The tamed pion cloud at work

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Motivation

- Light flavor sea asymmetry well-established by experiment
- Most natural explanation is meson cloud
- Results are model-dependent: choice of form factors, coupling constants, parton distributions in mesons

- Experimental results for larger $x$ expected soon; theory should make predictions.
- Can we remove some of the model-dependence?
But $\pi N$ vertex function $G_{\pi N}(t)$ is related to the pion decay constant $f_\pi$ and the nucleon axial form factor by the generalized Goldberger-Treiman relation

$$MG_A(t) = f_\pi G_{\pi N}(t)$$

$G_A$ constrained by $\nu$ experiments; can be used to control $G_{\pi N}$
Nucleon self-energy

∑_{\pi N} determined on light-front, integration over k to project intermediate nucleon on mass shell

Dipole form factor of G_A determines form factor for G_{\pi N}

\[ F(Q^2) = \frac{1}{1 + (Q^2 / M_A^2)^2} \]

axial mass \( M_A = 1.03 \pm 0.04 \) GeV (Thomas & Weise)
Similarly, the pionic coupling between nucleons and $\Delta$ particles has an off-diagonal Goldberger-Treiman relation, determined by the Adler form factor

$$2MC_5^A(t) = f_\pi G_{\pi N\Delta}(t)$$

We determine $\Sigma_\pi(\Delta)$ with the same procedure, to get the total pionic contribution to the nucleon mass

$$\Sigma_\pi = \Sigma_\pi(N) + \Sigma_\pi(\Delta)$$
• light-front determination of pionic contribution to the nucleon self-energy $\Sigma_\pi$

• dependence on pion mass $\mu$ consistent with $\chi$PT for small $\mu$,
  LHP lattice data for larger $\mu$ (approx linear)

Dashed line – $\chi$PT expansion; solid line – our calculation; lattice data PRD 79, 054502 (2009)
Pion content of the nucleon

\[ M_\pi = 2M \frac{\partial \Sigma_\pi}{\partial \mu^2} = \int_0^1 dy f_\pi(y), \quad y = \frac{k^+}{p^+} \]

Splitting functions

\[ f_\pi(y) \equiv f_\pi^N(y) + f_\pi^\Delta(y) \]

shown for \( M_A = 0.99, 1.03, 1.07 \) GeV

virtual pion contribution to DIS and determination of nucleon sea asymmetry
• best determination of violation of Gottfried sum rule (E866)

\[ D \equiv \int_0^1 [\bar{d}(x) - \bar{u}(x)]dx = 0.118 \pm 0.012 \]

• our result, for \( M_A = 1.03 \),

\[ D_\pi = \int_0^1 dy \, y \left[ \frac{2}{3} f_\pi^N (y) - \frac{1}{3} f_\pi^\Delta (y) \right] = 0.109 \]

Pion cloud contribution agrees with integrated asymmetry within experimental error
Flavor asymmetry

- New Muon Collaboration at CERN - DIS on proton and deuteron targets
- range in $x$ was $(0.004, 0.8)$
- NMC found $0.235 \pm 0.026$ at $Q^2 = 4 \text{ GeV}^2$ (1990)

$$\frac{2}{3} (\bar{u} - \bar{d}) = -0.098$$
$$\bar{d} - \bar{u} = 0.147$$

an excess of $d\bar{d}$ pairs in the sea!
Confirmed asymmetry
determined $\bar{d}(x), \bar{u}(x)$

$$\bar{d} = \int_{0}^{1} \bar{d}(x) dx, \quad \bar{u} = \int_{0}^{1} \bar{u}(x) dx$$

$$\bar{d} - \bar{u} = 0.118 \pm 0.012$$

http://p25ext.lanl.gov/e866/e866.html
Light sea flavor asymmetry

- Why a surprise? Expect a symmetric light sea
- Gluon splitting is flavor blind
- Mass difference of u, d too small to explain this
On reflection - not a surprise

Field/Feynman - 1977 – Pauli exclusion principle suppresses uubar pair creation relative to ddbar pairs

Thomas, Miller, Henley – importance of pion cloud

\[ p(uud) \rightarrow n(udd) + \pi^+ (ud\bar{d}) \text{ creates an excess of } \bar{d} \text{ over } \bar{u} \]

Natural to expand proton in terms of a meson cloud

\[ \pi\Delta \text{ intermediate states should also be included} \]
Momentum distribution measurements

• Drell-Yan production of $\mu^+\mu^-$ pairs
  
  – NA51 at CERN (1994) - at $x \sim 0.18$

  – E866/NuSea at Fermilab (1998)
    
    • 800 GeV proton beams
    • $x$ range (0.015, 0.345)

• Hermes - DIS

Asymmetry is $x$ - dependent
\[ \bar{d}(x) - \bar{u}(x) \]

Difference decreases with increasing \( x \)
\[ \frac{\bar{d}(x)}{\bar{u}(x)} \]

Ratio drops below 1 for \( x > 0.3 \) – what mechanism responsible?
Global pdfs differ greatly for $x > 0.2$.
Fit different experiments (BS15 includes no Drell-Yan).
Functional forms of pdfs at starting scale differ.
Bands represent evolution from starting scale to 100 (GeV/c)$^2$
At the starting scale, BS15 uses the Soffer statistical model – 21 parameters

\[
x q^h(x, Q_0^2) = \frac{A_q X_{0q}^h x^{b_q}}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}_q x^{\tilde{b}_q}}{\exp(x/\bar{x}) + 1},
\]

\[
x \tilde{q}^h(x, Q_0^2) = \frac{\tilde{A}_q (X_{0q}^{-h})^{-1} x^{\tilde{b}_q}}{\exp[(x + X_{0q}^{-h})/\bar{x}] + 1} + \frac{\tilde{A}_q x^{\tilde{b}_q}}{\exp(x/\bar{x}) + 1},
\]
Future experiments

E906 at Fermilab:

- 120 GeV, higher $\sigma$
- better statistics, higher $x$

Other experiments:

- DY at J-PARC, 50 GeV
- W production at RHIC
- EIC
Meson Cloud Model

The wave function of the proton is written in terms of a Fock State expansion

\[ |p> = \sqrt{Z} |p>_{\text{bare}} + \sum_{M,B} \int dy \, d^2k_{\perp} \phi_{MB}(y, k^2_{\perp}) |M(y, k_{\perp}) > |B(1 - y, -k_{\perp}) > \]

\( Z \) is a renormalization constant equal to the probability of finding the bare proton.


We include \( \pi N, \pi \Delta, \) and \( \omega N \) states.
Momentum conservation requires

The probability of creating a meson-baryon state is given by the integral of the splitting function:

\[
\int_0^1 dy f_{MB}(y) = <n>_{MB}
\]

Momentum conservation requires

\[
f_{BM}(1 - y) = f_{MB}(y)
\]
contributions to sea distributions

\[ f_{MB} \otimes q_M^f = \int_x^1 \frac{dy}{y} f_{MB}(y) q_M^f(\frac{x}{y}) \]

\[ q_N^f(x) = Z q_{N0}^f(x) + \sum_{B, M} f_{MB} \otimes q_M^f + \sum_{B, M} f_{BM} \otimes q_B^f. \]

\[ f_{\pi^+ n} = \frac{2}{3} f_{\pi N}, \quad f_{\pi^0 p} = \frac{1}{3} f_{\pi N}, \quad f_{\pi^- \Delta^{++}} = \frac{1}{2} f_{\pi \Delta}, \quad f_{\pi^0 \Delta^+} = \frac{1}{3} f_{\pi \Delta}, \quad f_{\pi^+ \Delta^0} = \frac{1}{6} f_{\pi \Delta} \]

\[ \bar{d}(x) = \left( \frac{5}{6} f_{\pi N} + \frac{1}{3} f_{\pi \Delta} + \frac{1}{2} f_{\omega N} \right) \otimes q_v^M + \sum_{B, M} f_{MB} \otimes q_s^M + \sum_{B, M} f_{BM} \otimes q_s^B + Z q_s^N(x) \]

\[ \bar{u}(x) = \left( \frac{1}{6} f_{\pi N} + \frac{2}{3} f_{\pi \Delta} + \frac{1}{2} f_{\omega N} \right) \otimes q_v^M + \sum_{B, M} f_{MB} \otimes q_s^M + \sum_{B, M} f_{BM} \otimes q_s^B + Z q_s^N(x) \]

leading to the asymmetry

\[ \bar{d}(x) - \bar{u}(x) = \left( \frac{2}{3} f_{\pi N} - \frac{1}{3} f_{\pi \Delta} \right) \otimes q_v^M \]
limits as $x \to 1$

As $x \to 1$, meson valence quark distributions dominate

$$\frac{\bar{d}(x)}{\bar{u}(x)} \approx \frac{\left(\frac{5}{6} f_{\pi N} + \frac{1}{3} f_{\pi \Delta} + \frac{1}{2} f_{\omega N}\right) \otimes q_v^M}{\left(\frac{1}{6} f_{\pi N} + \frac{2}{3} f_{\pi \Delta} + \frac{1}{2} f_{\omega N}\right) \otimes q_v^M}$$

The ratio will approach 5, 1/2 or 1 if one splitting function dominates.
Splitting functions

\[ f_{\pi N}(y) = 3M^2 \left( \frac{g_A}{f_\pi} \right)^2 y \int_{M^2y^2/(1-y)}^{\infty} \frac{dt}{16\pi^2} \frac{t}{(t + \mu^2)^2} f^2(t) \]

\[ f_{\Delta}(y) = 2y \left( \frac{g_{\pi N\Delta}}{2M} \right)^2 \frac{\pi}{(2\pi)^3} \frac{2}{3} \int_{y^2M^2+y(M^2_\Delta-M^2))/(1-y)}^{\infty} dt \frac{F^2(-t)}{(t + \mu^2)^2} \]

\[ \times \left( t + \frac{1}{4M^2_\Delta} (M^2 - M^2_\Delta + t)^2 \right) \frac{1}{2} ((M + M_\Delta)^2 + t) \]

\[ f_{\omega N}(y) = \pi g_\omega^2 y \int_{M^2y^2/(1-y)}^{\infty} dt \frac{(2M^2 - t)}{(t + M^2_\omega)^2} F_\omega(-t)^2 \]
$\pi N$ splitting:

- For the upper limit of the band we used $g_{\pi N} = 13.2$ (consistent with scattering data analysis of Perez et al., PRC 95 064001 (2017), and $\Lambda = 1.07$ GeV.

- For the lower limit, $g_{\pi N} = 12.8$ (from Goldberger-Treiman) and $\Lambda = 0.99$ GeV.

- Cutoffs determined by axial mass $M_A = 1.03 \pm 0.04$ GeV.
Splitting functions

$\pi\Delta$ splitting:

- For the upper limit of the band, $g_{\pi\Delta} = 22.4 \ (g_{\pi\Delta} = 1.7 \ g_{\pi N})$ and $\Lambda = 1.07 \ \text{GeV}$.
- For the lower limit, $g_{\pi\Delta} = 19.2 \ (\text{large } N_c \ \text{limit}, \ g_{\pi\Delta} = 1.5 \ g_{\pi N})$ and $\Lambda = 0.99 \ \text{GeV}$.
- Cutoffs determined by axial mass $M_A = 1.03 \pm 0.04 \ \text{GeV}$
For the upper limit of the band, $g^2_{\omega N}/4\pi = 8.1$, Dumbrajs et al., Nucl.Phys. B 216, 277 (1983).

For the lower limit, $g^2_{\omega N}/4\pi = 5.0$, Machleidt & Entem, Phys.Rept. 503, 1 (2011).

Cutoff $\Lambda_\omega = 1.09 \pm 0.05$ GeV (our dipole extraction from Machleidt & Entem analysis)
Peaks in momentum distributions are reasonable.
Parton distributions for the cloud

For our meson pdfs we use the pion pdfs of Aicher, Schafer and Vogelsang, PRL 105, 252003 (2010), evolved from their starting scale to $Q^2 = 54 \text{ GeV}^2$. The valence and sea distributions are given by

$$q^M_v(x) = 1.39 x^{-0.331} (1 - x)^{3.12} (7.18 x^2 + 1)$$

$$q^M_s(x) = 0.115 x^{-1.21} (1 - x)^{5.34} (1 - 2.38 \sqrt{x} + 4.28 x)$$

As pointed out by Holtmann et al., Nucl. Phys. A 596, 631 (1996), the bare proton sea cannot be determined from experimental data, which includes contributions from the meson cloud. They used a suppression factor $R_{sea} = 0.4$ to represent the bare proton sea as a fraction of the total proton sea. Szczurek et al., Nucl.Phys. A 624,495397(1997), used a fit to DIS data that included corrections for the meson cloud to determine the bare proton sea. We use their symmetric sea for the bare proton and the baryons in the cloud:

$$q^\bar{d}_{N0}(x) = q^\bar{u}_{N0}(x) = 0.217 x^{-1} (1 - x)^{15.6} (1. + 0.625 x)$$
composition of cloud: $Z = 0.56$ (bare proton), $\pi N = 0.24$, $\pi \Delta = 0.18$, $\omega N = 0.02$

ASV distributions used for $q_\pi(x/y)$

Bare nucleon sea distributions? Important, because of high probability of hitting this leading order term, and because there is no convolution that shifts the distribution to lower $x$. 
limits as $x \to 1$

As $x \to 1$, meson valence quark distributions dominate. All pdfs are very small, but where is the ratio heading?

\[
\frac{\bar{d}(x)}{\bar{u}(x)} \approx \frac{\left( \frac{5}{6} f_{\pi N} + \frac{1}{3} f_{\pi \Delta} + \frac{1}{2} f_{\omega N} \right)}{\left( \frac{1}{6} f_{\pi N} + \frac{2}{3} f_{\pi \Delta} + \frac{1}{2} f_{\omega N} \right)} \otimes q_v^M
\]

$f_{\pi \Delta}$ dominates

\[
\frac{\bar{d}(x)}{\bar{u}(x)} \to \frac{1}{2}
\]
Effects of Fock state contributions

Each successive term decreases the ratio; sum of all terms – black line

\[
\begin{align*}
\pi N \\
\pi N + N\pi \\
\pi N + N\pi + \pi\Delta \\
\pi N + N\pi + \pi\Delta + \Delta\pi \\
\pi N + N\pi + \pi\Delta + \Delta\pi + \omega N \\
\pi N + N\pi + \pi\Delta + \Delta\pi + \omega N + N\omega \\
\pi N + N\pi + \pi\Delta + \Delta\pi + \omega N + N\omega + \text{bare nucleon}
\end{align*}
\]

\[
\frac{\bar{d}(x)}{u(x)} = \frac{\bar{d}(x) - \bar{u}(x)}{\bar{u}(x)} + 1
\]
Conclusions

• The pion cloud contribution to the nucleon self-energy $\Sigma_{\pi}$ and DIS can be determined, to 2nd-order in the pion-baryon coupling constants, in a model-independent light-front calculation.

• The dependence of $\Sigma_{\pi}$ on the pion mass $\mu$ is consistent with $\chi$PT results for small $\mu$ and lattice results for larger $\mu$.

• The pion contribution to the Gottfried sum rule violation is in agreement with experiment.

• Explanation of the high-$x$ behavior of the light sea distributions is still a challenge. Many terms of the Fock state expansion are required. Careful treatment of form factors and baryon sea pdfs is important.