Taming the pion cloud

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Ancient History

NN force

Longest range component

Pion cloud

Fundamental Question: meson cloud or $4q\bar{q}$

Hidden color

Important implications for nuclear force and nuclear structure if meson cloud picture is shown to fail
Implications for sea

\[ p \rightarrow n\pi^+ , \ p \rightarrow \Delta^{++}\pi^- , \ldots \]
\[ \pi^+ \sim u\bar{d}, \ \pi^- \sim d\bar{u} \]

Thus \( \bar{d}(x) \neq \bar{u}(x) \)

Form factor \( \exp \left(-\frac{k^2}{\Lambda^2}\right) \)

Longest range component

H. Yukawa

A W Thomas PLB 126, 97 (1983)

Wandmolders et al PRL 66,2712 (1991)

\[ D = \int_0^1 [\bar{d}(x) - \bar{u}(x)] dx = 0.136 \pm 0.060 . \]

\( 0.16 \pm 0.01 \)

Henley & Miller PLB 251, 453 (1990)
Implications for sea

\[ p \rightarrow n\pi^+, \ p \rightarrow \Delta^{++}\pi^-, \ldots \]

\[ \pi^+ \sim u\bar{d}, \ \pi^- \sim d\bar{u} \]

Thus \( \bar{d}(x) \neq \bar{u}(x) \)

Form factor \( \exp(-k^2/\Lambda^2) \)

\[ \Lambda \rightarrow R_{\pi N} = R_{\pi N}^{\text{experiment}} = 0.66 \text{ fm} \]
Longest range component

Implications for sea

\[ p \rightarrow n\pi^+, \ p \rightarrow \Delta^{++}\pi^- \ldots \]
\[ \pi^+ \sim ud, \ \pi^- \sim d\bar{u} \]

Thus \( \bar{d}(x) \neq \bar{u}(x) \)

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Experimental Progress
Drell-Yan

- NMC measured integral quantity for Gottfried sum
- E866 FermiLab measured x-dependence

J. C. PENG et al. PHYSICAL REVIEW D 58 092004

FIG. 1. Comparison of the E866 $\bar{d}-\bar{u}$ results at $Q=7.35$ GeV with the predictions of various models as described in the text.

$\bar{d}(x) > \bar{u}(x)$ what about $\bar{d}/\bar{u}$?

Expect large ratio at large $x$
More data- E866 (1999)

Drell-Yan Measured \( \bar{d} - \bar{u} \) and \( \frac{\bar{d}}{\bar{u}} \)

Alberg, Henley and Miller PLB 471, 396 (2000)

With pions get too large a ratio \( \frac{\bar{d}}{\bar{u}} \)

Is there an isoscalar non-perturbative sea (omega meson)?

The omega represents any non-perturbative isoscalar sea

What’s going on at high x?

SeaQuest aims at better measurement, so we try to improve

Form factor

\[
G_M(t,u) = \exp\left(\frac{t - m^2_M}{2 \Lambda^2_M}\right) \exp\left(\frac{u - m^2_B}{2 \Lambda^2_M}\right),
\]

Theory problems

- Results depend on form factor parameter $\Lambda$
- Form factors enter as three dimensional functions even though expressed in terms of $t$ and $u$
- How to derive ????
- Why do we need form factors? Form factors oppose chiral perturbation theory
Why do we need form factors?

Using form factors opposes chiral perturbation theory

\[
\mathcal{L}_N^{(1)} = \bar{\psi}(i\gamma \cdot \partial - M)\psi - \frac{g_A}{2f_\pi} \bar{\psi} \gamma_\mu \gamma_5 \tau^a \psi \partial_\mu \pi^a - \frac{1}{f_\pi^2} \bar{\psi} \gamma_\mu \tau^a \psi \epsilon^{abc} \pi^b \partial_\mu \pi^c
\]

Non-renormalizable \( \mathcal{L} \), expand in powers of momentum
add counter terms order-by order \(-\) LECs.
results \textbf{INDEPENDENT OF CUTOFF} Not sufficient for DIS, IMHO because momenta \( \mathcal{O} \sim m_N \)

Form factor relates the LECs in a very specific way
different philosophy
Comment talk last week titled “\cdots with chiral perturbation theory”
is \textbf{NOT} – no LEC’s, but yes to cutoff dependence

pion-nucleon
Form factor takes composite
nature of pion and nucleon
into account
Constrain form factor using experimental input info from Thomas and Weise book

\[ q_\mu \langle p(\bar{p} \; | A^\mu_+(0) | n(P) \rangle = 2\bar{u}_p(P') \left[ MG_A(Q^2) - \frac{Q^2 f_{\pi} g_{\pi NN}(Q^2)}{Q^2 + m^2_{\pi}} \right] \gamma_5 u_n(P). \]

Thus the matrix element of the divergence of the axial current vanishes as \( m^2_{\pi} \to 0 \) if \( G_A(Q^2) \) and the pion-nucleon form factor \( g_{\pi NN}(Q^2) \) are related by

\[ MG_A(Q^2) = f_\pi g_{\pi NN}(Q^2). \tag{3.10} \]

At \( Q^2 = 0 \) this is known as the Goldberger-Treiman relation and it is satisfied at the level of 3 % (with \( g_A = G_A(0) = 1.267 \pm 0.004, g_{\pi NN} = g_{\pi NN}(0) = 13.2 \pm 0.1 \) and \( M = \)

\[ G_A(Q^2) = \frac{G_A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}, \]

\[
\begin{array}{c|c}
\text{Exp.} & M_A [\text{GeV}] \\
\hline
\text{BNL} & 1.07 \pm 0.06 \\
\text{ZGS} & 1.00 \pm 0.05 \\
\text{Fermilab} & 1.05 \pm 0.12 \\
\text{Average} & 1.03 \pm 0.04 \\
\end{array}
\]

Pion-Nucleon form factor determined for on-mass-shell nucleons, off shell pion
We present a light-front determination of the pionic contribution to the nucleon self-energy, \( g_{\pi NN}(Q^2) \propto G_A(Q^2) = \frac{G_A(0)}{(1+Q^2/M_A^2)^2} \)

Pion-Nucleon form factor determined for on-mass-shell nucleons, off shell pion

Nucleon self energy - intermediate nucleon and Delta

\[
D = \int_0^1 [\bar{d}(x) - \bar{u}(x)] dx = 0.118 \pm 0.012.
\]

Ours 0.109

Parameter free calculation!

So what's the problem?
Taming II- recent

To get \( \bar{u} \) and \( \bar{d} \) need to calculate the graphs:

Both pion and nucleon are off-shell in the Feynman graphs need to reconsider the formalism

\[
q_N^f(x) = Z_2 q_{N0}^f(x) + \sum_{B,M} \int_x^1 \frac{dy}{y} f_{MB}(y) q_M^f(\frac{x}{y}) + \sum_{B,M} \int_x^1 \frac{dy}{y} f_{BM}(y) q_B^f(\frac{x}{y}) \\
Z_2^{-1} - 1 = \sum_{B,M} \int dy f_{BM}(y),
\]

Brodsky-Lepage Fock space representation:

\[
|\pi N\rangle \propto \int_0^1 \frac{dy}{\sqrt{y}} d^2 k_{\perp \pi} \int_0^1 \frac{dy_N}{\sqrt{y_N}} d^2 k_{\perp N} \delta(1-y-y_N) \delta(\vec{k}_{\perp \pi} + \vec{k}_{\perp N}) \psi_{\pi N}(y, \vec{k}_{\perp \pi}; y_N, \vec{k}_{\perp N}) |\cdots\rangle \\
f_{\pi N}(y) = \int d^2 k_{\perp \pi} \left| \psi_{\pi N}(y, \vec{k}_{\perp \pi}; 1-y, -\vec{k}_{\perp \pi}) \right|^2 = f_{N\pi}(1-y)
\]
Taming II- recent

To get $\bar{u}$ and $\bar{d}$ need to calculate the graphs:

![Graphs](image)

Both pion and nucleon are off-shell in the Feynman graphs need to reconsider the formalism

$$q^f_N(x) = Z_2 q^f_{N0}(x) + \sum_{B,M} \int_x^1 \frac{dy}{y} f_{MB}(y)q^f_M(x/y) + \sum_{B,M} \int_x^1 \frac{dy}{y} f_{BM}(y)q^f_B(x/y)$$

$$Z_2^{-1} - 1 = \sum_{B,M} \int dy f_{BM}(y),$$

If $f_{MB}$ has $\delta(y)$

$Z_2$ would change,

but NO delta functions here!

Brodsky-Lepage Fock space representation:

$$|\pi N\rangle \propto \int_0^1 \frac{dy}{\sqrt{y}} d^2 k_{\perp \pi} \int_0^1 \frac{dy_N}{\sqrt{y_N}} d^2 k_{\perp N} \delta(1-y-y_N)\delta(\vec{k}_{\perp \pi} + \vec{k}_{\perp N})\psi_{\pi N}(y, \vec{k}_{\perp \pi}; y_N, \vec{k}_{\perp N})|\cdots\rangle$$

$$f_{\pi N}(y) = \int d^2 k_{\perp \pi} \left| \psi_{\pi N}(y, \vec{k}_{\perp \pi}; 1-y, -\vec{k}_{\perp \pi}) \right|^2 = f_{N\pi}(1-y)$$
\( \hat{P}^- \) is Hamiltonian operator, construct from energy-momentum tensor \( T^{+-} = \)
free particle kinetic energy \( M_0^2 \) plus interactions \( V \)

\[
\text{Schroedinger eq: } (\hat{P}^- \hat{P}^- - \hat{P}_\perp^2) |p\rangle = M_p^2 |p\rangle = (M_0^2 + V) |p\rangle
\]

\[
|p\rangle \approx Z \left( |p\rangle_0 + \frac{1}{M^2 - M_0^2} V |p\rangle_0 \right)
\]

\( |\pi N\rangle \) component

\[
\mathcal{L}_N^{(1)} = \bar{\psi} (i \gamma \cdot \partial - M) \psi - \frac{g_A}{2f_\pi} \bar{\psi} \gamma_\mu \gamma_5 \tau^a \psi \partial_\mu \pi^a - \frac{1}{f_\pi^2} \bar{\psi} \gamma_\mu \tau^a \psi \epsilon^{abc} \pi^b \partial_\mu \pi^c
\]

Form factors absent
Form factors

- Including form factors goes beyond usual LF treatment
- Need form factors in frame independent manner (4-space)
- Maintain momentum conservation, unique LF wave function

Keep experimental input
- For use in light front wave function-virtual N, π

- Product of pion and nucleon form factors:

\[ F(k, p, y) = \frac{1}{[1 - \frac{(k^2 - m^2_\pi)}{\Lambda^2}]} \frac{1}{[1 - \frac{y(p-k)^2 - m^2_N}{\Lambda^2}]} \]

\[ \psi_{LF} \propto \int_{-\infty}^{\infty} dk^- \psi_{Bethe-Salpeter}(k, p) \]

\[ \psi_{Bethe-Salpeter}(k, p) = \]

\[ \psi_{LF} \propto \int dk^- \Gamma(p, k) \frac{1}{k^2 - m^2_\pi + i\epsilon} \frac{1}{(p-k)^2 - m^2_N + i\epsilon} \]

\[ \Gamma \text{ contains form factor.} \]

Integrate over UH \( k^- \) plane = integrate over LH \( k^- \) plane w. stated form factor

\( n = 1 \) gives form factor very close to dipole, maintain experimental input!
Summary

• Have formalism to get light front wave functions and meson distribution functions needed for light flavor nucleon sea

• Meson-nucleon coupling constants are known

• Form factors included in frame independent manner that incorporates experimental input

• Given the meson cloud model can make calculations with reasonably-well understood uncertainties

• True test of meson cloud model!

• See Alberg’s talk