Huey-Wen Lin — The Flavor Structure of Nucleon Sea
Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

☞ Many ongoing/planned experiments
  (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

Electron Ion Collider: The Next QCD Frontier

Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701
Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

❖ Many ongoing/planned experiments
  (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

§ Important inputs to discern new physics at LHC

❖ Currently dominate errors in Higgs production

![Higgs Production Table](image)
Global Analysis

§ Experiments cover diverse kinematics of parton variables
☞ Global analysis takes advantage of all data sets

Choice of data sets and kinematic cuts
☞ Strong coupling constant $\alpha_s(M_Z)$
☞ How to parametrize the distribution

$$xf(x, \mu_0) = a_0 x^{a_1} (1 - x)^{a_2} P(x)$$

Assumptions imposed
☞ SU(3) flavor symmetry, charge symmetry, strange and sea distributions

$s = \bar{s} = \kappa(\bar{u} + \bar{d})$
Global Analysis

§ Discrepancies appear when data is scarce
§ Many groups have tackled the analysis
☞ CTEQ, MSTW, ABM, JR, NNPDF, etc.

What can we do on the lattice?
Lattice QCD 101

§ Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories

§ Physical observables are calculated from the path integral

\[
\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int DA \, D\bar{\psi} \, D\psi \, e^{iS(\bar{\psi}, \psi, A)} O(\bar{\psi}, \psi, A)
\]

in Euclidian space

♫ Quark mass parameter (described by \( m_\pi \))
♫ Impose a UV cutoff
discretize spacetime
♫ Impose an infrared cutoff
finite volume

§ Recover physical limit

\( m_\pi \rightarrow m_\pi^{\text{phys}}, \, a \rightarrow 0, \, L \rightarrow \infty \)
PDFs on the Lattice

§ Lattice calculations rely on operator product expansion, only provide moments

\[ \langle x^n \rangle_q = \int_{-1}^{1} dx \, x^n q(x) \]

\[ \langle x^n \rangle_{\Delta q} = \int_{-1}^{1} dx \, x^n \Delta q(x) \]

\[ \langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx \, x^n \delta q(x) \]

*Quark density/unpolarized*

*Helicity*  
longitudinally polarized

*Transversity*  
transversely polarized

§ True distribution can only be recovered with all moments

most well known

very poorly known

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Problem with Moments

For higher moments, ops mix with lower-dimension ops

Renormalization is difficult too

Relative error grows in higher moments

Calculation would be costly and difficult

Problem with Moments

Dolgov et al. PRD66, 034506 (2002)

Göckeler et al. PRD71, 114511 (2005)

LHPC (SCRI, SESAM): 2f, Wilson and clover

QCDSF: 0f

Huey-Wen Lin — The Flavor Structure of Nucleon Sea
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LHPC (SCRI, SESAM): 2f, Wilson and clover

QCDSF: 0f
State-of-the-Art Moments

§ FLAG rating system

§ New: excited-state rating

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>Ref.</th>
<th>$N_f$</th>
<th>chiral extrapolation</th>
<th>continuum extrapolation</th>
<th>finite volume</th>
<th>excited state</th>
<th>renormalization</th>
<th>$g_T$</th>
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<tbody>
<tr>
<td>PNDME'15</td>
<td>This work</td>
<td>P</td>
<td>2+1+1</td>
<td></td>
<td></td>
<td>1.020(76)$^a$</td>
<td></td>
<td>1.11(3)$^b$</td>
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<tr>
<td>ETMC’13</td>
<td>[30]</td>
<td>C</td>
<td>2+1+1</td>
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<td>1.037(20)$^c$</td>
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<tr>
<td>LHPC’12</td>
<td>[28]</td>
<td>A</td>
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<tr>
<td>RBC/UKQCD’10</td>
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<td>A</td>
<td>2+1</td>
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<td>1.005(17)(29)$^e$</td>
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<tr>
<td>RQCD’14</td>
<td>[31]</td>
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<td>[30]</td>
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<td>0.93(6)$^g$</td>
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<td>RBC’08</td>
<td>[32]</td>
<td>P</td>
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<td></td>
<td></td>
<td>0.93(6)$^g$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
(M_N - M_P)^{QCD} = 2.59(49) \text{ MeV}
\]
State-of-the-Art Moments

§ Improved transversity distribution with LQCD $g_T$

☞ Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
☞ Use to constrain the global analysis fits SIDIS $\pi^\pm$ production data from proton and deuteron targets

Lin, Melnitchouk, Prokudin, Sato, In preparation
State-of-the-Art Moments

FLAG-like rating system
Community averaging quantities

Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

§ Precision moments can be useful to improve PDFs!
 Inputs as constraint in global analysis, like gA

§ Whitepaper will
 Address precision needed for moments and their impacts
 Encourage more precision moment calculations in LQCD

Lin et al, In preparation
Beyond Traditional Moments?

§ Longstanding obstacle!

§ Holy grail of structure calculations

§ Applies to many structure quantities:
  ↘ Generalized parton distributions (GPDs)
  ↘ Transverse-momentum distributions (TMD)
  ↘ Meson distribution amplitudes...
Beyond Traditional Moments?

§ Many new developments
§ Reaching for higher moments

❖ Fictitious heavy quarks (Detmold and Lin, hep-lat/0507007)
  Wed. afternoon David Lin (NCTU)
❖ Smeared lat. ops (Davoudi et al. 1204.4146)

§ Direct calc. of $x$ dependence approach

❖ Hadronic tensor currents
  (Liu et al., hep-ph/9806491, ... 1603.07352) Next Monday, K.-F. Liu
❖ Inversion method/OPE without OPE (QCDSF, hep-lat/9809171, ...
  1703.01153) Wed. Morning, G. Schierholz (DESY)
❖ Euclidean correlation functions (RQCD, 1709.04325)
  A. Schäfer (Regensburg)
❖ LaMET This talk, Chen (NTU), F. Steffens (DESY)
A Promising New Direction

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013);
Details see J.-W. Chen’s talk this Wed.
Lattice Parton Physics Project (LP³)

https://www.pa.msu.edu/~hwlin/LP3/

Huey-Wen Lin — The Flavor Structure of Nucleon Sea
Large-Momentum Effective Theory for PDFs

1) Calculate nucleon matrix elements on the lattice (z dependence)

\[ Q^2 = 0 \]

\[ p(P_z) \]

\[ P_z \in \{0.43, 0.86, 1.29\} \text{ GeV} \]

Huey-Wen Lin — The Flavor Structure of Nucleon Sea
A New Direction

Large-Momentum Effective Theory for PDFs

2) Compute quasi-distribution via

\[ \tilde{q}(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left( A_\mu \right) \Gamma \exp \left( -i g \int_0^z dz' A_z(z') \right) \psi(0) | P \]

Uncorrected bare lattice results

\[ x = k_z / P_z \]

Distribution should sharper as \( P_z \) increases

Artifacts due to finite \( P_z \) on the lattice

HWL et al. 1402.1462

Huey-Wen Lin — The Flavor Structure of Nucleon Sea
A New Direction

Large-Momentum Effective Theory for PDFs

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x, \mu, P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z \left( \frac{x}{y} \frac{\mu}{P_z} \right) q(y, \mu) + O\left(\frac{M_N^2}{P_z^2}\right) + \cdots$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Removing $O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

Corrected distributions from the largest 2 $P_z$ show signs of convergence

HWL et al. 1402.1462
Large-Momentum Effective Theory for PDFs

4) Remove the leading high-twist effect \( \left( \frac{\Lambda_{\text{QCD}}^2}{P_z^2} \right) \)

\( N_f = 2+1+1 \) clover/HISQ lattices (MILC)

\( M_\pi \approx 310 \text{ MeV}, \ a \approx 0.12 \text{ fm} \ (M_\pi L \approx 4.5), \ O(10^3) \) measurements

Huey-Wen Lin — The Flavor Structure of Nucleon Sea
§ First time in LQCD history to study antiquark distribution!

\[ M_\pi \approx 310 \text{ MeV}, \ a \approx 0.12 \text{ fm} \]

\[ \bar{q}(x) = -q(-x) \]

Lost resolution in small-\(x\) region

Future improvement:
larger lattice volume

\[ \int dx \left( \bar{u}(x) - \bar{d}(x) \right) \approx -0.16(7) \]

\begin{table}
\begin{tabular}{|l|c|c|}
\hline
Experiment & \(x\) range & \(\int_0^1 [\bar{d}(x) - \bar{u}(x)] dx\) \\
\hline
E866 & 0.015 < \(x\) < 0.35 & 0.118 ± 0.012 \\
NMC & 0.004 < \(x\) < 0.80 & 0.148 ± 0.039 \\
HERMES & 0.020 < \(x\) < 0.30 & 0.16 ± 0.03 \\
\hline
\end{tabular}
\end{table}

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
First time in LQCD history to study antiquark distribution!

\[ M_\pi \approx 310 \text{ MeV}, \ a \approx 0.12 \text{ fm} \]

Sea Flavor Asymmetry

\[ \bar{q}(x) = -q(-x) \]

Lost resolution in small-\(x\) region

A milestone for lattice QCD!

First sea flavor asymmetry ever calculated!

HWL et al. 1402.1462

\[ \bar{n}(x) = -n(-x) \]

\[ x \]

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
§ Lattice exploratory study

\[ M_\pi \approx 310 \text{ MeV}, \ a \approx 0.12 \text{ fm} \]

Compared with E866
Too good to be true?

Lost resolution in small-\( x \) region

Similar results repeated by ETMC, at \( M_\pi \approx 373 \text{ MeV} \)

ETMC, 1504.07455

\[
\sum_{n} \langle n | \pi^+ (p_i) \pi^- (p_f) \rangle \approx 0.16 
\]

\[
\sum_{n} \langle n | \pi^+ (p_i) \pi^- (p_f) \rangle \approx 0.18 
\]

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
Exploratory study

\[ M_\pi \approx 310 \text{ MeV} \]

We see polarized “sea asymmetry”

\[ \int dx \left( \Delta \bar{u}(x) - \Delta \bar{d}(x) \right) \approx 0.14(9) \]

Both STAR and PHENIX at RHIC see \( \Delta \bar{u} > \Delta \bar{d} \)

1404.6880 and 1504.07451

Other experiments, Fermilab DY exp’ts (E1027/E1039), future EIC
§ Exploratory study  \( M_\pi \approx 310 \text{ MeV} \)

\[
\delta \bar{q}(x) = -\delta q(-x)^x
\]

\[
\int dx \left( \delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.10(8)
\]

\[
\int dx \left( \delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.082
\]

P. Schweitzer et al., PRD 64, 034013 (2001)

\( \approx \) SoLID at JLab, Drell-Yan exp’t at FNAL (E1027+E1039), EIC, ...

Removing \( O(M_N^n/P_z^n) + O(\Lambda_{QCD}^2/P_z^2) \) errors
Missing Ingredient: Renormalization (and Updates)

Recent progress:
1705.00246, 1705.11193, 1706.00265, 1706.01295,
1706.05373, 1706.08962, 1707.03107, 1707.07152,
1708.02458, 1708.05301 ...
Renormalization

§ Long-link operator

\[ O_\Gamma(z) = \bar{\psi}(z) \Gamma W_z(z, 0) \psi(0) \]

§ Vector operator mixing with scalar ones

\[
\begin{pmatrix}
O_{\gamma z}^R(z) \\
O_{\Pi}^R(z)
\end{pmatrix} =
\begin{pmatrix}
Z_{VV}(z) & Z_{VS}(z) \\
Z_{SV}(z) & Z_{SS}(z)
\end{pmatrix}
\begin{pmatrix}
O_{\gamma z}(z) \\
O_{\Pi}(z)
\end{pmatrix}
\]

§ RI/MOM renormalization scheme

\[ Z^{-1} = \frac{1}{12e^{-ipzz}} \left( \frac{\text{Tr}[\tilde{\Gamma}\Lambda(p, z, \gamma_z)]}{\text{Tr}[\Lambda(p, z, \gamma_z)]} \frac{\text{Tr}[\tilde{\Gamma}\Lambda(p, z, \Pi)]}{\text{Tr}[\Lambda(p, z, \Pi)]} \right)_{p^2 = \mu_R^2, p_z = P_z} \]

\[ \Lambda(p, z, \Gamma) = S(p)^{-1} \left( \sum_w S^+(p, w + zn) \Gamma W_z(w + zn) S(p, w) \right) S(p)^{-1} \]

projected with \( \tilde{\Gamma} = \gamma / p_z \)

\[ \text{Test case: } a \approx 0.12 \text{ fm, } M_\pi \approx 310 \text{ MeV, clover/HISQ} \]
§ RI/MOM renormalization scheme

$M_\pi \approx 310 \text{ MeV}$

$a \approx 0.12 \text{ fm}$

$h_R = Z_{VV} h_{\gamma z} + Z_{SV} h_{\parallel}$

Plot by Yi-Bo Yang; 1706.01295 (LP³)
§ Operator and mixing effect

Avoid mixing using different op: $h_R = Z_V h_{\gamma t}$

$p_z = \frac{6\pi}{L_S}$

$M_\pi \approx 310$ MeV, $a \approx 0.12$ fm

$h^R_t$: $\Gamma = \gamma_t$ ME

$h^R_z$, no mixing: $\Gamma = \gamma_z$ ME

$h^R_z$, including scalar mixing: $\Gamma = \gamma_z$ ME

Plot by Yi-Bo Yang
§ Effect on quasi-PDFs

\[
\tilde{q}_R(x, P_z, \mu_R) = \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{ixP_zz} \tilde{h}_R(z, P_z, \mu_R)
\]

\[p_z = \frac{4\pi}{L_s}\]

\[
P_z = \frac{6\pi}{L_s}
\]

\[M_\pi \approx 310 \text{ MeV}, \quad a \approx 0.12 \text{ fm}\]

Bare

Renormalized

Jian-Hui Zhang (Regensburg)
§ The problem persists/worsens at physical pion mass

\[ M_\pi \approx 135 \text{ MeV}, \ a \approx 0.09 \text{ fm}, \ L \approx 5.6 \text{ fm} \]
The problem persists/worsens at physical pion mass

\[ M_\pi \approx 135 \text{ MeV}, \ a \approx 0.09 \text{ fm}, \ L \approx 5.6 \text{ fm} \]
Physical Pion Mass

§ Not a lattice problem but Fourier transform issue

§ Simple exercise with CT14 PDF 1506.07443

1708.05301 (LP3) Fixed $L_z = 32$
Physical Pion Mass

§ Not a lattice problem but Fourier transform issue

§ Simple exercise with CT14 PDF 1506.07443

\[ P_z = \frac{4\pi}{L}, \frac{8\pi}{L}, \frac{12\pi}{L}, \frac{24\pi}{L} \]

\[ \chi \] Fixed \( P_z = 24\pi/L \)
Not a lattice problem but Fourier transform issue

Two possible solutions proposed (likely more)

Filter approach

\[
F(z_{\text{lim}}, z_{\text{wid}}) = \frac{1 + \text{erf}(\frac{z + z_{\text{lim}}}{z_{\text{wid}}})}{2} \cdot \frac{1 - \text{erf}(\frac{z - z_{\text{lim}}}{z_{\text{wid}}})}{2}
\]

Derivative approach

\[
q(x) = \int_{-z_{\text{max}}}^{+z_{\text{max}}} \frac{-P_z e^{ixP_z z}}{2\pi i P_z x} h'(z)
\]
§ Not a lattice problem but Fourier transform issue

§ Two possible solutions proposed (likely more)

Filter approach

\[ F(z_{\text{lim}}, z_{\text{wid}}) = \frac{1 + \text{erf}\left(\frac{z + z_{\text{lim}}}{z_{\text{wid}}}\right)}{2} \cdot \frac{1 - \text{erf}\left(\frac{z - z_{\text{lim}}}{z_{\text{wid}}}\right)}{2} \]

Derivative approach

\[ q(x) = \int_{-z_{\text{max}}}^{+z_{\text{max}}} dz \frac{-P_z e^{iP_z z}}{2\pi i P_z x} h'(z) \]

§ Larger momentum production is currently in progress
§ A variation of LaMET: \( \mathcal{P}(x, z^2, \mu, \epsilon) = \int dz (p_z/2\pi) e^{ix \cdot v} h(v, z^2, \mu, \epsilon) \)

§ Versus quasi-PDF \( \tilde{q}(x, p_z, \mu, \epsilon) = \int (dz/2\pi) e^{ix \cdot z} p_z h(zp_z, z^2, \mu, \epsilon) \)

§ Similarity and issues: \( h(v, z^2)/h(0, z^2) = M(v, z^2) \)

§ One of the numerical attractions

\( \Rightarrow \) Similar matrix elements; same problems we have

\( \Rightarrow \) Extension to other structures is not clear yet

Pseudo-PDF

Plot by Yi-Bo Yang

RI/MOM quasi-PDF
Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

22-24 March 2017, Oxford, UK

Discrepancies appear when data is scarce

Many groups have tackled the analysis

CTEQ, MSTW, ABM, JR, NNPDF, etc.


“A first joint workshop with global-fitting community to address key LQCD inputs

http://www.physics.ox.ac.uk/confs/PDFlattice2017

Whitepaper study the needed precision of lattice PDFs in the large-x region

Global Analysis

Discrepancies appear when data is scarce

Many groups have tackled the analysis

CTEQ, MSTW, ABM, JR, NNPDF, etc.

§ Implementing the pseudo-data from LQCD with $x = 0.7 – 0.9$

$$u(x_i, Q^2) - d(x_i, Q^2) \text{ and } \bar{u}(x_i, Q^2) - \bar{d}(x_i, Q^2)$$

Lin et al, In preparation

D: 12%
E: 6%
F: 3%
§ Implementing the pseudo-data from LQCD with $x = 0.7 – 0.9$

$$\Delta u(x_i, Q^2) - \Delta d(x_i, Q^2) \text{ and } \Delta \bar{u}(x_i, Q^2) - \Delta \bar{d}(x_i, Q^2)$$

D: 12%
E: 6%
F: 3%

Lin et al, In preparation
Exciting time for studying structure on the lattice

§ Overcoming longstanding obstacle to full $x$-distribution

☞ Most importantly, this can be done with today’s computers
☞ First lattice approach to study sea asymmetry

§ Moving on to remove the **systematics** of earlier study

☞ progress on renormalization,
further work on, larger momentum boost,
finer lattice spacing, ...
☞ Long-term future for lattice hadronic physics

§ LQCD impacts for current PDFs in the next few years

☞ Combined analysis with precision moments
☞ Large-$x$ isovector PDFs
Overcoming longstanding obstacle to $x$-distribution

New idea by Ji for studying full $x$ dependence of PDFs

Promising results on unpolarized and polarized sea asymmetry compared with experiments, even at non-physical pion mass

Caveats

Not a precision calculation yet

Need to complete the other $p_z$ corrections (on-going; possibly done in a couple weeks)

Systematics due to large momenta (some ideas to improve it)

Need improvement for large $q$ momentum sources

Better overlapping boosted hadron smearing (asymmetric source)

Applications: large $q$ form factors, hadronic and flavor physics, …

Exciting time for hadron structure on the lattice

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful
Backup Slides
Renormalization

§ Long-link operator

\[ O_\Gamma(z) = \bar{\psi}(z) \Gamma W_z(z, 0) \psi(0) \]

§ Vector operator mixing with scalar ones

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§ RI/MOM renormalization scheme

\[ Z^{-1} = \frac{1}{12e^{-ip_{zz}}} \left( \frac{1}{\text{Tr}[\bar{\Lambda}(p, z, \gamma_z)]} \frac{1}{\text{Tr}[\Lambda(p, z, \Pi)]} \right)_{p^2=\mu_R^2, p_z=p_z} \]

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projected with \[ \tilde{\Gamma} = \frac{\gamma}{p_z} \]

\[ \text{Test case: } a \approx 0.12 \text{ fm, } M_\pi \approx 310 \text{ MeV, clover/HISQ} \]
§ RI/MOM renormalization scheme

Momentum source vs point source for $|z| \leq 2$

$M_\pi \approx 310$ MeV

$a \approx 0.12$ fm

$p_z = \frac{6\pi}{L_S}$

$\mu_R^2 = p^2$

$= 5.74$ GeV$^2$

Plot by Yi-Bo Yang; 1706.01295 (LP3)
HI/MOM renormalization scheme

\[ M_\pi \approx 310 \text{ MeV} \]
\[ a \approx 0.12 \text{ fm} \]

\[ p_z = \frac{6\pi}{L_S} \]
\[ \mu_R^2 = p^2 \]
\[ = 5.74 \text{ GeV}^2 \]

\[ h_R = Z_{VV} h_{\gamma z} + Z_{SV} h_{\parallel} \]

Plot by Yi-Bo Yang; 1706.01295 (LP3)
Renormalization

§ Effect on nucleon matrix elements as function of $z$

\[ h_R \approx Z_{VV} h_{\gamma z} \]

\[ p_z = \frac{4\pi}{L_s} \]

\[ p_z = \frac{6\pi}{L_s} \]

\[ M_\pi \approx 310 \text{ MeV}, a \approx 0.12 \text{ fm} \]

Plot by Jianhui Zhang; 1706.01295 (LP3)

Jian-Hui Zhang (Regensburg)