{ud}} \sim 4 \text{ TeV}, \quad M_{\Delta_{dd}} \sim 50 \text{ TeV}, \quad M_{\Delta_{uu}} \sim 1 \text{ TeV}, \quad v_{B-L} \sim 300 \text{ TeV} \]

is a consistent choice

Flavor dependence of baryon asymmetry can be fixed via neutrino mass generation
Connection with neutrino oscillations

- Quark-Lepton symmetry implies that the Yukawa couplings entering $\eta$ decay and $n - \bar{n}$ oscillation is the same as in neutrino mass generation.

- In type-II seesaw mechanism for neutrino masses, $M_\nu \propto f$. A consistent choice of $f$ that yields inverted neutrino spectrum:

$$
    f = \begin{pmatrix}
        0 & 0.95 & 1 \\
        0.95 & 0 & 0.01 \\
        1 & 0.01 & 0.06
    \end{pmatrix}
$$

- This choice satisfies all flavor changing constraints mediated by color sextet scalars.

- The $(1,1)$ entry will be relevant for $n - \bar{n}$ oscillation. It is induced via a $W$ boson loop.
Flavor changing constraints

- $\Delta_{dd}, \Delta_{uu}, \Delta_{ud}$ fields lead to flavor violation, at tree level as well as at loop:

\[
\mathcal{H}_{\Delta F=2} = -\frac{1}{8} \frac{f_i f_k^*}{M_{\Delta_{dd}}^2} \left( \overline{d}_{kR}^\alpha \gamma_\mu d^\alpha_{iR} \right) \left( \overline{d}_{jR}^\beta \gamma_\mu d^\beta_{\ell R} \right) + \frac{1}{256\pi^2} \frac{[(ff^\dagger)_{ij} (ff^\dagger)_{\ell k} + (ff^\dagger)_{ik} (ff^\dagger)_{\ell j}]}{M_{\Delta_{dd}}^2} \\
\times \left[ \left( \overline{d}_{jR}^\alpha \gamma_\mu d^\alpha_{iR} \right) \left( \overline{d}_{kR}^\beta \gamma_\mu d^\beta_{\ell R} \right) + 5 \left( \overline{d}_{jR}^\alpha \gamma_\mu d^\alpha_{iR} \right) \left( \overline{d}_{kR}^\beta \gamma_\mu d^\beta_{\ell R} \right) \right]
\]
Flavor changing constraints

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Table: Constraints on the product of Yukawa couplings in the PSB model from $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$, $B_s^0 - \bar{B}_s^0$ and $B_d^0 - \bar{B}_d^0$ mixing.
Prediction for $n - \bar{n}$ oscillation

- We take all PSB constraints, neutrino mass and mixing constraints, and FCNC constraints to estimate $n - \bar{n}$ oscillation time.

- The flavor structure of $f$ has a zero in the (1,1) element. This entry is generated by a $W$ boson loop.
Prediction for $n - \bar{n}$ oscillation (cont.)

- **Amplitude for $n - \bar{n}$ oscillation:**

$$A_{n-\bar{n}}^\text{tree} \sim \frac{f_{11}g_{11}^2\lambda v_{BL}}{M_{\Delta_{dd}}^2 M_{\Delta_{ud}}^4} + \frac{f_{11}^2 h_{11}^2 \lambda' v_{BL}}{M_{\Delta_{dd}}^4 M_{\Delta_{uu}}^2}$$

- **Loop induced amplitude:**

$$A_{n-\bar{n}}^{1\text{-loop}} \sim \frac{g^2 g_{11}f_{13} V_{ub}^* V_{td} \lambda v_{BL}}{128\pi^2 M_{\Delta_{ud}}^2} \left( \frac{m_t m_b}{m_W^2} \right) F \langle \bar{n} | \mathcal{O}_{RLR}^2 | n \rangle$$

- **Loop function:**

$$F = \frac{1}{M_{\Delta_{ud}}^2 - M_{\Delta_{dd}}^2} \left[ \frac{1}{M_{\Delta_{ud}}^2} \ln \left( \frac{M_{\Delta_{ud}}^2}{m_W^2} \right) - \frac{1}{M_{\Delta_{dd}}^2} \ln \left( \frac{M_{\Delta_{dd}}^2}{m_W^2} \right) \right]$$

$$+ \frac{1}{M_{\Delta_{ud}}^2 M_{\Delta_{dd}}^2} \frac{1 - (m_t^2/4m_W^2)}{1 - (m_t^2/m_W^2)} \ln \left( \frac{m_t^2}{m_W^2} \right)$$
Effective operator:

\[ O_{RLR}^2 = (u_{iR}^T C d_{jR})(u_{kL}^T C d_{lL})(d_{mR}^T C d_{nR}) \Gamma_{ijklmn}^s \]

Matrix element in MIT bag model (Rao and Shrock):

\[ \langle \bar{n} | O_{RLR}^2 | n \rangle = -0.314 \times 10^{-5} \text{ GeV}^6 \]

QCD correction:

\[ c_{QCD}(\mu_\Delta, 1 \text{ GeV}) = \left[ \frac{\alpha_s(\mu_\Delta^2)}{\alpha_s(m_t^2)} \right]^{8/7} \left[ \frac{\alpha_s(m_t^2)}{\alpha_s(m_b^2)} \right]^{24/23} \left[ \frac{\alpha_s(m_b^2)}{\alpha_s(m_c^2)} \right]^{24/25} \left[ \frac{\alpha_s(1 \text{ GeV}^2)}{} \right]^{8/9} \]

\[ \tau_{n-\bar{n}}^{-1} \equiv \delta m = c_{QCD}(\mu_\Delta, 1 \text{ GeV}) \left| A_{n-\bar{n}}^{1-\text{loop}} \right| \]
Prediction for $n - \bar{n}$ oscillation (cont.)

**Figure**: Scatter plots for $\tau_{n-\bar{n}}$ as a function of the $\Delta$ masses $M_{\Delta_{ud}}, M_{\Delta_{dd}}$. 
Prediction for $n - \bar{n}$ oscillation (cont.)

![Graph showing likelihood probability for $\tau_{n-\bar{n}}/(10^8 \text{ sec})$]

**Figure**: The likelihood probability for a particular value of $\tau_{n-\bar{n}}$ as given by the model parameters.
Conclusions

- Post-sphaleron baryogenesis is an alternative to high scale leptogenesis

- Directly linked with $n - \bar{n}$ oscillation

- In quark-lepton symmetric models, post-sphaleron baryogenesis can lead to quantitative prediction for $n - \bar{n}$ oscillation time

- $\tau_{n-\bar{n}} \approx (10^9 - 10^{11})$ sec. is the preferred range from PSB

- Within a concrete model, an upper limit of $\tau_{n-\bar{n}} < 4 \times 10^{10}$ sec. is derived, which may be accessible to experiments