Double Beta Decay:
Status and connections to lattice QCD

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Weak interactions and nuclear physics:
• introduction to double beta decay
• single nucleon $g_A$ and form factor
• two nucleons: neutrino scattering from deuteron
• light ($A \leq 10$) and heavy nucleon beta decay
• light nuclei and heavy nuclei: neutrino scattering
• double beta decay

Intersections between lattice QCD and many-body
Summary and outlook
B. Neutrinoless Double-Beta Decay

\[ [T_{1/2}^{0\nu}]^{-1} = \sum_{\text{spins}} |Z_{0\nu}|^2 \delta(E_{e1} + E_{e2} + E_f - E_i) \frac{d^3p_1}{2\pi^3} \frac{d^3p_2}{2\pi^3}, \]

\[ m_{\beta\beta} \equiv \sum_k m_k U_{ek}^2 \]

\[ m_{\beta\beta} = |m_1 U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i(\alpha_2 - \alpha_1)} + m_3 |U_{e3}|^2 e^{i(-\alpha_1 - 2\delta)}, \]

\[ \sum_n \left[ \frac{\langle f|J_P^p(q)|n\rangle \langle n|J_P^p(-q)|i\rangle}{|q(E_n + |q| + E_{e2} - E_i)|} + \frac{\langle f|J_P^p(q)|n\rangle \langle n|J_P^p(-q)|i\rangle}{|q(E_n + |q| + E_{e1} - E_i)|} \right] \times 2\pi\delta(E_f + E_{e1} + E_{e2} - E_i), \]
The nuclear shell model is a well-established many-body theory that describes the collective behavior of nucleons in a nucleus. It is based on a Hamiltonian that includes terms for nuclear forces, such as the Reid soft-core potential, and it is formulated using a set of single-particle states, typically those of a harmonic-oscillator potential.

Many-body states are linear combinations of orthogonal Slater determinants of single-particle states, and the model is particularly useful for low-energy nuclear properties. The states of the full-space calculation are reproduced, e.g., those of the interacting boson model (IBM), the interacting boson-phonon model (IBP), and the interacting boson-fermion model (IBF).

The interacting boson model is formulated for a system of interacting bosons, and it is based on the assumption that the system is in a collective state. The model is particularly useful for the study of large nuclei, and it is compared against other models such as the interacting boson-phonon model and the interacting boson-fermion model.

Calculations of nuclear matrix elements at present carry large uncertainties. Matrix elements obtained with different variants of the quasiparticle random-phase approximation (QRPA) have been used to compute single- and two-neutrino decay lifetimes. The strengths and weaknesses of each model (IBM, IBM-2, SM) are reviewed.

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Theory Methods:

Radial Excitations

QRPA

Shell Model

many-body excitations (low \( hbar \omega \))

Everything seems to matter: deformation, n-p pairing, …

Combining methods should be very valuable
Why is this difficult?

- momentum dependence (light neutrino):
  Fermi matrix element: \( (\mathbf{T}_i^+ \mathbf{T}_j^+ / r_{ij}) \)
  Gamow-Teller: \( (\sigma_i \cdot \sigma_j \mathbf{T}_i^+ \mathbf{T}_j^+ / r_{ij}) \)

- low-energy transition to/from explicit states with moderate momentum transfer
- low-energy modes (deformation, etc.) important
- very small fraction (\(< < 1\%\)) of the relevant sum rules
First look at a single weak vertex:

- neutrino-nucleon scattering
- nuclear beta decay
- muon capture
- low-energy neutrino scattering
- quasi-elastic neutrino scattering

Not a lot of data on weak interactions at moderate q

- LSND (stopped pions) on Carbon
- muon capture
- more data would be very valuable
Scattering from a single nucleon

EM (vector) Nucleon Form Factors

Gonzalez-Jimenez, Caballero, Donnelly, Phys. Reports 2013
Axial Form Factor: Deuterium analysis

$$F_A(-Q^2)$$

- $N_a = 4$ z expansion
- $m_A = 1.014(14)$ dipole

$$r_A^2 = 0.46(0.22) \text{ fm}^2$$

Axial form factor from analysis of deuterium data
Meyer, Betancourt, Gran, Hill (2016)
Nucleon axial form factor in lattice QCD

Gupta et al 2017
Nuclear Beta Decay Calculations vs. Experiment

Quenching enters as 4th power in $\beta\beta$ decay:
up to a factor of $(1/1.27)^4 \sim 0.38$

2-Nucleon Currents critical to describe EM data

Form Factors

\[ |F_C(Q)| \]

\[ |F_M(Q)| \]

Magnetic Moments

EM Transitions

Wiringa, Pastore, Schiavilla, et al

NUCLEI
Nuclear Computational Low-Energy Initiative
**Nuclear Beta Decay: Light Nuclei**

*SNPA* currents

- Correlated w.f. (1b+2b)
- Correlated w.f. (1b)
- Simple w.f. (1b)

**chiral EFT currents**

- gfmc (1b)
- gfmc (1b+2b)

### Graphs

- **Ratio to EXPT**
  - $^{10}\text{C} \rightarrow ^{10}\text{B}$
  - $^{7}\text{Be} \rightarrow ^{7}\text{Li}$
  - $^{6}\text{He} \rightarrow ^{6}\text{Li}$
  - $^{3}\text{H} \rightarrow ^{3}\text{He}$

### Key Points

- *moderate quenching in light nuclei: 10-25%*
- *significant reduction from correlations*
- *modest enhancement from 2N currents*
- *much smaller effects than magnetic moments, transitions*
- *good reproduction of experimental results*

*from S. Pastore's talk at this program*
• significant quenching from 2N currents (different resolution in calculation)
• normal-ordering approximation (effective 1-body operator)
• need a consistent picture across momentum, A
Higher momentum transfer: neutrino scattering

Quasielastic Scattering: Sum Rules
Constructive Interference between 1- and 2-body

Large enhancement from combination of initial state

\[ \alpha \sigma_i \cdot k \sigma_i \cdot q (\sigma_j \cdot k)^2 (\tau_i \cdot \tau_j)^2 \psi^2(\pi) \]
Transverse response

Quasi-elastic electron scattering: $^{12}C$

$q = 300$ MeV/c

$q = 500$ MeV/c

Neutral current: sum rules in $^{12}\text{C}$

**Longitudinal**

**Transverse**

Note enhancement in axial charge; expected from non-relativistic nature of 2N currents (as opposed to V)

Lovato, et. al PRL 2014

Single Nucleon currents (open symbols) versus Full currents (filled symbols)
Double Beta Decay: test cases in light nuclei

- Standard light neutrino plus other possible mechanisms
- Note large cancellations in standard axial matrix element
Double Beta Decay: Heavy Nuclei

• compute effective operators in restricted spaces
• extending shell model spaces
• add correlations to DFT and QRPA approaches
• compare to calculations in light nuclei

Improved inputs at higher scale from lattice QCD
Lattice QCD / Nuclear Physics interface
many things well underway

- many things can be compared/contrasted
- strong interactions w/ 2- and 3-nucleons
- electroweak form factors of the nucleon
- inelastic scattering (π production, …)
Two Nucleons: beta decay

Pion mass dependence may be important:

- binding (momentum) of the system
- D-state in the deuteron (analogous to tritium beta decay)
Two Nucleons: double beta decay (2-neutrino)

- pion mass dependence nuclear interaction less important
- binding (momentum) of the system
- nn & pp states essentially s-wave
Inject momentum (neutrino scattering)

• Charge and neutral current processes
• Moderate momentum transfers (1 to 2 lattice units)
• Exclusive final states (?)
• Inclusive final states (?)
LQCD analogue of Euclidean response?

\[ \tilde{R}(q, \tau) = \langle 0 | j^\dagger \exp\left[-(H - E_0 - q^2/(2m))\tau\right] j | 0 \rangle > \]
Double beta decay

• nn to pp different than for real case
• useful to have the matrix element for different $q$
• want to understand pion - light neutrino matrix elements
Summary and Outlook

• $g_A$ quenching should be quantitatively understood
• $g_A$ enhancement (quasi-elastic) is solvable
• requires consistent treatment of interactions & currents and reliable matching to experiment and LQCD

• neutrinoless double beta decay more difficult
• involves multiple length scales that interfere
• quenching likely different in 0 and 2 neutrino cases
• more information needed from theory/lattice (2 weak vertices at NN level)
• more information at different q required
• significant progress by refining and combining methods