Extraction of unpolarized TMD PDFs at NNLO: analysis and result

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in collaboration with I.Scimemi
based on [1706.01473](ver 2)

Spatial and Momentum Tomography of Hadrons and Nuclei
Seattle
Sep.2017
Motivation

The theory of TMD made huge progress in recent years.

General theory

- Proof of collinear part of TMD factorization [Collins,84→11],[Becher,Neubert,Stewart...]
- Proof of rapidity divergences factorization [AV,1707.07606]
- $W$ to $Y$ matching [Collins,at al,1605.00671 ]

Perturbation theory

- TMD evolution kernels (anomalous dimensions)
  - UV evolution: 3 loop [Moch,et al,0505039]
  - rapidity evolution: 3 loop [Li & Zhu,1604.01404; AV,1610.05791]
- Hard coefficient functions: 3 loop [Moch,et al,0505039]
- Matching coefficients to PDFs [many]
- Structure of power suppressed terms of small-b OPE [Scimemi & AV,1609.06047]

Not widely used in TMD phenomenology!
Perturbation theory is important!

- Follow the world!
  All PDF extraction and $\alpha_s$ extraction are made at NNLO and higher. To use it in TMD phenomenology, one should use equivalent perturbative inputs.

- Mixing effects $\Leftrightarrow$ induced flavour dependence
  TMD hard part (and TMD evolution) is flavour-diagonal. All mixing effects comes from the matching, and they are large. (mixing with gluon at NLO, with see at NNLO)

- Quantitative effects
  They could be very large. E.g. DY-normalization: $0.85(\text{LO}) \rightarrow 0.95(\text{NLO}) \rightarrow 0.99(\text{NNLO})$
  Ultimately important for high-energy data, e.g. LHC

- Theoretical (perturbative) uncertainty
  It decreases from order to order. Very significant at low energies, due to large $\alpha_s$.

But requires a lot of work to include

- Involved coding
  TMD cross-section requires MANY different integrations. E.g. to analyse LHC $6(+2)(+2)(x_1, x_2, b, y, Q, p_T(+\text{evolution})(+\text{fid.}))$

- Requires some extra theory studies

- Reanalysis of data
  We cannot study $f_{1T}^{\perp}$ without $d_1$, and $d_1$ without $f_1$
That’s what we do!
That’s what we do!
But we just start.

Status

<table>
<thead>
<tr>
<th>Theory state</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Universal</strong></td>
</tr>
<tr>
<td><strong>Hard part</strong></td>
</tr>
<tr>
<td><strong>Evolution</strong></td>
</tr>
<tr>
<td><strong>Matching</strong></td>
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<tr>
<td>$f_1$</td>
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<tr>
<td>$g_1$</td>
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<tr>
<td>$h_1$</td>
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<tr>
<td>$h_{1T}$</td>
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<tr>
<td>$f_{1T}$</td>
</tr>
<tr>
<td>$h_{1L}$</td>
</tr>
<tr>
<td>$g_{1T}$</td>
</tr>
</tbody>
</table>

- **arTeMiDe** package for evaluation of TMDs and related cross-sections. Ver.1.1 includes $f_1$ (https://teorica.fis.ucm.es/artemide)
- We have extracted $f_1$ from DY and Z-boson production. (Presented here)
That's what we do!
But we just start.

Status

- **arTeMiDe** package for evaluation of TMDs and related cross-sections. Ver.1.1 includes $f_1$
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- We have extracted $f_1$ from DY and Z-boson production. (Presented here)
unpolarized Drell-Yan $\Rightarrow$ unpolarized TMDPDF

**Theory input**

\[
\frac{d\sigma}{dQdyd^2q_T} = H(Q,\mu) \int \frac{d^2b}{(2\pi)^2} e^{-ibq_T} F(x_A, b; \mu, \zeta) F(x_B, b; \mu, \zeta) + Y
\]

\[
F(x, b; \mu, \zeta) = R[b; (\mu, \zeta) \rightarrow (\mu_{low}, \zeta_{\mu})] F_{low}^1(x; b)
\]

\[
F_{low}^1(x, b) = \int_1^x \frac{dy}{y} C_{k \leftarrow l}(y, b; \mu) f_l \left( \frac{x}{y}, \mu \right) f_{NP}(y; b)
\]
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$$F(x, b; \mu, \zeta) = R[b; (\mu, \zeta) \rightarrow (\mu_{\text{low}}, \zeta_{\mu})] F_{\text{low}}(x; b)$$

$$F_{\text{low}}^k(x, b) = \int_x^1 \frac{dy}{y} C_{k-1}(y, b; \mu) f_l \left( \frac{x}{y}, \mu \right) f_{NP}(y; b)$$

We can define four successive orders:

| Name     | $|C_V|^2$ | $C_{f\leftarrow f'}$ | $\Gamma$ | $\gamma V$ | $D$ | PDF set | $a_s$(run) | $\zeta_{\mu}$ |
|----------|----------|----------------------|----------|------------|-----|---------|------------|-------------|
| NLL      | $a_0^s$  | $a_0^s$              | $a_2^s$  | $a_1^s$    | $a_1^s$ | nlo     | nlo        | NLL         |
| NLO      | $a_1^s$  | $a_1^s$              | $a_2^s$  | $a_2^s$    | $a_2^s$ | nlo     | nlo        | NLO         |
| NNLL     | $a_1^s$  | $a_1^s$              | $a_3^s$  | $a_2^s$    | $a_2^s$ | nnlo    | nnlo       | NNLL        |
| NNLO     | $a_2^s$  | $a_2^s$              | $a_3^s$  | $a_3^s$    | $a_3^s$ | nnlo    | nnlo       | NNLO        |
unpolarized Drell-Yan $\Rightarrow$ unpolarized TMDPDF

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pure TMD factorization $\Rightarrow$ small $q_T$

$$F(x, b; \mu, \zeta) = R[b; (\mu, \zeta) \to (\mu_{low}, \zeta_{low})] F_{low}^1(x; b)$$

To fit minory restricted

MHHT2014

We can define four successive orders

| Name   | $|C_V|^2$ | $C_{f \rightarrow f'}$ | $\Gamma$ | $\gamma_V$ | $D$ | PDF set | $a_s$ (run) | $\zeta_{\mu}$ |
|--------|----------|------------------------|----------|------------|----|---------|-------------|-------------|
| NLL    | $a_s^0$  | $a_s^0$                | $a_s^2$  | $a_s^1$   |     | nlo     | nlo         | NLL         |
| NLO    | $a_s^1$  | $a_s^1$                | $a_s^2$  | $a_s^1$   |     | nlo     | nlo         | NLO         |
| NNLL   | $a_s^1$  | $a_s^1$                | $a_s^2$  | $a_s^2$   |     | nnlo    | nnlo        | NNLL        |
| NNLO   | $a_s^2$  | $a_s^2$                | $a_s^3$  | $a_s^2$   |     | nnlo    | nnlo        | NNLO        |
unpolarized Drell-Yan $\Rightarrow$ unpolarized TMDPDF

**Theory input**

$$\frac{d\sigma}{dQ dy d^2 q_T} = H(Q,\mu) \int \frac{d^2 b}{(2\pi)^2} e^{-ibq_T} F(x_A, b; \mu, \zeta) F(x_B, b; \mu, \zeta) + \times$$

pure TMD factorization $\Rightarrow$ small $q_T$

$$F(x, b; \mu, \zeta) = R[b; (\mu, \zeta) \rightarrow (\mu_{\text{low}}, \zeta_{\mu})] F_{\text{low}}(x; b)$$

$F_{\text{low}}(x, b) = \int^1_x \frac{dy}{y} C_{k\rightarrow l}(y, b; \mu) f_l \left(\frac{x}{y}, \mu\right) f_{NP}(y; b)$

We can define four successive orders

| Name   | $|C_V|^2$ | $C_{f \leftarrow f'}$ | $\Gamma$ | $\gamma_V$ | $D$ | PDF set | $a_s$(run) | $\zeta_{\mu}$ |
|--------|---------|-------------------|--------|--------|-----|--------|---------|--------|
| NLL    | $a^0_s$ | $a^0_s$           | $a^2_s$| $a^1_s$| $a^1_s$| nlo    | nlo     | NLL    |
| NLO    | $a^1_s$ | $a^1_s$           | $a^2_s$| $a^1_s$| $a^1_s$| nlo    | nlo     | NLO    |
| NNLL   | $a^1_s$ | $a^1_s$           | $a^2_s$| $a^2_s$| $a^2_s$| nnlo   | nnlo    | NNLL   |
| NNLO   | $a^2_s$ | $a^2_s$           | $a^3_s$| $a^2_s$| $a^2_s$| nnlo   | nnlo    | NNLO   |
Non-perturbative input

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \ll Q^{-1}$</td>
<td>Perturbative</td>
</tr>
<tr>
<td>$b \ll B$</td>
<td>Perturbative</td>
</tr>
<tr>
<td>$b \sim B$</td>
<td>Perturbative</td>
</tr>
<tr>
<td>$b &gt; \Lambda^{-1}$</td>
<td>Non-perturbative</td>
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</tbody>
</table>
Non-perturbative input

<table>
<thead>
<tr>
<th>$b \ll Q^{-1}$</th>
<th>Perturbative</th>
<th>Not observable, deeply in $Y$-term dominated region</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \ll B$</td>
<td>Perturbative</td>
<td>Leading twist contribution $F(x, b) \sim C(x, b) \otimes f(x)$</td>
</tr>
<tr>
<td>$b \sim B$</td>
<td>Perturbative</td>
<td>Higher twist $F(x, b) \sim \sum_n \left(\frac{xb^2}{B^2}\right)^n C_n(x, b) \otimes f_n(x)$</td>
</tr>
<tr>
<td></td>
<td>but not calculable</td>
<td>the main scale parameter is $xb^2$ [I.Scimemi,AV, 1609.06047]</td>
</tr>
<tr>
<td>$b &gt; \Lambda^{-1}$</td>
<td>Non-perturbative</td>
<td>Nothing is know. Exponential? Gaussian?</td>
</tr>
</tbody>
</table>

Additionally, there can be non-perturbative contribution to the rapidity evolution only even powers can appear

$$D(b) = D^{\text{perp}}(b) + g_K b^2 + ...$$

Theory prediction: very small or zero $g_K = 0.01 \pm 0.03 \text{GeV}^2$ [I.Scimemi,AV, 1609.06047]
\[ \ln(\mu^2 b^2), \quad \ln(\zeta b^2) \]

- There are (potentially large) logs of \(b\). Some prescription is needed to handle it.
- Typically, \(b^*\)-prescription used \(\Rightarrow\) induces power corrections and new parameters
- \(\zeta\)-prescription does not introduce any artificial dependence

\(\zeta\)-prescription uses the freedom (granted to us by factorization theorem) to choose the scale in any convenient way.
\[
\ln(\mu^2 b^2), \quad \ln(\zeta b^2)
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- There are (potentially large) logs of $b$. Some prescription is needed to handle it.
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- $\zeta$-prescription does not introduce any artificial dependence

$\zeta$-prescription uses the freedom
(granted to us by factorization theorem)
to choose the scale in any convenient way.

This freedom has not been used yet.
TMD evolution is multi-scale evolution

\[
\mu^2 \frac{dF(x, b; \mu, \zeta)}{d\mu^2} = \frac{1}{2} \gamma K(\mu, \zeta) F(x, b; \mu, \zeta)
\]

\[
\zeta \frac{dF(x, b; \mu, \zeta)}{d\zeta} = -D(\mu, b) F(x, b; \mu, \zeta)
\]

The \((\mu, \zeta)\)-plane has a rich structure
TMD evolution is multi-scale evolution

\[ \mu^2 \frac{dF(x, b; \mu, \zeta)}{d\mu^2} = \frac{1}{2} \gamma_K(\mu, \zeta) F(x, b; \mu, \zeta) \]

\[ \zeta \frac{dF(x, b; \mu, \zeta)}{d\zeta} = -D(\mu, b) F(x, b; \mu, \zeta) \]

The \((\mu, \zeta)\)-plane has a rich structure

There are equi-evolution lines in the \((\mu, \zeta)\)-plane

\[ \mu^2 \frac{dF(x, b; \mu, \zeta(\mu))}{d\mu^2} = 0. \]
\[ F(x, b; Q, Q^2) = R[(Q, Q^2) \rightarrow (\mu_i, \zeta_i)] F(x, b; \mu_i, \zeta_i) \]

Typical choice

\[ \zeta_i = \mu_i^2 \]
\[ F(x, b; Q, Q^2) = R[(Q, Q^2) \to (\mu_i, \zeta_i)]F(x, b; \mu_i, \zeta_i) \]

Typical choice
\[ \zeta_i = \mu_i^2 \]

\( \zeta \)-prescription
\[ \zeta = \zeta(\mu) \text{ equi-evolution line} \]

WARNING: Picture is not entirely correct, because scales depend on \( b \) (i.e. it should be 3D)
\[ F(x, b; Q, Q^2) = R[(Q, Q^2) \rightarrow (\mu_i, \zeta_i)]F(x, b; \mu_i, \zeta_i) \]

**Typical choice**

\[ \zeta_i = \mu_i^2 \]

\[ (Q^2, Q^2), (\mu_1^2, \mu_1^2) \]

\[ \mu_2^2, \mu_2^2 \]

**\(\zeta\)-prescription**

\[ \zeta = \zeta(\mu) \text{ equi-evolution line} \]

\[ (Q^2, Q^2), (\mu_2^2, \zeta(\mu_2)) \]

\[ \mu_2^2, \zeta(\mu_2) \]

\[ \zeta \text{ extra logs} \]
ζ-presentation in practice

In the ζ-presentation a TMD is EVOLUTIONless

\[ \mu^2 \frac{d}{d\mu^2} F(x, b; \mu, \zeta_\mu) = 0 \iff \zeta_\mu = \frac{2\mu}{|b|} e^{-\gamma E} e^{3/2} + \ldots. \]

ζ-presentation eliminates large logs from the expressions. Good example is the coefficient function:

\[ F(x, b; \mu, \zeta) = C(x, b; \mu, \zeta) \otimes f(x, \mu) \]
\( \zeta \)-prescription in practice

In the \( \zeta \)-prescription a TMD is **EVOLUTIONless**

\[
\mu^2 \frac{d}{d\mu^2} F(x, b; \mu, \zeta) = 0 \quad \Leftrightarrow \quad \zeta = \frac{2\mu}{|b|} e^{-\gamma E} e^{3/2 + \ldots}.
\]

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\[
F(x, b; \mu, \zeta) = C(x, b; \mu, \zeta) \otimes f(x, \mu)
\]

\[
C = \delta(\bar{x}) + a_s C_F \left[ -2 \mathbf{L}_\mu p(x) + 2 \bar{x} + \delta(\bar{x}) \left( -\mathbf{L}_\mu^2 + \mathbf{L}_\mu 1_\zeta + 3 \mathbf{L}_\mu - \zeta \right) \right]
\]

usually large

never large

thanks to charge conservation

PT-calculable here \( \text{LO} \)

\( e^{3/2 + \ldots} \).
$\zeta$-prescription in practice

In the $\zeta$-prescription a TMD is EVOLUTIONless

\[
\mu^2 \frac{d}{d\mu^2} F(x, b; \mu, \zeta) = 0 \Leftrightarrow \zeta = \frac{2\mu}{|b|} e^{-\gamma_E} e^{3/2} + \ldots.
\]

$\zeta$-prescription eliminates large logs from the expressions. Good example is the coefficient function:

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F(x, b; \mu, \zeta) = C(x, b; \mu, \zeta) \otimes f(x, \mu)
\]

\[
C = \delta(\bar{x}) + a_s C_F \left[ -2 \left\{ L_\mu p(x) \right\} + 2\bar{x} + \delta(\bar{x}) \left( -L_\mu^2 + L_\mu L_\zeta + 3L_\mu - \zeta \right) \right]
\]

usually large

never large thanks to charge conservation

\[
= 0 \quad \text{at} \quad \zeta = \zeta_2
\]

charge conservation: \( \int_0^1 dx C(x, b) \otimes f(x) = \text{const} \)
ζ-prescription in practice

In the ζ-prescription a TMD is EVOLUTIONless

\[ \mu^2 \frac{d}{d\mu^2} F(x, b; \mu, \zeta_\mu) = 0 \iff \zeta_\mu = \frac{2\mu}{|b|} e^{-\gamma E} e^{3/2+\ldots}. \]

ζ-prescription eliminates large logs from the expressions. Good example is the coefficient function:

\[ F(x, b; \mu, \zeta) = C(x, b; \mu, \zeta) \otimes f(x, \mu) \]

No need for \( b^* \), since there are no large logs.

But \( b^* \) can be used, (we are not).

Our choose the simplest function: \( \mu_{\text{low}} = \mu_{\text{OPE}} = C_0 \left( \frac{1}{b} + 2 \right) \)

charge conservation: \[ \int_0^1 dx C(x, b) \otimes f(x) = \text{const} \]
- FORTRAN 90 code
- Module structure
- Convolutions, evolution (LO,NLO,NNLO)
- Fourier to $q_T$-space, integrations over phase space
- Scale-variation ($\zeta$-prescription)
- User defined PDFs, scales, $f_{NP}$
- Efficient code ($\sim 10^9$ TMDs $\sim 6$ min at NNLO)

Currently ver 1. (soon performance update to ver.1.1)

Available at: https://teorica.fis.ucm.es/artemide

Future plans: add modules for fragmentations, and polarized TMDs
High- & low-energy data are used

- High-energy $\Rightarrow$ precise fixation of asymptotic
- Low-energy $\Rightarrow$ better access to NP structure
- To start with we considered only "well-established" data
- In the final fit $309 = 163 + 146$ points used.

### Included data (at $q_T < 0.2Q$)

<table>
<thead>
<tr>
<th>reaction</th>
<th>$\sqrt{s}$</th>
<th>$Q$</th>
<th>comment</th>
<th>points</th>
</tr>
</thead>
<tbody>
<tr>
<td>E288 $p + Cu \rightarrow \gamma^* \rightarrow \mu\mu$</td>
<td>19.4 GeV</td>
<td>4-9 GeV</td>
<td>norm=0.8</td>
<td>35</td>
</tr>
<tr>
<td>E288 $p + Cu \rightarrow \gamma^* \rightarrow \mu\mu$</td>
<td>23.8 GeV</td>
<td>4-9 GeV</td>
<td>norm=0.8</td>
<td>45</td>
</tr>
<tr>
<td>E288 $p + Cu \rightarrow \gamma^* \rightarrow \mu\mu$</td>
<td>27.4 GeV</td>
<td>4-9 &amp; 11-14 GeV</td>
<td>norm=0.8</td>
<td>66</td>
</tr>
<tr>
<td>CDF+D0 $p + \bar{p} \rightarrow Z \rightarrow ee$</td>
<td>1.8 TeV</td>
<td>66-116 GeV</td>
<td>tiny errors!</td>
<td>44</td>
</tr>
<tr>
<td>CDF+D0 $p + \bar{p} \rightarrow Z \rightarrow ee$</td>
<td>1.96 TeV</td>
<td>66-116 GeV</td>
<td></td>
<td>43</td>
</tr>
<tr>
<td>ATLAS $p + p \rightarrow Z \rightarrow \mu\mu$</td>
<td>7 &amp; 8 TeV</td>
<td>66-116 GeV</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>CMS $p + p \rightarrow Z \rightarrow \mu\mu$</td>
<td>7 &amp; 8 TeV</td>
<td>60-120 GeV</td>
<td></td>
<td>14</td>
</tr>
<tr>
<td>LHCb $p + p \rightarrow Z \rightarrow \mu\mu$</td>
<td>7 &amp; 8 &amp; 13 TeV</td>
<td>60-120 GeV</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>ATLAS $p + p \rightarrow Z/\gamma^* \rightarrow \mu\mu$</td>
<td>8 TeV</td>
<td>46-66 GeV</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>ATLAS $p + p \rightarrow Z/\gamma^* \rightarrow \mu\mu$</td>
<td>8 TeV</td>
<td>116-150 GeV</td>
<td></td>
<td>9</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>309</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
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</table>
Limits of application of TMD factorization $\leftrightarrow$ size of Y-term

$$
\frac{d\sigma}{dQdy d^2q_T} = H(Q, \mu) \int \frac{d^2b}{(2\pi)^2} e^{-ibq_T} F(x_A, b; \mu, \zeta) F(x_B, b; \mu, \zeta) + Y
$$

TMD factorization derived at small $q_T$
the leading correction $\sim q_T^2/Q^2$

![Graph showing the distribution of $d\sigma/dq_T$ vs. $q_T$](image)
Limits of application of TMD factorization ↔ size of $Y$-term

\[
\frac{d\sigma}{dQ dy d^2 q_T} = H(Q, \mu) \int \frac{d^2 b}{(2\pi)^2} e^{-ibq_T} F(x_A, b; \mu, \zeta) F(x_B, b; \mu, \zeta) + Y
\]

We include all points with $q_T < \delta_T Q$

To find the value of $\delta_T$, we check the stability of the fit

- Make fits with increasing $\delta_T$ (0.1→0.3) (165→399 points)
- The value of $\chi^2/d.o.f.$ blows up for $\delta_T$ outside allowed region

No TMD factorization
Scans of $\delta_T$ (E288 not included)

\begin{align*}
\chi^2/\text{d.o.f.} & \quad f_{NP} = e^{-\lambda_1 b^2}, \text{NNLO} \\
\chi^2/\text{d.o.f.} & \quad f_{NP} = \cosh^{-1}(\lambda_1 b), \text{NNLO}
\end{align*}

- $\delta_T < 0.2$ save region,
- $\delta_T < 0.25$ un-save region,
- $\delta_T > 0.25$ TMD factorization does not work.

To be on the save side we used $\delta_T = 0.2$. There are 309 data points.
Scans of $\delta_T$ (E288 not included)

- $\delta_T < 0.2$ save region,
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To be on the save side we used $\delta_T = 0.2$
There are 309 data points
Selection of $f_{NP}$

We have tested many models with different behaviour.

Lessons

- Test at NNLO. Since at NLO (or NLL) all models are equally good/bad.
- High-energy experiments favour Gaussian-like
- Low-energy experiment favour exponent-like
- Need at least 2 parameters (to control $b^2$ correction and the tail)
Selection of $f_{NP}$

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- Need at least 2 parameters (to control $b^2$ correction and the tail)

Best models (2 parameters)$+g_K$

$$f_{NP} = \frac{\cosh(\lambda_2 b)}{\cosh(\lambda_1 b)}$$

$$f_{NP} = \exp \left[ - \frac{z \lambda_2 b^2}{\sqrt{1 + (zb \frac{\lambda_2}{\lambda_1})^2}} \right]$$

both

$$\frac{\chi^2}{dof} \approx 1.2$$

Just as expected from theory!
Perturbative uncertainties within TMD cross-section

There are four perturbative scale entries \( \Rightarrow \) four constants to vary \( \{c_1, c_2, c_3, c_4\} \).

\[
\frac{d\sigma}{dX} = H(c_2 \mu_{\text{hard}}) \left[ R(c_2 \mu_{\text{hard}} \to (c_3 \mu_{\text{low}}, \zeta c_3 \mu; c_1 \mu_0) F(c_4 \mu_{\text{OPE}}, \zeta c_4 \mu) \right]^2
\]
Perturbative uncertainties with in TMD cross-section

There are four perturbative scale entries ⇒ four constants to vary \( \{ c_1, c_2, c_3, c_4 \} \).

\[
\frac{d\sigma}{dX} = H(c_2 \mu_{\text{hard}}) \left[ R(c_2 \mu_{\text{hard}} \rightarrow (c_3 \mu_{\text{low}}, \zeta c_3 \mu; c_1 \mu_0) F(c_4 \mu_{\text{OPE}}, \zeta c_4 \mu) \right]^2
\]
Perturbative uncertainties with in TMD cross-section

There are four perturbative scale entries ⇒ four constants to vary \( \{c_1, c_2, c_3, c_4\} \).

\[
\frac{d\sigma}{dX} = H(c_2 \mu_{\text{hard}}) \left[ R(c_2 \mu_{\text{hard}} \rightarrow (c_3 \mu_{\text{low}}, \zeta c_3 \mu; c_1 \mu_0) F(c_4 \mu_{\text{OPE}}, \zeta c_4 \mu) \right]^2
\]
Perturbative uncertainties

\[ c_1 \rightarrow \text{uncertainty of RAD definition} \quad c_2 \rightarrow \text{uncertainty of hard matching} \quad c_3 \rightarrow \text{uncertainty of small-b matching} \]

\[ \int_{c_1 \mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1 \mu_0) \quad H(c_2 \mu)F(c_2 \mu)F(c_2 \mu) \quad C(c_3 \mu_{\text{low}}) \otimes f(c_3 \mu_{\text{low}}) \]

Total uncertainty is the maximum of three

\[ c_i \in (0.5, 2) \]

High-energy example: ATLAS 8 TeV (best precision)
Perturbative uncertainties

\[ c_1 \rightarrow \text{uncertainty of RAD definition} \quad c_2 \rightarrow \text{uncertainty of hard matching} \quad c_3 \rightarrow \text{uncertainty of small-b matching} \quad \text{Total uncertainty is the maximum of three} \]

\[ \int_{c_1 \mu_0}^{\mu} \Gamma + D_{\text{pert}}(c_1 \mu_0) \quad H(c_2 \mu)F(c_2 \mu)F(c_2 \mu) \quad C(c_3 \mu_{\text{low}}) \otimes f(c_3 \mu_{\text{low}}) \quad c_i \in (0.5, 2) \]

High-energy example: ATLAS 8 TeV (best precision)
Perturbative uncertainties

\[ c_1 \rightarrow \text{uncertainty of RAD definition} \]
\[ \int_{c_1 \mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1 \mu_0) \]
\[ c_2 \rightarrow \text{uncertainty of hard matching} \]
\[ H(c_2 \mu) F(c_2 \mu) F(c_2 \mu) \]
\[ c_3 \rightarrow \text{uncertainty of small-b matching} \]
\[ C(c_3 \mu_{\text{low}}) \otimes f(c_3 \mu_{\text{low}}) \]
\[ c_i \in (0.5, 2) \]

High-energy example: ATLAS 8 TeV (best precision)
Perturbative uncertainties

\[ c_1 \rightarrow \text{uncertainty of RAD definition} \]
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\[ \int_{c_1 \mu_0}^{\mu} \Gamma + \mathcal{D}_{\text{pert}}(c_1 \mu_0) \quad H(c_2 \mu) F(c_2 \mu) F(c_2 \mu) \quad C(c_3 \mu_{\text{low}}) \otimes f(c_3 \mu_{\text{low}}) \]

Total uncertainty is the maximum of three

\[ c_i \in (0.5, 2) \]

High-energy example: ATLAS 8 TeV (best precision)
Perturbative uncertainties

\[ \int_{c_1 \mu_0}^{\mu} \Gamma + \mathcal{D}_{\text{pert}}(c_1 \mu_0) \]

- \( c_1 \rightarrow \) uncertainty of RAD definition
- \( c_2 \rightarrow \) uncertainty of hard matching
- \( c_3 \rightarrow \) uncertainty of small-b matching
- Total uncertainty is the maximum of three

Low-energy example: \( E288 \sqrt{s} = 19.4 \text{ GeV}, Q = 4 - 5 \text{ GeV} \)
Perturbative uncertainties

\[ c_1 \rightarrow \text{uncertainty of RAD definition} \]

\[ \int_{c_1 \mu_0}^{\mu} \Gamma + \mathcal{D}_{\text{pert}}^{c_1 \mu_0} \]

\[ c_2 \rightarrow \text{uncertainty of hard matching} \]

\[ H(c_2 \mu) F(c_2 \mu) F(c_2 \mu) \]

\[ c_3 \rightarrow \text{uncertainty of small-b matching} \]

\[ C(c_3 \mu_{\text{low}}) \otimes f(c_3 \mu_{\text{low}}) \]

Total uncertainty is the maximum of three

\[ c_i \in (0.5, 2) \]

Low-energy example: E288 \( \sqrt{s} = 19.4 \text{ GeV}, \ Q = 4 - 5 \text{ GeV} \)
Perturbative uncertainties

\[ c_1 \rightarrow \text{uncertainty of RAD definition} \]
\[ \int_{c_1 \mu_0}^\mu \Gamma + \mathcal{D}^{\text{pert}}(c_1 \mu_0) \]
\[ c_2 \rightarrow \text{uncertainty of hard matching} \]
\[ H(c_2 \mu)F(c_2 \mu)F(c_2 \mu) \]
\[ c_3 \rightarrow \text{uncertainty of small-b matching} \]
\[ C(c_3 \mu_{\text{low}}) \otimes f(c_3 \mu_{\text{low}}) \]

Total uncertainty is the maximum of three
\[ c_i \in (0.5, 2) \]

Low-energy example: E288 \(\sqrt{s} = 19.4\) GeV, \(Q = 4 - 5\) GeV

\[ \frac{d\sigma}{du}(\text{GeV}^{-1}) \]

\[ \frac{q_T}{(\text{GeV})} \]

\[ \text{E288(200) 5-6GeV NNLL variation} \]
\[ \text{E288(200) 5-6GeV NNLL variation} \]
\[ \text{E288(200) 5-6GeV NNLL variation} \]
\[ \text{E288(200) 5-6GeV NNLL variation} \]
\[ \text{E288(200) 5-6GeV NNLL max uncertainty} \]
Perturbative uncertainties

\[ c_1 \rightarrow \text{uncertainty of RAD definition} \]
\[ c_2 \rightarrow \text{uncertainty of hard matching} \]
\[ c_3 \rightarrow \text{uncertainty of small-b matching} \]

Total uncertainty is the maximum of three
\[ c_i \in (0.5, 2) \]

Low-energy example: E288 \( \sqrt{s} = 19.4 \text{ GeV}, Q = 4 - 5 \text{ GeV} \)
Uncertainties in TMD

\[ f_1(x, b) \]

\[ x = 10^{-2} \]

Band is obtained by \( c_4 \) variation.
Uncertainties in TMD

\[ f_1(x, q) \]

\[ x = 10^{-2} \]

Band is obtained by $c_4$ variation

NNLO

NLL

$1$
Results of fit

<table>
<thead>
<tr>
<th></th>
<th>$\chi^2$/dof</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$gK$</th>
<th>norm</th>
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<tbody>
<tr>
<td>NLL</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>NLO</td>
<td>1.18</td>
<td>0.20</td>
<td>0.43</td>
<td>0.021</td>
<td>$\sim 0.94$</td>
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<tr>
<td>NNLL</td>
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<td>0.17</td>
<td>1.30</td>
<td>0.012</td>
<td>$\sim 0.97$</td>
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<tr>
<td>NNLO</td>
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<td>0.244</td>
<td>0.307</td>
<td>0.006</td>
<td>$\sim 0.99$</td>
</tr>
</tbody>
</table>

$u$, $d$, $\bar{u}$, $\bar{d}$

$A.Vladimirov$

NNLO, $x = 10^{-1}$
ATLAS 7TeV
model 2 NNLO
$\chi^2$/points=2.28
N=1.00

ATLAS 8TeV
model 2 NNLO
$\chi^2$/points=2.80
N=0.97

CMS 7TeV
model 2 NNLO
$\chi^2$/points=1.36
$\sigma$=387 pb

CMS 8TeV
model 2 NNLO
$\chi^2$/points=1.58
$\sigma$=429 pb
ATLAS 7TeV model 2 NNLO
\( \chi^2/\text{points} = 2.28 \),
\( N = 1.00 \)

ATLAS 8TeV model 2 NNLO
\( \chi^2/\text{points} = 2.80 \),
\( N = 0.97 \)

CMS 7TeV model 2 NNLO
\( \chi^2/\text{points} = 1.36 \),
\( \sigma = 387 \text{ pb} \)

CMS 8TeV model 2 NNLO
\( \chi^2/\text{points} = 1.58 \),
\( \sigma = 429 \text{ pb} \)

\( \sigma^{-1} d\sigma/dq_T \) at various values of \( q_T \):

- ATLAS 7TeV, \( \chi^2/\text{points} = 2.28 \),
  \( N = 1.00 \)

- ATLAS 8TeV, \( \chi^2/\text{points} = 2.80 \),
  \( N = 0.97 \)

- CMS 7TeV, \( \chi^2/\text{points} = 1.36 \)
  \( \sigma = 387 \text{ pb} \)

- CMS 8TeV, \( \chi^2/\text{points} = 1.58 \)
  \( \sigma = 429 \text{ pb} \)

\( \sigma^{-1} d\sigma/dq_T \) at various values of \( q_T \):

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  \( N = 0.97 \)

- CMS 7TeV, \( \chi^2/\text{points} = 1.36 \)
  \( \sigma = 387 \text{ pb} \)

- CMS 8TeV, \( \chi^2/\text{points} = 1.58 \)
  \( \sigma = 429 \text{ pb} \)
$d\sigma/dq_T \ [pb \cdot GeV^{-1}]$ theory/data
$q_T \ [GeV]$
CDF run 1
model 2 NNLO
χ²/points = 0.64

CDF run 2
model 2 NNLO
χ²/points = 1.33

D0 run 1
model 2 NNLO
χ²/points = 0.62

D0 run 2
model 2 NNLO
χ²/points = 2.78

σ - 1dσ/dq_T [fb · GeV⁻¹]

q_T [GeV]

1.1 1.0 0.9 1.1 1.0 0.9

0.02 0.04 0.06 0.08 0.10

5 10 15 20 25 5 10 15 20 25
Conclusion

- Perturbative input significantly affect extraction of non-perturbative part.
- At least NLO is needed (better go NNLO).
- $\zeta$-prescription (as a clever distribution of logs between parts of factorization theorem) help to reduce the theory uncertainty.

Ongoing work/Future plans

- Include SIDIS
- arTeMiDe updates
- Polarized TMDs.