Bjorken-\(x\) Dependent

Hadron Structure from LQCD

Huey-Wen Lin
PDFs are universal quark/gluon distributions of nucleon

- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

Imaging of the proton

How are the *sea* quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701
PDFs are universal quark/gluon distributions of nucleon

- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

Important inputs to discern new physics at LHC

- Currently dominate errors in Higgs production

(J. Campbell, HCP2012)
Global Analysis

§ Experiments cover diverse kinematics of parton variables

📈 Global analysis takes advantage of all data sets

- Theory Input
- Expt Input

Global Analysis of PDFs

§ Some choices made for the analysis

🔍 Choice of data sets and kinematic cuts
🔍 Strong coupling constant $\alpha_s(M_Z)$
🔍 How to parametrize the distribution

$$xf(x, \mu_0) = a_0 x^{a_1} (1 - x)^{a_2} P(x)$$

🔍 Assumptions imposed

SU(3) flavor symmetry, charge symmetry, strange and sea distributions

$$s = \tilde{s} = \kappa(\bar{u} + \bar{d})$$

Huey-Wen Lin — Hadron Imaging at Jlab and future EIC
Global Analysis

Discrepancies appear when data is scarce
Many groups have tackled the analysis

CTEQ, MSTW, ABM, JR, NNPDF, etc.

What can we do on the lattice?
PDFs on the Lattice

§ Lattice calculations rely on operator product expansion, only provide moments

\[ \langle x^n \rangle_q = \int_{-1}^{1} dx \ x^n q(x) \]

\[ \langle x^n \rangle_{\Delta q} = \int_{-1}^{1} dx \ x^n \Delta q(x) \]

\[ \langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx \ x^n \delta q(x) \]

True distribution can only be recovered with all moments

Quark density/unpolarized

Helicity

longitudinally polarized

Transversity

transversely polarized

most well known

very poorly known

Huey-Wen Lin — Hadron Imaging at Jlab and future EIC
For higher moments, ops mix with lower-dimension ops. Renormalization is difficult too.

Relative error grows in higher moments. Calculation would be costly and difficult.

Problem with Moments

Dolgov et al. PRD66, 034506 (2002)
Göckeler et al. PRD71, 114511 (2005)

\[ \langle x^2 \rangle_q \]

LHPC (SCRI, SESAM): 2f, Wilson and clover
QCDSF: 0f

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Problem with Moments

§ For higher moments, ops mix with lower-dimension ops
.flip
Renormalization is difficult too

§ Relative error grows in higher moments
.flip
Calculation would be costly and difficult

Dolgov et al. PRD66, 034506 (2002)
Göckeler et al. PRD71, 114511 (2005)

LHPC (SCRI, SESAM): 2f, Wilson and clover
QCDSF: 0f
§ FLAG rating system

§ New: excited-state rating

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>Ref.</th>
<th>$N_f$</th>
<th>chiral extrapolation</th>
<th>continuum extrapolation</th>
<th>finite volume</th>
<th>excited state</th>
<th>renormalization</th>
<th>$g_T$</th>
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<tbody>
<tr>
<td>PNDME'15</td>
<td>This work</td>
<td>P 2+1+1</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>1.020(76)$^a$</td>
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<tr>
<td>ETMC'13</td>
<td>[30]</td>
<td>C 2+1+1</td>
<td>■</td>
<td>○</td>
<td>○</td>
<td>■</td>
<td>■</td>
<td>1.11(3)$^b$</td>
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<tr>
<td>LHPC’12</td>
<td>[28]</td>
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<td>★</td>
<td>○</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>1.037(20)$^c$</td>
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<tr>
<td>RBC/UKQCD’10</td>
<td>[29]</td>
<td>A 2+1</td>
<td>○</td>
<td>■</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>1.10(7)$^d$</td>
</tr>
<tr>
<td>RQCD’14</td>
<td>[31]</td>
<td>P 2</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>1.005(17)(29)$^e$</td>
</tr>
<tr>
<td>ETMC’13</td>
<td>[30]</td>
<td>C 2</td>
<td>■</td>
<td>■</td>
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<td>1.114(46)$^f$</td>
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<td>RBC’08</td>
<td>[32]</td>
<td>P 2</td>
<td>■</td>
<td>■</td>
<td>■</td>
<td>■</td>
<td>■</td>
<td>0.93(6)$^g$</td>
</tr>
</tbody>
</table>

$m_T$

$m_A$

PNDME, 1506.06411; 1606.07049

Huey-Wen Lin — Hadron Imaging at Jlab and future EIC
State-of-the-Art Moments

§ Improved transversity distribution with LQCD $g_T$

※ Global analysis with 12 extrapolation forms, gives $g_T = 1.058$

※ Use to constrain the global analysis fits SIDIS $\pi^\pm$ production data from proton and deuteron targets

Lin, Melnitchouk, Prokudin, Sato, In preparation
State-of-the-Art Moments

FLAG-like rating system
Community averaging quantities

White paper in progress with representatives from each collaboration

Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

22-24 March 2017, Oxford, UK

§ Precision moments can be useful to improve PDFs!

☞ Inputs as constraint in global analysis, like $gA$

§ Whitepaper will

☞ Address precision needed for moments and their impacts
☞ Encourage more precision moment calculations in LQCD

Lin et al, In preparation
A Promising New Direction

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)
A New Direction

Large-Momentum Effective Theory for PDFs

1) Calculate nucleon matrix elements on the lattice (z dependence)

\[ p(P_z) \]

\[ Q^2 = 0 \quad \Gamma \in \{ \gamma_z, \gamma_z \gamma_5, \gamma_z \gamma_\perp \gamma_5 \} \]

\[ P_z \in \{0.43, 0.86, 1.29\} \text{ GeV} \]
Large-Momentum Effective Theory for PDFs

2) Compute quasi-distribution via

\[ \tilde{q}(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left( P \left| \bar{\psi}(z) \right\rangle \Gamma \exp \left( -ig \int_0^z dz' A_z(z') \right) \psi(0) \right| P \]
A New Direction

Large-Momentum Effective Theory for PDFs

3) Recover true distribution (take $P_z \to \infty$ limit)

$\tilde{q}(x, \mu, P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z \left( \frac{x}{y}, \frac{\mu}{P_z} \right) q(y, \mu) + O\left(\frac{M_N^2}{P_z^2}\right) + \cdots$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Removing $O(M_N^n / P_z^n)$ errors + $O(\alpha_s)$

Corrected distributions from the largest 2 $P_z$ show signs of convergence

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4) Remove the leading high-twist effect $\left(\frac{\Lambda_{QCD}^2}{P_z^2}\right)$

- $N_f = 2+1+1$ clover/HISQ lattices (MILC)
- $M_\pi \approx 310$ MeV, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$), $O(10^3)$ measurements
Sea Flavor Asymmetry

First time in LQCD history to study antiquark distribution!

\[ M_\pi \approx 310 \text{ MeV}, \ a \approx 0.12 \text{ fm} \]

\( \bar{q}(x) = -q(-x) \)

Lost resolution in small-\( x \) region

Future improvement: larger lattice volume

\[ \int \! dx \ (\bar{u}(x) - \bar{d}(x)) \approx -0.16(7) \]

<table>
<thead>
<tr>
<th>Experiment</th>
<th>( x ) range</th>
<th>( \int_0^1 !d(x) - \bar{u}(x)\int d(x)dx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E866</td>
<td>( 0.015 &lt; x &lt; 0.35 )</td>
<td>( 0.118 \pm 0.012 )</td>
</tr>
<tr>
<td>NMC</td>
<td>( 0.004 &lt; x &lt; 0.80 )</td>
<td>( 0.148 \pm 0.039 )</td>
</tr>
<tr>
<td>HERMES</td>
<td>( 0.020 &lt; x &lt; 0.30 )</td>
<td>( 0.16 \pm 0.03 )</td>
</tr>
</tbody>
</table>

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
Sea Flavor Asymmetry

§ First time in LQCD history to study antiquark distribution!

\[ M_\pi \approx 310 \text{ MeV}, \quad a \approx 0.12 \text{ fm} \]

HWL et al. 1402.1462

\[ \bar{q}(x) = -q(-x) \]

Lost resolution in small-x region

A milestone for lattice QCD!

First sea flavor asymmetry ever calculated!

\[ \int_{-0.16(7)}^{0.16(7)} \]

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
§ Lattice exploratory study

\[ M_\pi \approx 310 \text{ MeV}, \ a \approx 0.12 \text{ fm} \]

Compared with E866

Too good to be true?

Lost resolution in small-\( x \) region

Similar results repeated by ETMC, at \( M_\pi \approx 373 \) MeV

ETMC, 1504.07455

Sea Flavor Asymmetry

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
Exploratory study \( M_{\pi} \approx 310 \text{ MeV} \)

- We see polarized “sea asymmetry” \( \int dx (\Delta \bar{u}(x) - \Delta \bar{d}(x)) \approx 0.14(9) \)
- Both STAR and PHENIX at RHIC see \( \Delta \bar{u} > \Delta \bar{d} \)
- 1404.6880 and 1504.07451
- Other experiments, Fermilab DY exp’ts (E1027/E1039), future EIC

Removing \( O(M_N^n/P_z^n) + O(\Lambda_{QCD}^2/P_z^2) \) errors
Exploratory study

$M_\pi \approx 310 \text{ MeV}$

Transversity Distribution

$\delta \bar{q}(x) = -\delta q(-x)^x$

Removing $O(M^n_N/P^n_z) + O(\Lambda^2_{QCD}/P^2_z)$ errors

We found sea asymmetry of

$\int dx \left( \delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.10(8)$

Chiral quark-soliton model

$\int dx \left( \delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.082$

P. Schweitzer et al., PRD 64, 034013 (2001)

SoLID at JLab, Drell-Yan exp’t at FNAL (E1027+E1039), EIC, ...
Missing Ingredient: Renormalization (and Updates)

Recent progress:
1705.00246, 1705.11193, 1706.00265, 1706.01295,
1706.05373, 1706.08962, 1707.03107, 1707.07152,
1708.02458, 1708.05301 ...
Renormalization

§ Effect on quasi-PDFs

\[ \tilde{q}_R(x, P_z, \mu_R) = \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{ixP_zz} \tilde{h}_R(z, P_z, \mu_R) \]

\[ M_\pi \approx 310 \text{ MeV}, \quad a \approx 0.12 \text{ fm} \]

\[ p_z = \frac{4\pi}{L_s} \]

\[ p_z = \frac{6\pi}{L_s} \]

\[ \nu - d \]

\[ x \]

Bare

RI/MOM renormalized

Plot by Jianhui Zhang

§ Avoid mixing using different op for quark distribution

\[ \Upsilon h_R = Z_V h_{\gamma_t} \]
§ The problem persists/worsens at physical pion mass

\[ M_\pi \approx 135 \text{ MeV}, \ a \approx 0.09 \text{ fm}, \ L \approx 5.6 \text{ fm} \]
Physical Pion Mass

The problem persists/worsens at physical pion mass

\[ M_\pi \approx 135 \text{ MeV}, \ a \approx 0.09 \text{ fm}, \ L \approx 5.6 \text{ fm} \]

J. Green et al (ETMC)

\[ M_\pi \approx 375 \text{ MeV} \]
Physical Pion Mass

§ Not a lattice problem but Fourier transform issue
§ Simple exercise with CT14 PDF

\[ P_z = \frac{4\pi}{L} \]
\[ P_z = \frac{8\pi}{L} \]
\[ P_z = \frac{12\pi}{L} \]
\[ P_z = \frac{24\pi}{L} \]

1708.05301 (LP3) Fixed \( L_z = 32 \)
Physical Pion Mass

§ Not a lattice problem but Fourier transform issue
§ Simple exercise with CT14 PDF 1506.07443

\[ P_z = \frac{4\pi}{L} \]

\[ \frac{8\pi}{L} \]

\[ \frac{12\pi}{L} \]

\[ \frac{24\pi}{L} \]

\[ \chi \]

Fixed \( P_z = 24\pi/L \)

1708.05301 (LP3)
Physical Pion Mass

§ Not a lattice problem but Fourier transform issue
§ Two possible solutions proposed (likely more)

Filter approach

\[
F(z_{\text{lim}}, z_{\text{wid}}) = \frac{1 + \text{erf}\left(\frac{z + z_{\text{lim}}}{z_{\text{wid}}}\right)}{2} \frac{1 - \text{erf}\left(\frac{z - z_{\text{lim}}}{z_{\text{wid}}}\right)}{2}
\]

Derivative approach

\[
q(x) = \int_{-z_{\text{max}}}^{+z_{\text{max}}} dz \frac{-P_z e^{iP_z z}}{2\pi} \frac{1}{iP_z x} h'(z)
\]

\[
F_{1708.05301 (LP3)}
\]

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Physical Pion Mass

§ Not a lattice problem but Fourier transform issue

§ Two possible solutions proposed (likely more)

Filter approach

\[
F(z_{\text{lim}}, z_{\text{wid}}) = \frac{1 + \text{erf}\left(\frac{z + z_{\text{lim}}}{z_{\text{wid}}}\right)}{2} - \frac{1 - \text{erf}\left(\frac{z - z_{\text{lim}}}{z_{\text{wid}}}\right)}{2}
\]

Derivative approach

\[
q(x) = \int_{-z_{\text{max}}}^{+z_{\text{max}}} dz \frac{-P_z \, e^{iP_z z}}{2\pi i P_z x} h'(z)
\]

§ Larger momentum production is currently in progress
Discrepancies appear when data is scarce. Many groups have tackled the analysis, including CTEQ, MSTW, ABM, JR, NNPDF, etc. 


A first joint workshop with the global-fitting community to address key LQCD inputs.

http://www.physics.ox.ac.uk/confs/PDFlattice2017

Whitepaper study the needed precision of lattice PDFs in the large-\(x\) region.
Implementing the pseudo-data from LQCD with $x=0.7–0.9$

$u(x_i, Q^2) - d(x_i, Q^2)$ and $\bar{u}(x_i, Q^2) - \bar{d}(x_i, Q^2)$

Lin et al, In preparation
§ Implementing the pseudo-data from LQCD with $x=0.7–0.9$

$$\Delta u(x_i, Q^2) - \Delta d(x_i, Q^2) \text{ and } \Delta \bar{u}(x_i, Q^2) - \Delta \bar{d}(x_i, Q^2)$$

\[\delta(\Delta \bar{u}) \text{ at } Q^2=4 \text{ GeV}^2, \text{ NNPDFpol1.1}\]

\[\delta(\Delta \bar{d}) \text{ at } Q^2=4 \text{ GeV}^2, \text{ NNPDFpol1.1}\]

D: 12%
E: 6%
F: 3%

Lin et al, In preparation
Exciting time for studying structure on the lattice

§ Overcoming longstanding obstacle to full $x$-distribution

☞ Most importantly, this can be done with today’s computers
☞ First lattice approach to study sea asymmetry
☞ First look into PDA 1702.00008

§ Moving on to remove the systematics of earlier study

☞ Working on renormalization, statistics (all-mode averaging?), larger momentum boost, finer lattice spacing, ...
☞ Long-term future for lattice hadronic physics

§ Small-$x$ physics for EIC

☞ Combined analysis with precision moments

Summary & Outlook

Huey-Wen Lin — Hadron Imaging at Jlab and future EIC
Overcoming longstanding obstacle to $x^*$-distribution

New idea by Ji for studying full $x$ dependence of PDFs

Promising results on unpolarized and polarized sea asymmetry compared with experiments, even at non-physical pion mass

Caveats

Not a precision calculation yet

Need to complete the other $p_z$ corrections (on-going; possibly done in a couple weeks)

Systematics due to large momenta (some ideas to improve it)

Need improvement for large $-q$ form factors, hadronic and flavor physics, ...

Exciting time for hadron structure on the lattice

Huey-Wen Lin — Hadron Imaging at Jlab and future EIC
Backup Slides
§ Long-link operator

\[ O_\Gamma(z) = \bar{\psi}(z) \Gamma W_z(z, 0) \psi(0) \]

§ Vector operator mixing with scalar ones

\[
\begin{pmatrix}
O_{\gamma z}^R(z) \\
O_{\Pi}^R(z)
\end{pmatrix} = 
\begin{pmatrix}
Z_{VV}(z) & Z_{VS}(z) \\
Z_{SV}(z) & Z_{SS}(z)
\end{pmatrix}
\begin{pmatrix}
O_{\gamma z}(z) \\
O_{\Pi}(z)
\end{pmatrix}
\]

§ RI/MOM renormalization scheme

\[
Z^{-1} = 
\frac{1}{12 e^{-i p_z z}} \begin{pmatrix}
\text{Tr}[\tilde{\Gamma} \Lambda(p, z, \gamma_z)] & \text{Tr}[\tilde{\Gamma} \Lambda(p, z, \Pi)] \\
\text{Tr}[\Lambda(p, z, \gamma_z)] & \text{Tr}[\Lambda(p, z, \Pi)]
\end{pmatrix}
p^2 = \mu_R^2, p_z = p_z
\]

\[
\Lambda(p, z, \Gamma) = S(p)^{-1} \left( \sum_w S^+(p, w + zn) \Gamma W_z(w + zn) S(p, w) \right) S(p)^{-1}
\]

projected with \( \tilde{\Gamma} = \phi/p_z \)

\[ \text{Test case: } a \approx 0.12 \text{ fm, } M_\pi \approx 310 \text{ MeV, clover/HISQ} \]
§ RI/MOM renormalization scheme

Momentum source vs point source for $|z| \leq 2$

\[ M_\pi \approx 310 \text{ MeV} \]
\[ a \approx 0.12 \text{ fm} \]

\[ p_z = \frac{6\pi}{L_S} \]
\[ \mu_R^2 = p^2 = 5.74 \text{ GeV}^2 \]

Plot by Yi-Bo Yang; 1706.01295 (LP3)
§ RI/MOM renormalization scheme

\[ M_\pi \approx 310 \text{ MeV} \]
\[ a \approx 0.12 \text{ fm} \]

\[ p_z = \frac{6\pi}{L_s} \]
\[ \mu_R^2 = p^2 = 5.74 \text{ GeV}^2 \]

\[ h_R = Z_{VV} h_{\gamma_z} + Z_{SV} h_\parallel \]

Plot by Yi-Bo Yang; 1706.01295 (LP3)
§ Effect on nucleon matrix elements as function of \( z \)

\[ h_R \approx Z_{VV} h_{\gamma z} \]

\[ p_z = 4\pi/L_s \]

\[ p_z = 6\pi/L_s \]

\[
\begin{align*}
M_\pi &\approx 310 \text{ MeV, } a \approx 0.12 \text{ fm} \\
M_\pi &\approx 310 \text{ MeV, } a \approx 0.12 \text{ fm}
\end{align*}
\]

Plot by Jianhui Zhang; 1706.01295 (LP3)