Baryon spectrum and structure, nucleon Compton scattering

Gernot Eichmann

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Seattle, WA, USA
Why?

QCD Lagrangian: \[ \mathcal{L} = \bar{\psi} (\bar{\psi} + igA + m) \psi + \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \]

- if it only were that simple...
  - we don’t measure quarks and gluons, but **hadrons**

mesons  

baryons  

mesons  

baryons

- origin of **mass generation** and **confinement**?

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>d</th>
<th>s</th>
<th>c</th>
<th>b</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current mass [GeV]</td>
<td>0.003</td>
<td>0.005</td>
<td>0.1</td>
<td>1</td>
<td>4</td>
<td>175</td>
</tr>
<tr>
<td>„Constituent“ mass [GeV]</td>
<td>0.35</td>
<td>0.35</td>
<td>0.5</td>
<td>1.5</td>
<td>4.5</td>
<td>175</td>
</tr>
</tbody>
</table>

- need to understand **spectrum and interactions**!
Compton scattering

- **Two-photon corrections to form factors:**
  can explain difference between Rosenbluth and polarization transfer measurements

- **Proton radius puzzle:**
  can TPE explain discrepancy between $e$ & $\mu$ measurements? So far: probably not, but . . .

- **Nucleon polarizabilities:**
  efforts from ChPT & dispersion relations
  Hagelstein, Miskimen, Pascalutsa, Prog. Part. Nucl. Phys. 88 (2016)
Compton scattering

- **Forward limit:**
  determined by photoabsorption cross section and nucleon structure functions

- **Virtual CS:** generalized polarizabilities,
  DVCS: factorization & handbag dominance, extraction of GPDs

- **Real CS:** dominant quark-level mechanism in WACS?

- **Timelike CS:**
  p\bar{p} annihilation @ PANDA

### Diagrams

- [Compton scattering diagram](image_url)
- [Forward limit illustration](image_url)
- [Virtual CS illustration](image_url)
- [Real CS illustration](image_url)
- [Timelike CS illustration](image_url)

Hamilton et al., PRL 94 (2005)
Compton scattering ...

Compton amplitude = sum of **Born terms + 1PI structure part:**

\[ \text{Born terms: determined by nucleon form factors} \]

\[ \text{Polarizabilities: structure information} \]

\[ + \]

\[ \text{t-channel meson exchange } (\pi, \sigma, a_1, \ldots) \]

\[ + \]

\[ \text{s/u-channel nucleon resonances } (\Delta, N^*, \ldots) \]

**"Pion cloud" (ChPT)**

but also:

\[ \Rightarrow \text{is there a common underlying quark-level description?} \]

# The hadron zoo

## Mesons

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>$0^-$</th>
<th>$0^+$</th>
<th>$1^-$</th>
<th>$1^+$</th>
<th>$1^-$</th>
<th>$2^+$</th>
<th>$2^+$</th>
<th>$3^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi(140)$</td>
<td>$\pi(1300)$</td>
<td>$\pi(1800)$</td>
<td>$\rho(770)$</td>
<td>$\rho(1450)$</td>
<td>$\rho(1570)$</td>
<td>$\rho(1700)$</td>
<td>$\rho(1900)$</td>
<td>$\rho_2(1400)$</td>
</tr>
<tr>
<td>$\rho_1(1450)$</td>
<td>$\rho_2(1420)$</td>
<td>$\rho_2(1640)$</td>
<td>$\eta_1(1260)$</td>
<td>$\eta_2(1235)$</td>
<td>$\eta_2(1870)$</td>
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<td>$\eta_2(1700)$</td>
<td>$\eta_2(1900)$</td>
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<tr>
<td>$\eta(1600)$</td>
<td>$\eta(1900)$</td>
<td>$\eta(1950)$</td>
<td>$\phi(950)$</td>
<td>$\phi(1020)$</td>
<td>$\phi(1420)$</td>
<td>$\phi(1510)$</td>
<td>$\phi(1680)$</td>
<td>$\phi(1850)$</td>
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## Baryons

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>$\frac{1}{2}^+$</th>
<th>$\frac{3}{2}^-$</th>
<th>$\frac{3}{2}^+$</th>
<th>$\frac{1}{2}^+$</th>
<th>$\frac{3}{2}^-$</th>
<th>$\frac{3}{2}^+$</th>
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<tr>
<td>$\Delta(1910)$</td>
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<tr>
<td>$\Lambda(1116)$</td>
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<td>$\Lambda(1670)$</td>
<td>$\Lambda(1800)$</td>
<td>$\Lambda(1800)$</td>
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<td>$\Sigma(1189)$</td>
<td>$\Sigma(1385)$</td>
<td>$\Sigma(1385)$</td>
<td>$\Sigma(1670)$</td>
<td>$\Sigma(1670)$</td>
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<td>$\Sigma(1315)$</td>
<td>$\Sigma(1315)$</td>
<td>$\Sigma(1530)$</td>
<td>$\Sigma(1530)$</td>
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<td>$\Sigma(1670)$</td>
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<td>$\Xi(1315)$</td>
<td>$\Xi(1315)$</td>
<td>$\Xi(1530)$</td>
<td>$\Xi(1530)$</td>
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<td>$\Omega(1672)$</td>
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Gernot Eichmann (IST Lisboa)
The hadron zoo
The hadron zoo

\[
\begin{align*}
N & \Delta & \Lambda & \Sigma & \Xi & \Omega \\
1^+ & 1^- & 3^+ & 3^- & 5^+ & 5^- \\
\chi & \phi & \Lambda & \tau & \sigma & \rho
\end{align*}
\]
Light baryons

$M \ [\text{GeV}]$

$J^P = \frac{1}{2}^+ \quad \frac{1}{2}^- \quad \frac{3}{2}^+ \quad \frac{3}{2}^- \quad \frac{3}{2}^+ \quad \frac{3}{2}^- \quad \frac{1}{2}^+ \quad \frac{1}{2}^-$

- Extraction of resonances?
- Gluon exchange vs. flavor dependence?
- “Quark core” vs. chiral dynamics?
- Nature of Roper?
- qqq vs. quark-diquark?
- Hybrid baryons?
QCD

QCD’s classical action:

\[ S = \int d^4x \left[ \bar{\psi} \left( \partial^\mu \phi + igA^\mu + m \right) \psi + \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}_\sigma \right] \]

Quantum “effective action”:

\[ \int \mathcal{D}[\psi, \bar{\psi}, A] e^{-S} = e^{-\Gamma} \]

DSEs = quantum equations of motion:
derived from path integral, relate n-point functions

- infinitely many coupled equations
- reproduce perturbation theory, but nonperturbative!
- systematic truncations: neglect higher n-point functions to obtain **closed system**

Reviews:
Roberts, Williams, Prog. Part. Nucl. Phys. 33 (1994),
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer,
QCD

QCD’s classical action:
\[ S = \int d^4x \left[ \bar{\psi} \left( \partial_t + igA + m \right) \psi + \frac{1}{4} F_{\mu\nu}^a F_{a\mu\nu} \right] \]
\[ = \Gamma - \chi \phi, \tau \bar{\phi}, \]
\[ \mid D \int \mid = -1 = -1 + \]

Quantum “effective action”:
\[ \int D[\psi, \bar{\psi}, A] e^{-S} = e^{-\Gamma} \]

DSEs = quantum equations of motion:
derived from path integral, relate n-point functions

Quark propagator:
DCSB generates ‘constituent-quark masses’

Gernot Eichmann (IST Lisboa)

Aug 30, 2017
QCD

QCD’s classical action:

\[
S = \int d^4x \left[ \bar{\psi} (\hat{\phi} + igA + m) \psi + \frac{1}{4} F_{\mu \nu}^a F_{a \mu \nu} \right]
\]

Quantum “effective action”:

\[
\int D[\psi, \bar{\psi}, A] e^{-S} = e^{-\Gamma}
\]

- Gluon propagator

\[
\frac{D(p^2)}{p^2} \left( \delta^{\mu \nu} - \frac{p^\mu p^\nu}{p^2} \right)
\]

Williams, Fischer, Heupel, PRD 93 (2016)

- Quark-gluon vertex

\[
f_1 \gamma^\mu + f_2 i p^\mu + f_3 p^\mu \hat{\phi} + \ldots
\]

Williams, Fischer, Heupel, PRD 93 (2016)

- Three-gluon vertex

\[
F_1 \left[ \delta^{\mu \nu} (p_1 - p_2)^\rho + \delta^{\nu \rho} (p_2 - p_3)^\mu + \delta^{\rho \mu} (p_3 - p_1)^\nu \right] + \ldots
\]

GE, Williams, Alkofer, Vujinovic, PRD 84 (2014)

Agreement between lattice, DSE & FRG within reach!
Hadrons?

- Simplest n-point function that encodes information on **baryons**: quark 6-point correlator
  \[
  \langle \psi_\alpha(x_1) \psi_\beta(x_2) \psi_\gamma(x_3) \bar{\psi}_\rho(y_1) \bar{\psi}_\sigma(y_2) \bar{\psi}_\tau(y_3) \rangle
  \]
Hadrons?

- Simplest n-point function that encodes information on **baryons**: quark 6-point correlator
  \[ \langle \psi_\alpha(x_1) \psi_\beta(x_2) \psi_\gamma(x_3) \bar{\psi}_\rho(y_1) \bar{\psi}_\sigma(y_2) \bar{\psi}_\tau(y_3) \rangle \]

- Spectral decomposition:
  \[ \sum_\lambda |\lambda\rangle \langle \lambda | \rightarrow \sum_\lambda \frac{\cdots}{P^2 + m_i^2} \]
  \[ \Rightarrow \text{Same singularity structure as in} \]

- Bethe-Salpeter wave function:
  residue at pole, contains all information about baryon

\[ \Rightarrow \text{extract gauge-invariant baryon poles from gauge-fixed 6-quark function} \]
**DSEs & BSEs**

- **Homogeneous Bethe-Salpeter equation** for BS wave function:

  \[ P \chi = -m \chi \]

  \[ G \]

  \[ K \chi = \]

  Depends on QCD's n-point functions as input, satisfy **DSEs = quantum equations of motion**

  \[ \chi^{-1} = \chi^{-1} + \chi^{-1} \]

  \[ \chi^{-1} = \chi^{-1} + \chi^{-1} + \chi^{-1} + \chi^{-1} + \chi^{-1} + \ldots \]

- Kernel can be derived in accordance with **chiral symmetry**:

  \[ = \]

  \[ = \]

  **Quark propagator**

  \[ A(p^2) (i\gamma + M(p^2))^{-1} \]

  - Dynamical chiral symmetry breaking generates 'constituent-quark masses'

  - **Quark mass function [GeV]**:
    - Bottom
    - Charm
    - Strange
    - Up/down
    - Chiral limit

  \[ p^2 [GeV^2] \]

  \[ 10^{-2} 10^{-1} 10^0 10^1 10^2 10^3 \]

  \[ 350 \text{ MeV} \]

  \[ 3 \text{ MeV} \]
DSEs & BSEs

Kernel can be derived in accordance with chiral symmetry:

\[
\begin{align*}
-1 &= -1 \\
+ &= + \\
\end{align*}
\]

Light meson spectrum beyond rainbow-ladder:

- Williams, Fischer, Heupel, PRD 93 (2016)
- GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- Kernel can be derived in accordance with **chiral symmetry**:
Kernel can be derived in accordance with chiral symmetry:

\[
\begin{align*}
-1 &= -1 \\
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Light meson spectrum beyond rainbow-ladder:

- Williams, Fischer, Heupel, PRD 93 (2016)
- GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- Kernel can be derived in accordance with \textbf{chiral symmetry}:

  \[
  \begin{align*}
  \alpha(k^2) &= \alpha(k^2) \\
  \end{align*}
  \]

Rainbow-ladder:

effective gluon exchange

\[
\alpha(k^2) = \alpha_{1R}(k^2/A^2, \eta) + \alpha_{UV}(k^2)
\]

adjust scale $A$ to observable, keep width $\eta$ as parameter

- Maris, Tandy, PRC 60 (1999)
- Qin et al., PRC 84 (2011)
**Baryons**

**Covariant Faddeev equation** for baryons:

GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

\[
\begin{align*}
\text{Baryons} & = \text{3-gluon diagram vanishes} \Rightarrow \text{3-body effects small?} \\
\text{Relativistic bound states} & \text{carry OAM:} \\
\text{Octet & decuplet baryons, pion cloud effects,} \\
\text{Baryon form factors:} \\
\text{Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), 1606.09602}
\end{align*}
\]
### DSE / Faddeev landscape

\[ N \rightarrow N^* \gamma \]

#### Quark-diquark

![Quark-diquark Diagram]

<table>
<thead>
<tr>
<th>Contact interaction</th>
<th>QCD-based model</th>
<th>DSE (RL)</th>
<th>RL</th>
<th>bRL</th>
<th>bRL + 3q</th>
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<tr>
<td>( N, \Delta ) masses</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>( N, \Delta ) em. FFs</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>( N \rightarrow \Delta \gamma )</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>Roper</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>( N \rightarrow N^* \gamma )</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>( N^*(1535), \ldots )</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>( N \rightarrow N^* \gamma )</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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#### Three-quark

![Three-quark Diagram]

<table>
<thead>
<tr>
<th>Interaction</th>
<th>RL</th>
<th>bRL</th>
<th>bRL + 3q</th>
</tr>
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<td>✓</td>
<td>...</td>
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<td>( N, \Delta ) em. FFs</td>
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<td>...</td>
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<tr>
<td>( N \rightarrow \Delta \gamma )</td>
<td>✓</td>
<td>✓</td>
<td>...</td>
</tr>
<tr>
<td>Roper</td>
<td>✓</td>
<td>✓</td>
<td>...</td>
</tr>
<tr>
<td>( N \rightarrow N^* \gamma )</td>
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<td>✓</td>
<td>...</td>
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<tr>
<td>( N^*(1535), \ldots )</td>
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<td>...</td>
</tr>
<tr>
<td>( N \rightarrow N^* \gamma )</td>
<td>✓</td>
<td>✓</td>
<td>...</td>
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</table>

**Authors:** Roberts, Bashir, Segovia, Chen, Wilson, Lu, ... Oettel, Alkofer, Roberts, Cloet, Segovia, ... GE, Alkofer, Nicmorus, ... GE, Sanchis-Alepuz, Fischer, Alkofer, Williams, ...
The role of diquarks

Mesons and ‘diquarks’ closely related: after taking traces, only factor 1/2 remains ⇒ diquarks ‘less bound’ than mesons

Pseudoscalar & vector mesons already good in rainbow-ladder
Scalar & axialvector mesons too light, repulsion beyond RL

Scalar & axialvector diquarks sufficient for nucleon and Δ
Pseudoscalar & vector diquarks important for remaining channels
The role of diquarks

Simulate beyond-RL effects:

Insert factor $0 < c < 1$ in ‘bad’ meson and diquark channels ⇒ increases masses, adjusted in meson sector ($\rho-a_1$ splitting)

⇒ reduces strength of $ps + v$ diquarks
Baryon spectrum I


- M [GeV]
- qqq and q-dq agrees: N, Δ, Roper, N(1535)
- # levels compatible with experiment: no states missing
- N, Δ and their 1st excitations (including Roper) agree with experiment
- But remaining states too low ⇒ wrong level ordering between Roper and N(1535)
Baryon spectrum

Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

Quantitative agreement with experiment
- $N(\frac{1}{2}^+)$ and $\Delta(\frac{3}{2}^+)$ depend on sc + av diquarks; remaining ones “polluted” by ps + v diquarks
- Correct level ordering between Roper and $N(1535)$
- Scale $\Lambda$ set by $f_\pi$
- Current-quark mass $m_q$ set by $m_\pi$
- $c$ adjusted to $\rho-a_1$ splitting
- $\eta$ doesn’t change much
**Baryon spectrum**

**Quark-diquark** with reduced pseudoscalar + vector diquarks: GE, FBS 58 (2017)

**Orbital angular momentum content:**

- in nonrelativistic quark model: N, Δ ~ **s waves**, negative-parity states ~ **p waves**, etc.
- Here: ‘quark-model forbidden’ contributions are always present, e.g. **Roper**: dominated by **p waves** ⇒ relativity is important!
Strange baryons

\[
\begin{array}{c}
[nn] \\
\{nn\} \\
[ns] \\
\{ns\} \\
\{ss\}
\end{array}
\begin{pmatrix}
\begin{array}{c}
\text{Diagram 1} \\
\text{Diagram 2} \\
\text{Diagram 3} \\
\text{Diagram 4} \\
\text{Diagram 5}
\end{array}
\end{pmatrix}
= \\
\begin{pmatrix}
\begin{array}{c}
\text{Diagram 6} \\
\text{Diagram 7} \\
\text{Diagram 8} \\
\text{Diagram 9} \\
\text{Diagram 10}
\end{array}
\end{pmatrix}
\begin{pmatrix}
\begin{array}{c}
\text{Diagram 11} \\
\text{Diagram 12} \\
\text{Diagram 13} \\
\text{Diagram 14} \\
\text{Diagram 15}
\end{array}
\end{pmatrix}
\]
Strange baryons

Nucleon

\[
\begin{array}{c}
\text{[nn]} \\
\text{\{nn\}} \\
\text{Nucleon}
\end{array}
\]
Strange baryons

\[
\begin{array}{c}
\{nn\} \\
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{Delta}
\end{array}
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{nn}
\end{array}
\end{array}
\end{array}
\]
Strange baryons
Strange baryons

\[
\begin{align*}
\begin{bmatrix}
[nn] \\
[ns] \\
{ns}
\end{bmatrix}
&= 
\begin{bmatrix}
\text{Diagram 1} \\
\text{Diagram 2} \\
\text{Diagram 3}
\end{bmatrix}

\text{Lambda}
\end{align*}
\]
Strange baryons

\[
\begin{array}{ccc}
\{nn\} & = & \sigma \\
[ns] & & \\
\{ns\} & & \\
\end{array}
\]

Sigma
Strange baryons

Strange baryons include:
- N(1440)
- N(1535)
- N(1520)
- N(1650)
- N(1710)
- N(1875)

Delta baryons include:
- Δ(1232)
- Δ(1600)
- Δ(1620)
- Δ(1700)
- Δ(1900)
- Δ(1910)
- Δ(1920)
- Δ(1940)

Omega baryons include:
- Ω(1672)
- Ω(2250)~
- Ω(2470)?
- Ω(2380)?
- Ω(2500)?

Masses (in GeV):
- N(1710): 1.0
- N(1440): 1.8
- Ω(1672): 2.0
- Δ(1600): 1.4
- Δ(1910): 2.2
- Ω(2250)?: 2.8
Strange baryons
Strange baryons
**Strange baryons**

- Strange baryons similar to **light baryons:**
  
  \[
  \Omega \rightarrow \Delta \\
  \Sigma, \Xi \rightarrow N + \Delta \quad \rightarrow \text{rich spectrum!} \\
  \Lambda \rightarrow N + \text{singlets}
  \]

- Roper, \(\Lambda(1600), \Lambda(1405), \Lambda(1520)\):
  levels are there, but additional dynamics?

- **Structure information?**
  OAM, decays, form factors!
Form factors

Sketch of a generic electromagnetic form factor:

- **timelike:** $e^+e^- \rightarrow NN$
- **spacelike:** $e^-N \rightarrow e^-N$

How can we calculate this from the **quark level**?

- 'rainbow-ladder'
- **quark-photon vertex**
- **quark propagator**
- Faddeev amplitude
Matrix elements

Insert **spectral decomposition** in \( \langle \cdots \psi(x_1) \cdots \psi(y_1) \cdots j^\mu(z) \cdots \rangle \)

- Use properties of (functional) derivative, obtain general expression for **current matrix elements** and **scattering amplitudes**:

  \[ \mathcal{J}^\mu = -\overline{\Psi}_f (G^{-1})^\mu_{\cdot i} \quad \quad \mathcal{M}^{\mu\nu} = \overline{\Psi}_f \left[ (G^{-1})^{\mu\nu} G (G^{-1})_{\cdot \nu} - (G^{-1})^{\mu\nu} \right] \Psi_i \]

- Relate \( G \) to elementary propagators, vertices and kernels:

  \( (G^{-1})^\mu = (G_0^{-1})^\mu - K^\mu = \left[ \Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^\mu_{(2)} + \text{perm.} \right] - K^\mu_{(3)} \)

  \( (G^{-1})^{\mu\nu} = (G_0^{-1})^{\mu\nu} - K^{\mu\nu} = \left[ \Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} + \Gamma^{\mu} \otimes K^{\nu}_{(2)} \otimes S^{-1} - \Gamma^{\mu} \otimes K^{\nu}_{(2)} - S^{-1} \otimes K^{\mu\nu}_{(2)} + \text{perm.} \right] - K^{\mu\nu}_{(3)} \)
Matrix elements

Current matrix element:
- impulse approximation + coupling to kernels
- **gauge invariance** is automatic, as long as all ingredients calculated from same symmetry-preserving kernel

\[ \mathcal{J}^\mu = \ldots \]

Use properties of (functional) derivative, obtain general expression for current matrix elements and scattering amplitudes:

\[ \mathcal{J}^\mu = -\overline{\Psi}_f (G^{-1})^\mu \overline{\Psi}_i \]
\[ \mathcal{M}^{\mu\nu} = \overline{\Psi}_f \left[ (G^{-1})^{\mu} G (G^{-1})^{\nu} - (G^{-1})^{\mu\nu} \right] \overline{\Psi}_i \]

Relate \( G \) to elementary propagators, vertices and kernels:

\[ (G^{-1})^\mu = (G_0^{-1})^\mu - K^\mu = \left[ \Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K_{(2)}^\mu + \text{perm.} \right] - K_{(3)}^\mu \]
\[ (G^{-1})^{\mu\nu} = (G_0^{-1})^{\mu\nu} - K^{\mu\nu} = \left[ \Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} - \Gamma^{(\mu \otimes \Gamma^\nu)} \otimes S^{-1} - \Gamma^{(\mu \otimes K_{(2)}^\nu)} - S^{-1} \otimes K_{(2)}^{\mu\nu} + \text{perm.} \right] - K_{(3)}^{\mu\nu} \]
Matrix elements

Current matrix element:

- impulse approximation + coupling to kernels
- **gauge invariance** is automatic,
  as long as all ingredients calculated from same symmetry-preserving kernel

\[
\mathcal{J}^\mu = \begin{array}{c}
\text{diagram 1} \\
+ \text{diagram 2} \\
+ \text{diagram 3} \\
+ \text{diagram 4}
\end{array}
\]

Use properties of (functional) derivative, obtain general expression for current matrix elements and scattering amplitudes:

\[
\mathcal{J}^\mu = -\overline{\Psi}_f (G^{-1})^\mu \Psi_i \quad \quad \mathcal{M}^{\mu\nu} = \overline{\Psi}_f \left[ (G^{-1})^{\mu\nu} \Phi (G^{-1})^{\nu\rho} - (G^{-1})^{\mu\rho} \right] \Psi_i
\]

Relate \( G \) to elementary propagators, vertices and kernels:

\[
(G^{-1})^\mu = (G_0^{-1})^\mu - K^\mu = \left[ \Gamma^{\mu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu} \otimes K_{(2)} - S^{-1} \otimes K^{\mu}_{(2)} + \text{perm.} \right] - K_{(3)}^\mu
\]
\[
(G^{-1})^{\mu\nu} = (G_0^{-1})^{\mu\nu} - K^{\mu\nu} = \left[ \Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} + \Gamma^{(\mu} \otimes \Gamma^{\nu)} \otimes S^{-1} - \Gamma^{(\mu} \otimes K^{\nu)}_{(2)} - S^{-1} \otimes K^{\mu\nu}_{(2)} + \text{perm.} \right] - K_{(3)}^{\mu\nu}
\]
**Form factors**

**Nucleon em. form factors**
from three-quark equation

GE, PRD 84 (2011)

- **Timelike vector-meson poles**
  generated in quark-photon vertex

- "Quark core without pion-cloud"

- **similar**: $N \rightarrow \Delta \gamma$ transition,
  axial & pseudoscalar FFs,
  octet & decuplet em. FFs

  **Review**: GE, Sanchis-Alepuz, Williams,
  Fischer, Alkofer, PPNP 91 (2016), 1606.09602

- $\pi \rightarrow \gamma \gamma^*$ transition: vm. poles
  modify asymptotic scaling!

GE, Fischer, Weil, Williams, 1704.05774 [hep-ph]

---

**Equation**

$$J^\mu = \begin{array}{c}
\text{diagram 1} \\
\text{diagram 2} \\
\text{diagram 3}
\end{array}$$

**Graphs**

- $G_E^p / G_D$
- $G_E^n (Q^2)$
- $G_M^p (Q^2)$
- $|G_M^n (Q^2)|$
Pion form factor

- Form factor from

- Timelike vector meson poles automatically generated by quark-photon vertex BSE!

⇒ $\Gamma^\mu = \text{Ball-Chiu}$ (em. gauge invariance)

+ Transverse part (vm. poles & dominance)

- Form factor at large $Q^2$

  Chang, Cloet, Roberts, Schmidt, Tandy, PRL 111 (2013)

- Include pion cloud effects:

  GE, Fischer, Kubrak, Williams, in preparation

A. Krassnigg (Schladming 2010),
Compton scattering ...

Compton amplitude = sum of Born terms + 1PI structure part:

\[ \text{Compton amplitude} = \text{Born terms} + \text{1PI structure part} \]

Born terms: determined by nucleon form factors

\[ \text{Polarizabilities:} \quad \text{structure information} \]

\[ \text{"Pion cloud" (ChPT)} \]

\[ \text{t-channel meson exchange} \quad (\pi, \sigma, a_1, \ldots) \]

\[ \text{s/u-channel nucleon resonances} \quad (\Delta, N^*, \ldots) \]

but also:

\[ \text{GPD} \]

\[ \Rightarrow \text{is there a common underlying quark-level description?} \]

Extracting resonances

Hadronic coupled-channel equations:

Microscopic effects?
What is an “offshell hadron”?

Sato-Lee/EBAC/ANL-Osaka, Dubna-Mainz-Taiwan, Valencia, Jülich-Bonn, GSI, JLab, MAID, SAID, KSU, Giessen, Bonn-Gatchina, JPAC,...

Suzuki et al., PRL 104 (2010)
Matrix elements

Scattering amplitude:

\[ \mathcal{M}^{\mu\nu} = \begin{array}{c}
\text{Nucleon resonances} \\
\text{quark Born: reproduces perturbative handbag}
\end{array} \]

Use properties of (functional) derivative, obtain general expression for current matrix elements and scattering amplitudes:

\[ \mathcal{J}^\mu = -\bar{\Psi}_f (G^{-1})^\mu \Psi_i \]

\[ \mathcal{M}^{\mu\nu} = \bar{\Psi}_f \left[ (G^{-1})^{\mu
\nu} - (G^{-1})^{\mu\nu} \right] \Psi_i \]

Relate \( G \) to elementary propagators, vertices and kernels:

\[ (G^{-1})^\mu = (G_0^{-1})^\mu - K^\mu = \left[ \Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K_{(2)}^\mu + \text{perm.} \right] - K_{(3)}^\mu \]

\[ (G^{-1})^{\mu\nu} = (G_0^{-1})^{\mu\nu} - K^{\mu\nu} = \left[ \Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} + \Gamma^{\mu \otimes \Gamma^{\nu}} \otimes S^{-1} - \Gamma^{\mu \otimes K_{(2)}^{\nu}} - S^{-1} \otimes K_{(2)}^{\mu\nu} + \text{perm.} \right] - K_{(3)}^{\mu\nu} \]
Matrix elements

Scattering amplitude:

\[ M^{\mu \nu} = \begin{array}{c}
\text{Nucleon resonances} \\
\text{quark Born: reproduces perturbative handbag} \\
\text{quark 1PI: reproduces t-channel meson poles}
\end{array} \]

Use properties of (functional) derivative, obtain general expression for current matrix elements and scattering amplitudes:

\[ J^\mu = -\bar{\Psi}_f (G^{-1})^\mu \Psi_i \]

\[ M^{\mu \nu} = \bar{\Psi}_f \left( (G^{-1})^\mu \Gamma \left( (G^{-1})^\nu \right) - (G^{-1})^{\mu \nu} \right) \Psi_i \]

Relate \( G \) to elementary propagators, vertices and kernels:

\[ (G^{-1})^\mu = (G_0^{-1})^\mu - K^\mu = \left[ \Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^\mu_{(2)} + \text{perm.} \right] - K^\mu_{(3)} \]

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Matrix elements

Scattering amplitude:

\[ \mathcal{M}^{\mu\nu} = \begin{array}{c}
\text{Nucleon resonances} \\
\text{quark Born: reproduces perturbative handbag} \\
\text{quark 1PI: reproduces t-channel meson poles} \\
\text{cat's ears diagrams}
\end{array} \]

Use properties of (functional) derivative, obtain general expression for **current matrix elements** and **scattering amplitudes**:

\[ \mathcal{J}^\mu = -\bar{\Psi}_f (G^{-1})^\mu \Psi_i \]

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Matrix elements

Scattering amplitude:

\[ \mathcal{M}^{\mu\nu} = \]  
\[ \text{Nucleon resonances} \] + \[ \text{quark Born: reproduces perturbative handbag} \] + \[ \text{quark 1PI: reproduces t-channel meson poles} \] + \[ \text{cat's ears diagrams} \]

Use properties of (functional) derivative, obtain general expression for \textbf{current matrix elements} and \textbf{scattering amplitudes}:

\[ \mathcal{J}^\mu = -\bar{\Psi}_f (G^{-1})^\mu \Psi_i \quad \mathcal{M}^{\mu\nu} = \bar{\Psi}_f \left( (G^{-1})^{\mu} G (G^{-1})^{\nu} - (G^{-1})^{\mu\nu} \right) \Psi_i \]

Relate \( G \) to elementary propagators, vertices and kernels:

\[ (G^{-1})^\mu = (G_0^{-1})^\mu - K^\mu = \left[ \Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^\mu_{(2)} + \text{perm.} \right] - K^\mu_{(3)} \]
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Matrix elements

Scattering amplitude:

\[ \mathcal{M}^{\mu\nu} = \text{Nucleon resonances} + \text{quark Compton vertex: reproduces perturbative handbag & t-channel meson poles} + \text{cat's ears diagrams} \]

Use properties of (functional) derivative, obtain general expression for **current matrix elements** and **scattering amplitudes**:

\[ \mathcal{J}^\mu = -\bar{\Psi}_f (G^{-1})^\mu f \Psi_i \]

\[ \mathcal{M}^{\mu\nu} = \bar{\Psi}_f \left[ (G^{-1})^{\{\mu} G (G^{-1})^{\nu\}} - (G^{-1})^{\mu\nu} \right] \Psi_i \]

Relate \( G \) to elementary propagators, vertices and kernels:

\( (G^{-1})^\mu = (G_0^{-1})^\mu - K^\mu = \left[ \Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^{\mu}_{(2)} + \text{perm.} \right] - K^{\mu}_{(3)} \)

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Matrix elements

**Scattering amplitude:**  

\[ M_{\mu \nu} = N + T + \text{GPD} \]

- **Poincaré covariance** and **crossing symmetry** are automatic
- **gauge invariance** and **chiral symmetry** are automatic, as long as all ingredients calculated from same symmetry-preserving kernel
- **perturbative processes** are included
- **s, t, u channel poles** are generated dynamically, no need for “offshell hadrons”
- hadronic rescattering is implicit

Nucleon resonances

quark **Compton vertex**: reproduces perturbative handbag & t-channel meson poles

**cat’s ears** diagrams
**Kinematics**

**Electromagnetic current:**

\[ J^\mu(p, Q) = e \bar{u}(p_f) \Gamma^\mu(p, Q) u(p_i) \]

\[ F_1(Q^2) \gamma^\mu + F_2(Q^2) \frac{i}{4m} [\gamma^\mu, Q] \]

2 form factors (Dirac + Pauli),
1 kinematic variable \( Q^2 \)

**CS amplitude:**

\[ M(p, Q, Q') = \frac{e^2}{m} \varepsilon^{\mu}(Q') \bar{u}(p_f) \Gamma^{\mu\nu}(p, Q, Q') u(p_i) \varepsilon^\nu(Q) \]

18 Compton form factors (CFFs),
4 kinematic variables:

\[ \eta_+ = \frac{Q^2 + Q'^2}{2m^2}, \quad \eta_- = \frac{Q \cdot Q'}{m^2}, \quad \omega = \frac{Q^2 - Q'^2}{2m^2}, \quad \lambda = -\frac{p \cdot Q}{m^2} = -\frac{p \cdot Q'}{m^2} \]

\[ \Rightarrow \sum_{i=1}^{18} c_i(\eta_+, \eta_-, \omega, \lambda) \bar{u}(p_f) \tau_i^{\mu\nu}(p, Q, Q') u(p_i) \]
Kinematics

Kinematic domain for two-photon exchange: cone around $\eta_+$

$$t = \frac{\Delta^2}{4m^2} = \frac{\eta_+ - \eta_-}{2}, \quad \sigma = \frac{\Sigma^2}{m^2} = \frac{\eta_+ + \eta_-}{2}, \quad \tau = \frac{Q^2}{4m^2}, \quad \tau' = \frac{Q'^2}{4m^2} \tag{12}$$

$$\mathcal{M}(p, Q, Q') = \frac{e^2}{m} \varepsilon^\mu(Q') \bar{u}(p_f) \Gamma^{\mu\nu}(p, Q, Q') u(p_i) \varepsilon^\nu(Q) \tag{13}$$


18 Compton form factors (CFFs),

4 kinematic variables:

$$\eta_+ = \frac{Q^2 + Q'^2}{2m^2}, \quad \eta_- = \frac{Q \cdot Q'}{m^2}, \quad \omega = \frac{Q^2 - Q'^2}{2m^2}, \quad \lambda = -\frac{p \cdot Q}{m^2} = -\frac{p \cdot Q'}{m^2}$$

$$\Rightarrow \sum_{i=1}^{18} c_i(\eta_+, \eta_-, \omega, \lambda) \bar{u}(p_f) \tau_i^{\mu\nu}(p, Q, Q') u(p_i)$$
Forward CS

Forward limit: \( \Delta^\mu = 0 \) \( \Rightarrow \) 2 variables:
\[
\eta = \eta_+ = \eta_- = \frac{Q^2}{m^2}, \quad \lambda = -\frac{p \cdot Q}{m^2}, \quad \omega = 0
\]
\[\Rightarrow 4 \text{ CFFs: } \bar{u}(p) \left( \frac{c_1}{m^4} t^{\mu \alpha}_Q t_{\rho \nu}^Q + \frac{c_2}{m^2} t^{\mu \nu}_Q + \frac{c_3}{m} i \epsilon^{\mu \nu \gamma} Q_\gamma + \frac{c_4}{m^2} \lambda [t^{\mu \alpha}_Q, t_{\gamma Q}^{\alpha \nu}] \right) u(p)\]
Forward CS

Forward limit: $\Delta^\mu = 0 \Rightarrow 2$ variables: 

$\eta = \eta_+ = \eta_- = \frac{Q^2}{m^2}$, $\lambda = -\frac{p \cdot Q}{m^2}$, $\omega = 0$ 

$\Rightarrow 4$ CFFs: 

$\bar{u}(p) \left( \frac{c_1}{m^4} t^\mu_\alpha t^\nu_\mu + \frac{c_2}{m^2} t^\mu_\mu + \frac{c_3}{m} i \varepsilon^\mu_\nu Q_\gamma + \frac{c_4}{m^2} \lambda [t^\mu_\alpha, t^\nu_\gamma] \right) u(p)$

- Low-energy expansion:
  
  $c_1(\eta, \lambda) = c_1^{\text{Born}}(\eta, \lambda) + \alpha(\eta) + \beta(\eta) + \mathcal{O}(\lambda^2)$

  $c_2(\eta, \lambda) = c_2^{\text{Born}}(\eta, \lambda) + \beta(\eta) + \mathcal{O}(\lambda^2)$

- Nucleon resonances at $s, u > m^2$, $N\pi$ branch cuts for $s, u > (m + m_n)^2$

- TPE region $\to$ proton radius puzzle

- $\text{Im} \ c_i$ for $x = \eta / (2\lambda) \in [0, 1]$ known from $N\gamma^* \to X$ cross section

- Use dispersion relations for rest:

  $c_i(\eta, \lambda) = \frac{1}{\pi} \int_{\lambda_i^2}^{\infty} d\lambda^2 \frac{\text{Im} \ c_i(\eta, \lambda^2)}{\lambda^2 - \lambda^2 - i\epsilon}$

$\Rightarrow$ Baldin sum rule for $\alpha + \beta$, but $\beta$ unconstrained (need subtracted DR)

$\Rightarrow$ ChPT + pQCD, but result much too small to explain discrepancy

Birse, McGovern, EPJ A 48 (2012)
Forward CS

**Forward limit:** $\Delta^\mu = 0 \Rightarrow 2$ variables: $\eta = \eta_+ = \eta_- = \frac{Q^2}{m^2}$, $\lambda = -\frac{p\cdot Q}{m^2}$, $\omega = 0$

$\Rightarrow 4$ CFFs: $\bar{u}(p) \left( \frac{c_1}{m^4} t^\mu_\alpha t^\nu_\beta + \frac{c_2}{m^2} t^\mu_\alpha t^\nu_\beta + \frac{c_3}{m} \varepsilon^{\mu_\nu} + \frac{c_4}{m^2} \lambda [t^\mu_\alpha t^\nu_\beta] \right) u(p)$

Singularity structure of quark propagator prevents **direct kinematic access** to all relevant regions ...

- if amplitudes free of kinematic singularities: **only phys. poles and cuts**, extrapolate from unphysical regions
- clean solution (expensive): **contour deformations**
Pion transition form factor

\[ Q' = \Sigma - \frac{\Delta}{2} \quad Q = \Sigma + \frac{\Delta}{2} \]

\[ \eta_+ = \frac{Q^2 + Q'^2}{2} \]
\[ \omega = \frac{Q^2 - Q'^2}{2} \]
\[ \eta_- = Q \cdot Q' \]

\[ \langle \pi p | J_{\mu\nu}^{\pi} | \Sigma \rangle = e^2 \frac{F(Q^2, Q'^2)}{4\pi^2 f_{\pi}} \varepsilon^{\mu\nu\alpha\beta} Q'^\alpha Q^\beta \]

- \( F(0, 0) = 1 \) in chiral limit
- \( \frac{\eta_+ F(Q^2, Q'^2)}{4\pi^2 f_{\pi}^2} \xrightarrow{\eta_+ \to \infty} \frac{2}{3} \ldots 1(?) \)

Lepage, Brodsky, PRD 22 (1980)

\[ \eta_+ [\text{GeV}^2] \]

Belle
BaBar
CLEO
CELLO
Pion transition form factor

\[ Q' = \Sigma - \frac{\Delta}{2} \]
\[ Q = \Sigma + \frac{\Delta}{2} \]

\[ \eta_+ = \frac{Q^2 + Q'^2}{2} \]
\[ \omega = \frac{Q^2 - Q'^2}{2} \]
\[ \eta_- = Q \cdot Q' \]

**Quark singularities** complicate matters: symmetric limit ok, but asymmetric limit only up to \( \sim 4 \text{ GeV}^2 \)  

Maris, Tandy, PRC 65 (2002)

exploit Lorentz invariance to change frame

Weil, GE, Fischer, Williams, 1704.06046 [hep-ph]
Pion transition form factor

Idea:
- calculate FF inside cone
- interpolate to physical plane using VM pole as constraint
- can be done for arbitrary $Q^2$

\[
\eta_+ = \frac{Q^2 + Q'^2}{2}
\]
\[
\omega = \frac{Q^2 - Q'^2}{2}
\]
\[
\eta_- = Q \cdot Q'
\]
Pion transition form factor

\[ Q' = \Sigma - \frac{\Delta}{2} \]

\[ Q = \Sigma + \frac{\Delta}{2} \]

\[ p + \Sigma \]

\[ p - \frac{\Delta}{2} \]

\[ p + \frac{\Delta}{2} \]

\[ \Delta \]

\[ \eta_+ = \frac{Q^2 + Q'^2}{2} \]

\[ \omega = \frac{Q^2 - Q'^2}{2} \]

\[ \eta_- = Q \cdot Q' \]

VM poles modify asymptotic scaling!

GE, Fischer, Weil, Williams, 1704.05774 [hep-ph]

Gernot Eichmann (IST Lisboa)

Aug 30, 2017 25 / 30
Rare pion decay $\pi^0 \to e^+e^-$

- Depends on pion transition FF as input: GE, Fischer, Weil, Williams, 1704.05774

$$A(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p + \Sigma)^2 + m^2} \frac{F(Q^2, Q'^2)}{Q^2 Q'^2}$$

- cannot be calculated directly in Euclidean kinematics because of photon and lepton poles

- After reanalysis of radiative corrections still $2\sigma$ discrepancy in branching ratio between exp and theory:

  KTeV Collab.: Abouzaid et al., PRD 75 (2007); Husek, Kampf, Novotny, EPJ C74 (2014)

  Dorokhov, JETP Lett. 91 (2010), Masjuan, Sanchez-Puertas, 1504.07001
Rare pion decay \( \pi^0 \rightarrow e^+ e^- \)

- Depends on pion transition FF as input: GE, Fischer, Weil, Williams, 1704.05774

\[
A(t) = \frac{1}{2\pi^2 t} \int d^4 \Sigma \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p + \Sigma)^2 + m^2} \frac{F(Q^2, Q^2')}{Q^2 Q^2'}
\]

- cannot be calculated directly in Euclidean kinematics because of photon and lepton poles

- workaround with dispersion relations:

\[
\text{Im } A^{LO}(t) = \frac{\pi \ln \gamma(t)}{2\beta(t)} F(0, 0) \quad \Rightarrow \quad \text{Re } A(t) = A(0) + \frac{\ln^2 \gamma(t) + \frac{1}{3} \pi^2 + 4 \operatorname{Li}_2(-\gamma(t))}{4\beta(t)}
\]

- After reanalysis of radiative corrections still 2\(\sigma\) discrepancy in branching ratio between exp and theory:

<table>
<thead>
<tr>
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<th>KTeV Collab.: Abouzaid et al., PRD 75 (2007); Husek, Kampf, Novotny, EPJ C74 (2014)</th>
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<td>Result for the rare decay ( \pi^0 \rightarrow e^+ e^- )</td>
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<td>6.87(36) (\times 10^{-8})</td>
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Rare pion decay $\pi^0 \to e^+ e^-$

\[
A(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p + \Sigma)^2 + m^2} \frac{F(Q^2, Q'^2)}{Q^2 Q'^2}.
\]

Photon and lepton poles produce branch cuts in complex $\Sigma^2 = \sigma$ plane:

- ‘Euclidean integration’: $0 < \sigma < \infty$
Rare pion decay $\pi^0 \rightarrow e^+ e^-$

Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

- ‘Euclidean integration’: $0 < \sigma < \infty$
- not possible: circular photon cut
Rare pion decay $\pi^0 \rightarrow e^+ e^-$

Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

- ‘Euclidean integration’: $0 < \sigma < \infty$
- not possible: circular photon cut
- deform integration contour: cut opens at $t$
Rare pion decay $\pi^0 \to e^+ e^-$

$Q' = \Sigma - \frac{\Delta}{2}$

$Q = \Sigma + \frac{\Delta}{2}$

$\Delta$

$p + \frac{\Delta}{2}$

$p + \Sigma$

$p - \frac{\Delta}{2}$

$A(t) = \frac{1}{2\pi^2 t} \int d^4 \Sigma \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p + \Sigma)^2 + m^2} \frac{F(Q^2, Q'^2)}{Q^2 Q'^2}$.

Photon and lepton poles produce branch cuts in complex $\Sigma^2 = \sigma$ plane:

- ‘Euclidean integration’: $0 < \sigma < \infty$
- not possible: circular photon cut
- deform integration contour: cut opens at $t$
- but lepton cut does not open at $t$!
Rare pion decay $\pi^0 \rightarrow e^+ e^-$

\[ A(t) = \frac{1}{2\pi^2 t} \int d^4 \Sigma \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p + \Sigma)^2 + m^2} \frac{F(Q^2, Q'^2)}{Q^2 Q'^2}. \]

Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

- ‘Euclidean integration’: $0 < \sigma < \infty$
- not possible: circular photon cut
- deform integration contour: cut opens at $t$
- but lepton cut does **not** open at $t$!
- **deform contour** such that it never crosses any cut!
Rare pion decay $\pi^0 \rightarrow e^+ e^-$

$$A(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p + \Sigma)^2 + m^2} \frac{F(Q^2, Q'^2)}{Q^2 Q'^2}.$$
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$Q' = \Sigma - \frac{\Delta}{2}$

$Q = \Sigma + \frac{\Delta}{2}$

$\Delta$

$p + \frac{\Delta}{2}$

$p + \Sigma$

$p - \frac{\Delta}{2}$

$A(t) = \frac{1}{2\pi^2 t} \int d^4 \Sigma \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p + \Sigma)^2 + m^2} \frac{F(Q^2, Q'^2)}{Q^2 Q'^2}$.

- Algorithm is stable & efficient
- Can be applied to any integral as long as singularity locations known
- Useful for treating resonances!

Weil, GE, Fischer, Williams, PRD 96 (2017)
Tetraquarks

- Light scalar mesons $\sigma, \kappa, a_0, f_0$ as tetraquarks: solution of four-body equation reproduces mass pattern
  GE, Fischer, Heupel, PLB 753 (2016)

BSE dynamically generates meson poles in wave function:

\[
\begin{align*}
 f_i \left( S_0, \bigtriangleup, \bigtriangleup, \bigcirc \right) & \rightarrow 1500 \text{ MeV} \\
 f_i \left( S_0, \bigtriangleup, \bigtriangleup, \bigcirc \right) & \rightarrow 1500 \text{ MeV} \\
 f_i \left( S_0, \bigtriangleup, \bigtriangleup, \bigcirc \right) & \rightarrow 1200 \text{ MeV} \\
 f_i \left( S_0, \bigtriangleup, \bigtriangleup, \bigcirc \right) & \rightarrow 350 \text{ MeV} !!
\end{align*}
\]

- Similar in meson-meson / diquark-antidiquark approximation (analogue of quark-diquark for baryons)
  Heupel, GE, Fischer, PLB 718 (2012)
Towards multiquarks

Transition from quark-gluon to nuclear degrees of freedom:

- 6 ground states, one of them deuteron
  Dyson, Xuong, PRL 13 (1964)
- Dibaryons vs. hidden color?
  Bashkanov, Brodsky, Clement, PLB 727 (2013)
- Deuteron FFs from quark level?

Microscopic origins of nuclear binding?

- only quarks and gluons
- quark interchange and pion exchange automatically included
- dibaryon exchanges

Compton scattering

Nucleon polarizabilities:
ChPT & dispersion relations
Hagelstein, Miskimen, Pascalutsa, PPNP 88 (2016)

In total: polarizabilities ≈
Quark-level effects ↔ Baldin sum rule
+ nucleon resonances (mostly Δ)
+ pion cloud (at low $\eta_+$)?

First DSE results:
GE, FBS 57 (2016)

- Quark Compton vertex
  (Born + 1PI) calculated,
  added $\Delta$ exchange

- compared to DRs
  Pasquini et al., EPJ A11 (2001),
  Downie & Fonvieille, EPJ ST 198 (2011)

- $\alpha_E$ dominated by handbag,
  $\beta_M$ by $\Delta$ contribution

⇒ large “QCD background”!
Hadron physics with functional methods

Understand properties of elementary \( n \)-point functions \( \leftrightarrow \) Calculate hadronic observables:
- mass spectra, form factors, scattering amplitudes, …

- QCD
- symmetries intact (Poincare invariance & chiral symmetry important)
- access to all momentum scales & all quark masses
- compute mesons, baryons, tetraquarks, … from same dynamics

- systematic construction of truncations
- technical challenges: coupled integral equations, complex analysis, structure of 3-, 4-, … point functions, need lots of computational power!

access to underlying nonperturbative dynamics!
Backup slides
QED

QED’s classical action:

\[
S = \int d^4x \left[ \bar{\psi} (\slashed{\partial} + igA + m) \psi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]
\]

Quantum “effective action”:

\[
\int \mathcal{D}[\psi, \bar{\psi}, A] e^{-S} = e^{-\Gamma}
\]
QED

QED’s classical action:
\[ S = \int d^4x [\bar{\psi} (\partial + igA + m) \psi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu}] \]

Quantum “effective action”:
\[ \int D[\psi, \bar{\psi}, A] e^{-S} = e^{-\Gamma} \]

Perturbation theory: expand Green functions in powers of the coupling

- Mass function:
  \[ A(p^2) \left( i\phi + M(p^2) \right) = \frac{1}{i\phi + m} + \cdots \]

- Running coupling:
  \[ D^{-1}(p^2)(p^2\delta^{\mu\nu} - p^\mu p^\nu) = \frac{1}{p^2\delta^{\mu\nu} - p^\mu p^\nu} + \cdots \]

- Anomalous magnetic moment:
  \[ F_1(0) - \frac{F_2(0)}{2m} \sigma^{\mu\nu} Q^\nu + \cdots \]

- Additional diagrams for expansions.
QED

QED’s classical action:

\[ S = \int d^4x \left[ \bar{\psi} \left( \slash{\partial} + igA + m \right) \psi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right] \]

Quantum “effective action”:

\[ \int D[\psi, \bar{\psi}, A] e^{-S} = e^{-\Gamma} \]

Perturbation theory: expand Green functions in powers of the coupling

Moller scattering

Compton scattering

Light-by-light scattering

 ⟹ extremely precise theory predictions!
QCD

QCD's classical action:

\[
S = \int d^4x \left[ \bar{\psi} \left( \hat{\phi} + igA + m \right) \psi + \frac{1}{4} F_{\mu \nu}^a F_{a \mu \nu} \right] = \frac{1}{g^2} \quad \frac{1}{g} \quad \frac{1}{g^2}
\]

Quantum “effective action”:

\[
\int D[\psi, \bar{\psi}, A] e^{-S} = e^{-\Gamma}
\]

Perturbation theory: expand Green functions in powers of the coupling
QCD

QCD’s classical action:

\[ S = \int d^4 x \left[ \bar{\psi} (\not{\partial} + i g \not{A} + m) \psi + \frac{1}{4} F_{\mu \nu}^a F_{a \mu \nu} \right] \]

Quantum “effective action”:

\[ \int D[\psi, \bar{\psi}, A] e^{-S} = e^{-\Gamma} \]

Perturbation theory: expand Green functions in powers of the coupling

But ... \( \alpha(Q^2) \) becomes large at low momenta

\( \alpha(Q^2) \) decreases with distance

large distances: all these can contribute with same magnitude!

\[ QCD \Rightarrow \text{need non-perturbative methods!} \]

Gernot Eichmann (IST Lisboa)
Bethe-Salpeter equations

- Example pion: quark-antiquark bound state $\iff$ Goldstone boson of DCSB

$$\gamma_5 \left( f_1 + f_2 \not{p} + f_3 q \cdot P \not{q} + f_4 [q, \not{P}] \right) \otimes \text{Color} \otimes \text{Flavor}$$

most general Dirac-Lorentz structure, Lorentz-invariant dressing functions:

$$f_i = f_i(q^2, q \cdot P, P^2 = -m^2) \implies \text{pion is made of s waves and p waves!}$$

(relative momentum $\sim$ orbital angular momentum)

- Homogeneous BSE becomes

$$f_i(q^2, z) = \int d^4q' K_{ij}(q^2, q'^2, z, z', q \cdot q') f_j(q'^2, z')$$

Eigenvalue spectrum of BS kernel:

$$K_{ijqq'zz'} f_{ijq'zz'}^{(n)} = \lambda_n(P^2) f_{iqz}^{(n)} \quad \lambda_n \xrightarrow{P^2 \rightarrow -m^2} 1$$

Example pion: quark-antiquark bound state $\iff$ Goldstone boson of DCSB

\[\begin{array}{c}
p \text{[GeV]} \\
\hline
\text{f1} & \text{f2} & \text{f3} & \text{f4} \\
\hline
0.0 & 0.5 & 1.0 & 1.5 & 2.0 \\
0 & 1 & 2 & 3 & 4 \\
\end{array}\]
Bethe-Salpeter equations

- Example pion: quark-antiquark bound state $\Leftrightarrow$ Goldstone boson of DCSB

$$\gamma_5 \left( f_1 + f_2 \not{p} + f_3 q \cdot P \not{q} + f_4 [\not{q}, \not{P}] \right) \otimes \text{Color} \otimes \text{Flavor}$$

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- Homogeneous BSE becomes

$$f_i(q^2, z) = \int d^4q' K_{ij}(q^2, q'^2, z, z', q \cdot q') f_j(q'^2, z')$$

**Eigenvalue spectrum** of BS kernel:

$$K_{ijqq'zz'} f^{(n)}_{jq'z'} = \lambda_n(P^2) f^{(n)}_{iqz} \quad \lambda_n \rightarrow \frac{1}{\lambda_n}$$

- Eigenvectors = BS amplitudes

![Graphs showing the eigenvalue spectrum and eigenvectors of the Bethe-Salpeter kernel](image)
Mesons

- **Pion is Goldstone boson**: $m_\pi^2 \sim m_q$

- **Light meson spectrum** beyond rainbow-ladder

- **Charmonium spectrum**
  
  Fischer, Kubrak, Williams,  EPJ A 51 (2015)

- **Pion transition form factor**

  GE, Fischer, Weil, Williams, 1704.05774 [hep-ph]

---

Williams, Fischer, Heupel, PRD 93 (2016)

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)
Light baryons

Nonrelativistic quark model:

\[ P = (-1)^L \]

\[ J^P = 1/2^+ \quad 1/2^- \quad 3/2^+ \quad 3/2^- \]

**M [GeV]**

- N(1440)
- N(1480)
- N(1520)
- N(1535)
- N(1600)
- N(1650)
- N(1700)
- N(1720)
- N(1880)
- N(1900)
- N(1910)

**N = 1:** 70

**L = 1, P = -**

“p wave”

**N = 0:** 56

**L = 0, P = +**

“s wave”

\[ S \cdot S \]

\[ L \cdot S \]
Tetraquarks in charm region?

- Can we distinguish different tetraquark configurations?

- Four quarks dynamically rearrange themselves into dq-dq, molecule, hadroquarkonium; strengths determined by four-body BSE:
nPI effective action

nPI effective actions provide symmetry-preserving closed truncations. 3PI at 3-loop: all two- and three-point functions are dressed; 4, 5, ... do not appear.

\[ \Gamma_2 = - \begin{array}{c}
\text{closed truncations.}
\end{array} \]

Self-energy:
\[ \Sigma = \frac{\delta \Gamma_2}{\delta D} = - \begin{array}{c}
\begin{array}{c}
\text{closed truncations.}
\end{array}
\end{array} \]

Vertex:
\[ \frac{\delta \Gamma_2}{\delta V} = 0 \Rightarrow - \begin{array}{c}
\begin{array}{c}
\text{closed truncations.}
\end{array}
\end{array} \]

Vacuum polarization:
\[ \Sigma' = \frac{\delta \Gamma_2}{\delta D'} = - \begin{array}{c}
\begin{array}{c}
\text{closed truncations.}
\end{array}
\end{array} \]

BSE kernel:
\[ -K = \frac{\delta \Sigma}{\delta D} = - \begin{array}{c}
\begin{array}{c}
\text{closed truncations.}
\end{array}
\end{array} \]

nPI effective action

nPI effective actions provide symmetry-preserving closed truncations. 3PI at 3-loop: all two- and three-point functions are dressed; 4, 5, ... do not appear.

\[
\Gamma_2 = - \begin{array}{c}
\text{Diagram 1} \\
\text{Diagram 2} \\
\text{Diagram 3}
\end{array} + \frac{1}{2} \begin{array}{c}
\text{Diagram 1} \\
\text{Diagram 2} \\
\text{Diagram 3}
\end{array} + \frac{1}{4} \begin{array}{c}
\text{Diagram 1} \\
\text{Diagram 2} \\
\text{Diagram 3}
\end{array}
\]


So we arrive at a closed system of equations:

- Crossed ladder cannot be added by hand, requires vertex correction!
nPI effective action

nPI effective actions provide symmetry-preserving closed truncations.
3PI at 3-loop: all two- and three-point functions are dressed; 4, 5, ... do not appear.

So we arrive at a closed system of equations:

- Crossed ladder cannot be added by hand, requires vertex correction!
- without 3-loop term: rainbow-ladder with tree-level vertex ⇒ 2PI
- but still requires DSE solutions for propagators!
- Similar in QCD. nPI truncation guarantees chiral symmetry, massless pion in chiral limit, etc.

A toy model

Scattering amplitude for two **massive scalar particles** (mass $m$) with **massive exchange particle** (mass $\mu$):

$$T(p, \Sigma, \Delta) = K(p, \Sigma) + \int \frac{\,d^4k}{\sqrt{2m^2\Delta^2}} \, T(p, k, \Delta) \, D(k_+) \, D(k_-) \, K(k, \Sigma)$$

**Onshell amplitude:** Mandelstam plane

- $t = \frac{\Delta^2}{4m^2}$, $\lambda = -\frac{p \cdot \Sigma}{m^2}$
- Born terms for exchange particle produce s- and u-channel poles
- Bound state pole in t channel
A toy model

Scattering amplitude for two massive scalar particles (mass $m$) with massive exchange particle (mass $\mu$):

$$T(p, \Sigma, \Delta) = K(p, \Sigma) + \int_k T(p, k, \Delta) D(k^+) D(k^-) K(k, \Sigma)$$

Onshell amplitude: Mandelstam plane

- $t = \frac{\Delta^2}{4m^2}$, $\lambda = -\frac{p \cdot \Sigma}{m^2}$
- Born terms for exchange particle produce s- and u-channel poles
- Bound state pole in t channel
- **Poles** in propagators and exchange particle pose restrictions:
  $$-1 < t < \delta, \quad |\lambda| < 1 + t, \quad \delta = \frac{\mu^2}{m^2} - 1$$
**A toy model**

- Born terms for exchange particle produce s- and u-channel poles
- Bound state pole in t channel
- **Poles** in propagators and exchange particle pose restrictions:
  \[-1 < t < \delta, \quad |\lambda| < 1 + t, \quad \delta = \frac{\mu^2}{m^2} - 1\]

**Subtract Born terms** to get rid of s- and u-channel poles ( ↔ 1PI part):
- rise is due to t-channel bound state
- outside blue region: naive integration over poles (wrong)
- scattering amplitude almost independent of \( \lambda \)!

---

Gernot Eichmann (IST Lisboa)
Baryon spectrum

Level ordering between Roper and N(1535):

dynamics of ps diquark produces 2 nearby states: \(N(1535), N(1650)\)

Gernot Eichmann (IST Lisboa)
Eigenvalue spectra

- $N(\frac{1}{2}^+)$ and $\Delta(\frac{3}{2}^+)$ channels hardly affected by ps, v diquarks

- all other channels: sc, av $\rightarrow$ masses too high
- sc, av, ps, v $\rightarrow$ masses too low

- not all eigenvalues extrapolate to masses below 2 GeV

- some are complex conjugate (but imaginary parts small), some split into 2 real branches: numerical or truncation artifact?
Resonances

- **Current-mass evolution** of Roper:
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

- Branch cuts & widths generated by **meson-baryon interactions**: Roper → \( N\pi \), etc.

- **Lattice**: finite volume, DSE (so far): bound states

- ‘**Pion cloud**’ effects difficult to implement at quark-gluon level:

  Resonance dynamics shifts poles into complex plane, but effects on real parts small?
# Nucleon em. form factors

## Nucleon charge radii:
- isovector (p-n) Dirac (F1) radius

\[(r_1^V)^2 \ [fm^2]\]

\[m_{\pi}^2 \ [GeV^2]\]

- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.

## Nucleon magnetic moments:
- isovector (p-n), isoscalar (p+n)

\[\kappa^V \ [\mu_N]\]

\[m_{\pi}^2 \ [GeV^2]\]

\[\kappa^S \ [\mu_N]\]

\[m_{\pi}^2 \ [GeV^2]\]

- **But:** pion-cloud **cancels** in \(\kappa^S \leftrightarrow \) quark core

Exp: \(\kappa^s = -0.12\)  
Calc: \(\kappa^s = -0.12(1)\)  
GE, PRD 84 (2011)
Nucleon-Δ-γ transition

- Magnetic dipole transition ($G_M^*$) dominant: quark spin flip (s wave). “Core + 25% pion cloud”

- Electric & Coulomb quadrupole ratios small & negative, encode deformation.
  Reproduced without pion cloud: OAM from p waves!
  GE, Nicmorus, PRD 85 (2012)

- First three-body results similar

GE, Nicmorus, PRD 85 (2012)
Resonances?

Branch cuts & widths generated by **meson-baryon interactions**: Roper → $N\pi$, etc.

Without them: **bound states without widths**

To generate resonances dynamically at **quark level**: complicated topologies beyond rainbow-ladder

cf. $\rho$ meson: bound state vs. resonance below / above $\pi\pi$ threshold

resonance dynamics shifts pole into complex plane, effect on real part small?

References:
see GE et al., PPNP 91 (2016) 1606.09602
Complex eigenvalues?

**Excited states:** some EVs are complex conjugate?

Typical for *unequal-mass* systems, already in Wick-Cutkosky model

Wick 1954, Cutkosky 1954

Connection with “anomalous” states?


$K(M) \quad G(M) \quad \phi_i(M)$

$\phi_i(M)$

$K$ and $G$ are Hermitian (even for unequal masses!) but $KG$ is not

If $G = G^\dagger$ and $G > 0$:

Cholesky decomposition $G = L^\dagger L$

$K L^\dagger L \phi_i = \lambda_i \phi_i$

$(LKL^\dagger)(L\phi_i) = \lambda_i (L\phi_i)$

⇒ Hermitian problem with same EVs!
Complex eigenvalues?

**Excited states:** some EVs are complex conjugate?

Typical for **unequal-mass** systems, already in Wick-Cutkosky model

Wick 1954, Cutkosky 1954

Connection with “**anomalous**” states?


If \( G = G^\dagger \) and \( G > 0 \):

Cholesky decomposition \( G = L^\dagger L \)

\[
K L^\dagger L \phi_i = \lambda_i \phi_i \\
(LKL^\dagger) (L\phi_i) = \lambda_i (L\phi_i)
\]

⇒ Hermitian problem with same EVs!

⇒ all EVs strictly **real**
⇒ level repulsion
⇒ “anomalous states” removed?

\[
before: \\
after:
\]

\[
\begin{array}{c}
\text{GE, FBS 58 (2017)} \\
\text{only pos. EVs in G} \\
\text{only neg. EVs in G}
\end{array}
\]
Complex eigenvalues?

Excited states: some EVs are complex conjugate?

Typical for unequal-mass systems, already in Wick-Cutkosky model

Wick 1954, Cutkosky 1954

Connection with “anomalous” states?


If \( G = G^\dagger \) and \( G > 0 \):

Cholesky decomposition \( G = L^\dagger L \)

\[ K L^\dagger L \phi_i = \lambda_i \phi_i \]

\[ (LKL^\dagger) (L\phi_i) = \lambda_i (L\phi_i) \]

⇒ Hermitian problem with same EVs!

⇒ all EVs strictly real
⇒ level repulsion
⇒ “anomalous states” removed?
Extracting resonances

Photoproduction of exotic mesons at JLab/GlueX:

What if exotic mesons are relativistic $q\bar{q}$ states? ⇒ study with DSE/BSE!

Scattering amplitudes at quark-gluon level:

$T$

GPD

$N, N^*, \Delta, \ldots$

$\pi, \rho, \ldots$

$N, N^*, \Delta, \ldots$

Meson electroproduction

3 independent variables ($\leftrightarrow s$, $t$, $u$):

$$
\tau = \frac{Q^2}{4m^2}, \quad \eta = \frac{K \cdot Q}{m^2}, \quad \lambda = -\frac{P \cdot Q}{m^2} = -\frac{P \cdot K}{m^2}
$$

Amplitude depends on 6 Lorentz-invariant “FFs”

$$
M^\mu(P, K, Q) = \bar{u}(P_f) \left( \sum_{i=1}^{6} A_i(\tau, \eta, \lambda) M_i^\mu(P, K, Q) \right) u(P_i)
$$

with appropriate tensor basis: no kinematic singularities

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

(lin contrast to Dennery amplitudes)

Fubini, Nambu, Wataghin 1958, Dennery 1961
Meson electroproduction

Singularity structure of quark propagator prevents direct kinematic access to all relevant regions

Strategies:

- if amplitudes free of kinematic singularities, only physical poles and cuts ⇒ extrapolate from unphysical regions (or offshell kinematics)
- clean solution (expensive): use contour deformations
Scattering amplitudes from quark level:

- **$\pi\pi$ scattering**
  Bicudo et al., PRD 65 (2002),

- **Nucleon Compton scattering**
  Goecke, Fischer, PRD 85 (2012) &
  PRD 87 (2013), GE, FBS 57 (2016)

- **Hadronic light-by-light scattering**
  Goecke, Fischer, Williams, PLB 704 (2011),
  GE, Fischer, Heupel, PRD 92 (2015)
Muons

• Muon anomalous magnetic moment: total SM prediction deviates from exp. by \( \sim 3\sigma \)

\[
i e \bar{u}(p') \left[ F_1(q^2) \gamma^\mu - F_2(q^2) \frac{\sigma^{\mu\nu}q^\nu}{2m} \right] u(p)
\]

• Theory uncertainty dominated by QCD: Is QCD contribution under control?

• LBL amplitude: ENJL & MD model results

\[ a_\mu \left[ 10^{-10} \right] \]

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<td>VP (LO+HO) 685.1 (4.3)</td>
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Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by $\sim 3\sigma$

\[
\frac{\gamma}{q} = \frac{ie}{m_p} \bar{u}(p') \left[ F_1(q^2) \gamma^\mu - F_2(q^2) \frac{\sigma^{\mu\nu}q_\nu}{2m_p} \right] u(p)
\]

- **Theory uncertainty dominated by QCD:**
  Is QCD contribution under control?

- **LBL amplitude** at quark level, derived from gauge invariance:

  \[ GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013) \]

\[ a_\mu [10^{-10}] \]

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- **no double-counting, gauge invariant!**
- need to understand **structure of amplitude**

GE, Fischer, Heupel, PRD 92 (2015)