Tritium $\beta$ decay in pionless EFT

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Recipe for EFT(\(\pi\))

- For momenta \(p < m_\pi\) pions can be integrated out as degrees of freedom and only nucleons and external currents are left.
- Write down all possible terms of nucleons and external currents that respect symmetries (rotational, isospin).
- Develop a power counting to organize terms by their relative importance.
  - Organized by counting powers of momentum.
  - Ensure order-by-order results are renormalization group invariant (converge to finite values for \(\Lambda \to \infty\)).
  - Check that various sets of observables converge as expected.
- Calculate respective observables up to a given order in the power counting.
Two-body inputs for EFT(\(\pi\)):

- LO scattering lengths in \(a_1\) (\(^3S_1\)) and \(a_0\) (\(^1S_0\)) non-perturbative
- NLO range corrections \(r_1\) and \(r_0\) perturbative
- \(N^2\)LO SD-mixing term perturbative

Three-body inputs for EFT(\(\pi\)):

- LO three-body force \(H_0\) fit to doublet S-wave \(nd\) scattering length non-perturbative (Bedaque et al.) nucl-th/9906032
- NNLO three-body energy dependent three-body force \(H_2\) fit to triton binding energy perturbative

Total of 6 NNLO parameters, ignoring SD-mixing.
The LO dressed deuteron propagator is given by a bubble sum

\[ \text{Im}[p] = \ \pmatrix{\ldots} \]

\( ^3S_1(S - \text{matrix}) \)

\( \text{Re}[p] \)

\( m_\pi \)

\( i\gamma_t \)

\( (\gamma_t \approx 45\text{MeV}) \)

\((Z\text{-parametrization})\) At LO coefficients are fit to reproduce the deuteron pole and at NLO to reproduce the residue about the deuteron pole (Phillips et al. (2000)) nucl-th/9908054.
Triton vertex function given by infinite sum of diagrams at LO

\[ \begin{align*}
\begin{array}{c}
\text{LO} \\
\text{Triton Vertex function}
\end{array}
\end{align*} \]

\[ = \begin{array}{c}
\text{LO} \\
\text{Triton Vertex function}
\end{array} + \begin{array}{c}
\text{LO} \\
\text{Triton Vertex function}
\end{array} + \begin{array}{c}
\text{LO} \\
\text{Triton Vertex function}
\end{array} + \cdots \]

Infinite sum is represented by integral equation that is solved numerically

\[ \begin{align*}
\begin{array}{c}
\text{LO} \\
\text{Triton Vertex function}
\end{array} = \begin{array}{c}
\text{LO} \\
\text{Triton Vertex function}
\end{array} + \begin{array}{c}
\text{LO} \\
\text{Triton Vertex function}
\end{array} + \begin{array}{c}
\text{LO} \\
\text{Triton Vertex function}
\end{array} + \cdots \]

\[ \begin{array}{c}
\text{LO} \\
\text{Triton Vertex function}
\end{array} = \begin{array}{c}
\text{LO} \\
\text{Triton Vertex function}
\end{array} + \begin{array}{c}
\text{LO} \\
\text{Triton Vertex function}
\end{array} + \begin{array}{c}
\text{LO} \\
\text{Triton Vertex function}
\end{array} + \cdots \]
NLO Triton Vertex function

NLO triton vertex function given by sum of diagrams

\[
\begin{align*}
1 & = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \\
1 & = \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6}
\end{align*}
\]

where

\[
\begin{align*}
\text{Diagram 1} & = \text{Diagram 7} + \text{Diagram 8} + \text{Diagram 9} \\
\text{Diagram 2} & = \text{Diagram 10} + \text{Diagram 11} + \text{Diagram 12}
\end{align*}
\]

Can also be solved by set of integral equations

\[
\begin{align*}
1 & = \text{Diagram 13} + \text{Diagram 14} + \text{Diagram 15} \\
1 & = \text{Diagram 16} + \text{Diagram 17} + \text{Diagram 18}
\end{align*}
\]
Defining

\[
\Sigma_0 = \quad + \quad \Sigma_0 \Sigma_0
\]

The dressed three-nucleon propagator is given by the sum of diagrams

\[
\quad = \quad \Sigma_0 \Sigma_0 \quad + \quad \Sigma_0 \Sigma_0 \quad + \quad \cdots
\]

which yields

\[
i \Delta_3(E) = \frac{i}{\Omega} - \frac{i}{\Omega} H_{LO} \Sigma_0(E) \frac{i}{\Omega} + \cdots \\
= \frac{i}{\Omega} \frac{1}{1 - H_{LO} \Sigma_0(E)},
\]
Higher-Order Three-Nucleon Propagator

Defining the functions

\[ \Sigma_1 = 1 + 1 \]
\[ \Sigma_2 = 2 + 2 \]

The NNLO three-nucleon propagator is

\[
\begin{align*}
\Sigma_1 &+ H_{\text{NLO}} \Sigma_0 \\
\Sigma_2 &+ H_{\text{NLO}} \Sigma_1 + H_{\text{NNLO}} \Sigma_0 \\
&+ (H_{\text{NLO}})^2 \Sigma_0 \Sigma_0 + h_2 
\end{align*}
\]
Properly Renormalized Vertex Function

- Three-body forces are fit to ensure triton propagator has pole at triton binding energy.
- Three-nucleon wavefunction renormalization given by the residue of the three-nucleon propagator about the pole.
- LO three-nucleon wavefunction renormalization is

\[ Z_{\psi}^{LO} = \frac{\pi}{\Sigma'_{0}(B)}. \]

- NNLO three-body force \( h_2 \) fit to triton binding energy and NNLO correction to \( H_0 \) fit to doublet \( S \)-wave \( nd \) scattering length.
- Total of 2 three-body inputs at NNLO.
Three-Nucleon “generic” Form Factor

Three-nucleon LO “generic” form factor given by the three diagrams

(a) (b) (c)

NLO “generic” form factor

(a) (b) (c)

(d) (e)
Form Factor Couplings

Generic form factor can be expanded as

\[ F(Q^2) = a \left( 1 - \frac{1}{6} \langle r^2 \rangle Q^2 + \cdots \right) \]

The couplings for the form factors of interest are given by

<table>
<thead>
<tr>
<th></th>
<th>Charge</th>
<th>Magnetic</th>
<th>Axial</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B LO</td>
<td>( -e \hat{N}^\dagger \hat{N} \hat{A}_0 )</td>
<td>( \hat{N}^\dagger (\kappa_0 + \tau_3 \kappa_1) \vec{\sigma} \cdot \vec{B} \hat{N} )</td>
<td>( \frac{g_A}{\sqrt{2}} \hat{N}^\dagger \sigma_i \tau_+ \hat{N} \hat{A}_i^+ )</td>
</tr>
<tr>
<td>2B NLO</td>
<td>( ec_{0t} \hat{t}_i^\dagger \hat{t}_i \hat{A}_0 )</td>
<td>( e \frac{L_1}{2} \hat{t}_i^\dagger \hat{s}_3 \hat{B}_j - e \frac{L_2}{2} i \epsilon^{ijk} \hat{t}_i^\dagger \hat{t}_j \hat{B}_k )</td>
<td>( l_{1,A} \hat{s}_-^\dagger \hat{t}_i \hat{A}_i^- )</td>
</tr>
<tr>
<td>( a )</td>
<td>( Z )</td>
<td>( \mu )</td>
<td>( \langle \text{GT} \rangle )</td>
</tr>
</tbody>
</table>

**Table:** List of couplings for form factors of interest and their physical values at \( Q^2 = 0 \).
The LO form factor for $Q^2 = 0$ is

$$F_0(0) = 2\pi M_N \left( \tilde{\Gamma}_0(q) \right)^T \otimes \left\{ \frac{\pi}{2} \frac{\delta(q - \ell)}{q^2 \sqrt{\frac{3}{4} q^2 - M_NB}} \left( \begin{array}{cc} c_{11} + a_{11} & c_{12} \\ c_{21} & c_{22} + a_{22} \end{array} \right) \right\} \otimes \tilde{\Gamma}_0(\ell),$$

$$+ \frac{1}{q^2\ell^2 - (q^2 + \ell^2 - M_NB)^2} \left( \begin{array}{cc} b_{11} - 2a_{11} & b_{12} + 3(a_{11} + a_{22}) \\ b_{21} + 3(a_{11} + a_{22}) & b_{22} - 2a_{22} \end{array} \right) \right\} \otimes \tilde{\Gamma}_0(\ell),$$

The coefficients for various form factors are given by

<table>
<thead>
<tr>
<th>Form factor</th>
<th>$a_{11}$</th>
<th>$a_{22}$</th>
<th>$b_{11}$</th>
<th>$b_{12}$</th>
<th>$b_{21}$</th>
<th>$b_{22}$</th>
<th>$c_{11}$</th>
<th>$c_{12}$</th>
<th>$c_{21}$</th>
<th>$c_{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{3^3H}^C(Q^2)$</td>
<td>0</td>
<td>$\frac{2}{3}$</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>$\frac{1}{3}$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$F_{3^3He}^C(Q^2)$</td>
<td>1</td>
<td>$\frac{1}{3}$</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>$-\frac{4}{3}$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$\frac{5}{3}$</td>
</tr>
<tr>
<td>$F_{3^3H}^{GT}(Q^2)$</td>
<td>$-\frac{1}{3}$</td>
<td>$-\frac{1}{3}$</td>
<td>$\frac{5}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{5}{3}$</td>
<td>0</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{2}{3}$</td>
<td>0</td>
</tr>
<tr>
<td>$F_{3^3He}^{GW}(Q^2)$</td>
<td>1</td>
<td>$-\frac{1}{3}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$-\frac{5}{3}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\frac{4}{3}$</td>
</tr>
</tbody>
</table>

Table: Values of coefficients for the LO $^3$H and $^3$He axial and charge form factors.
NLO correction to “generic” form factor at \( Q^2 = 0 \)

\[
F_1(0) = 2\pi M_N \left( \tilde{\Gamma}_1(q) \right)^T \otimes \left\{ \frac{\pi}{2} \frac{\delta(q - \ell)}{q^2 \sqrt{\frac{3}{4} q^2 - M_N B_0}} \begin{pmatrix} c_{11} + a_{11} & c_{12} \\ c_{21} & c_{22} + a_{22} \end{pmatrix} \right. \\
+ \frac{1}{q^2 \ell^2 - (q^2 + \ell^2 - M_N B_0)^2} \left( \begin{pmatrix} b_{11} - 2a_{11} & b_{12} + 3(a_{11} + a_{22}) \\ b_{21} + 3(a_{11} + a_{22}) & b_{22} - 2a_{22} \end{pmatrix} \right) \left. \right\} \otimes \tilde{\Gamma}_0(\ell) \\
+ 2\pi M_N \left( \tilde{\Gamma}_0(q) \right)^T \otimes \left\{ \frac{\pi}{2} \frac{\delta(q - \ell)}{q^2 \sqrt{\frac{3}{4} q^2 - M_N B_0}} \begin{pmatrix} c_{11} + a_{11} & c_{12} \\ c_{21} & c_{22} + a_{22} \end{pmatrix} \right. \\
+ \frac{1}{q^2 \ell^2 - (q^2 + \ell^2 - M_N B_0)^2} \left( \begin{pmatrix} b_{11} - 2a_{11} & b_{12} + 3(a_{11} + a_{22}) \\ b_{21} + 3(a_{11} + a_{22}) & b_{22} - 2a_{22} \end{pmatrix} \right) \left. \right\} \otimes \tilde{\Gamma}_1(\ell) \\
- 4\pi M_N \left( \tilde{\Gamma}_0(q) \right)^T \otimes \left\{ \frac{\pi}{2} \frac{\delta(q - \ell)}{q^2} \begin{pmatrix} \frac{1}{2} \rho_t a_{11} + d_{11} & d_{12} \\ d_{21} & \frac{1}{2} \rho_s a_{22} + d_{22} \end{pmatrix} \right\} \otimes \tilde{\Gamma}_0(\ell),
\]

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NLO Form Factor (cont.)

\[
\begin{align*}
F_{\text{C}}^{3\text{H}}(Q^2) & = \frac{1}{2} \rho_t \\
F_{\text{C}}^{3\text{He}}(Q^2) & = \frac{1}{2} \rho_t \\
F_{\text{M}}^{3\text{H}}(Q^2) & = -\frac{2}{3} L_2 + \frac{1}{3} L_1 \\
F_{\text{M}}^{3\text{He}}(Q^2) & = -\frac{2}{3} L_2 - \frac{1}{3} L_1 \\
F_{\text{GT}}^{W}(Q^2) & = 0
\end{align*}
\]

<table>
<thead>
<tr>
<th>Form factor</th>
<th>(d_{11})</th>
<th>(d_{12})</th>
<th>(d_{21})</th>
<th>(d_{22})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F_{\text{C}}^{3\text{H}}(Q^2))</td>
<td>(\frac{1}{2} \rho_t)</td>
<td>0</td>
<td>0</td>
<td>(\frac{1}{3} \frac{1}{2} \rho_s)</td>
</tr>
<tr>
<td>(F_{\text{C}}^{3\text{He}}(Q^2))</td>
<td>(\frac{1}{2} \rho_t)</td>
<td>0</td>
<td>0</td>
<td>(\frac{5}{3} \frac{1}{2} \rho_s)</td>
</tr>
<tr>
<td>(F_{\text{M}}^{3\text{H}}(Q^2))</td>
<td>(-\frac{2}{3} L_2)</td>
<td>(\frac{1}{3} L_1)</td>
<td>(\frac{1}{3} L_1)</td>
<td>0</td>
</tr>
<tr>
<td>(F_{\text{M}}^{3\text{He}}(Q^2))</td>
<td>(-\frac{2}{3} L_2)</td>
<td>(-\frac{1}{3} L_1)</td>
<td>(-\frac{1}{3} L_1)</td>
<td>0</td>
</tr>
<tr>
<td>(F_{\text{GT}}^{W}(Q^2))</td>
<td>0</td>
<td>(\frac{1}{3} l_{1,A})</td>
<td>(\frac{1}{3} l_{1,A})</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table:** Values of coefficients for the NLO corrections to (d)-type diagrams for the \(^3\text{H}\) and \(^3\text{He}\) magnetic, charge, and axial form factors.
Radii

Generic form factor given by

\[ F(Q^2) = a \left( 1 - \frac{1}{6} \langle r^2 \rangle Q^2 + \cdots \right) \]

Calculating only \( Q^2 \) contribution for diagram-(a) gives

\[
\frac{1}{2} \left. \frac{\partial^2}{\partial Q^2} F_n^{(a)}(Q^2) \right|_{Q^2=0} = Z_{LO}^{\psi} \sum_{i,j=0}^{i+j \leq n} \left\{ \tilde{G}_i^T(p) \otimes A_{n-i-j}(p,k) \otimes \tilde{G}_j(k) + 2 \tilde{G}_i^T(p) \otimes A_{n-i}(p) \delta_{j0} + A_n \delta_{i0} \delta_{j0} \right\},
\]
LO EFT(\(\chi\)) \(r_C = 2.1 \pm .6 \text{ fm}\) (Platter and Hammer (2005)) \text{nucl-th/0509045}

NLO EFT(\(\chi\)) \(r_C = 1.6 \pm .2 \text{ fm}\) (Kirscher et al. (2010)) \text{arXiv:0903.5538}

NNLO EFT(\(\chi\)) \(r_C = 1.62 \pm .03 \text{ fm}\) (Vanasse (2016)) \text{arXiv:1512.03805}
The iso-scalar and iso-vector combination of magnetic moments are

\[ \mu_s = \frac{1}{2} \left( \mu^{3\text{He}} + \mu^{3\text{H}} \right), \quad \mu_v = \frac{1}{2} \left( \mu^{3\text{He}} - \mu^{3\text{H}} \right), \]

\( \mu_s \) only depends on \( \kappa_0 \) and \( L_2 \), while \( \mu_v \) only depends on \( \kappa_1 \) and \( L_1 \) at NLO.

<table>
<thead>
<tr>
<th></th>
<th>( \mu_s )</th>
<th>( \mu_v )</th>
<th>( L_1 ) fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>0.440(152)</td>
<td>-2.31(78)</td>
<td>N/A</td>
</tr>
<tr>
<td>NLO</td>
<td>0.421(50)</td>
<td>-2.20(26)</td>
<td>( \sigma_{np} )</td>
</tr>
<tr>
<td>NLO</td>
<td>0.421(50)</td>
<td>-2.56(31)</td>
<td>( \mu^{3\text{H}} )</td>
</tr>
<tr>
<td>NLO</td>
<td>0.421(50)</td>
<td>-2.50(30)</td>
<td>( \sigma_{np} ) and ( \mu^{3\text{H}} )</td>
</tr>
<tr>
<td>Exp</td>
<td>0.426</td>
<td>-2.55</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**Table:** Table of three-nucleon iso-scalar and iso-vector magnetic moments compared to experiment. The different NLO rows are different fits for \( L_1 \) and are organized the same as the previous table.
NLO two-body magnetic currents given by

\[ \mathcal{L}_{2}^{\text{mag}} = \left( e^{\frac{L_{1}}{2}} \hat{t}^{j^{\dagger}} \hat{s}_{3} \mathbf{B}_{j} + \text{H.c} \right) - e^{\frac{L_{2}}{2}} i \epsilon^{ijk} \hat{t}^{i^{\dagger}} \hat{t}^{j} \mathbf{B}_{k}. \]

\( L_{2} \) is fit to deuteron magnetic moment and \( L_{1} \) is typically fit to cold \( np \) capture cross section \( (\sigma_{np}) \)

Magnetic moments and polarizabilities also calculated to NLO by (Kirscher et al. (2017)) arXiv:1702.07268

<table>
<thead>
<tr>
<th></th>
<th>( \mu_{3}^{\text{H}} )</th>
<th>( \mu_{3}^{\text{He}} )</th>
<th>( r_{M}^{\text{\MakeLowercase{He}}} ) ( \text{fm} )</th>
<th>( r_{M}^{\text{\MakeLowercase{He}}} ) ( \text{fm} )</th>
<th>( \sigma_{np} ) ( \text{mb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>2.75(95)</td>
<td>-1.87(73)</td>
<td>1.40(24)</td>
<td>1.49(26)</td>
<td>325.2 ± 225.6</td>
</tr>
<tr>
<td>NLO</td>
<td>2.62(31)</td>
<td>-1.78(33)</td>
<td>1.83(11)</td>
<td>1.92(11)</td>
<td>334.2 ± 79.7</td>
</tr>
<tr>
<td>NLO</td>
<td>2.98(36)</td>
<td>-2.14(26)</td>
<td>1.77(11)</td>
<td>1.83(11)</td>
<td>370.47 ± 88.4</td>
</tr>
<tr>
<td>NLO</td>
<td>2.92(35)</td>
<td>-2.08(25)</td>
<td>1.78(11)</td>
<td>1.85(11)</td>
<td>364.5 ± 87.0</td>
</tr>
<tr>
<td>Exp</td>
<td>2.979</td>
<td>-2.127</td>
<td>1.84(18)</td>
<td>1.97(15)</td>
<td>334.2(5)</td>
</tr>
</tbody>
</table>

Table: Values of magnetic moments and magnet radii for three-nucleon systems and \( \sigma_{np} \) to NLO compared to experiment. The first NLO row is for \( L_{1} \) fit to \( \sigma_{np} \), the second NLO row for \( L_{1} \) fit to the \( ^{3}\text{H} \) magnetic moment \( (\mu_{3}^{\text{H}}) \), and the final NLO row is \( L_{1} \) fit to both \( \sigma_{np} \) and \( \mu_{3}^{\text{H}} \).
Bound State Observables for 3N Systems


<table>
<thead>
<tr>
<th>Observable</th>
<th>LO</th>
<th>NLO</th>
<th>NNLO</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3$H: $r_C$ [fm]</td>
<td>1.14(19)</td>
<td>1.59(8)</td>
<td>1.62(3)</td>
<td>1.5978(40)</td>
</tr>
<tr>
<td>$^3$He: $r_C$ [fm]</td>
<td>1.26(21)</td>
<td>1.72(8)</td>
<td>1.74(3)</td>
<td>1.7753(54)</td>
</tr>
<tr>
<td>$^3$H: $r_m$ [fm]</td>
<td>1.40(24)</td>
<td>1.78(11)</td>
<td>–</td>
<td>1.840(181)</td>
</tr>
<tr>
<td>$^3$He: $r_m$ [fm]</td>
<td>1.49(26)</td>
<td>1.85(11)</td>
<td>–</td>
<td>1.965(153)</td>
</tr>
<tr>
<td>$^3$H: $\mu_m$ [$\mu_N$]</td>
<td>2.75(92)</td>
<td>2.92(35)</td>
<td>–</td>
<td>2.98</td>
</tr>
<tr>
<td>$^3$He: $\mu_m$ [$\mu_N$]</td>
<td>-1.87(73)</td>
<td>-2.08(25)</td>
<td>–</td>
<td>-2.13</td>
</tr>
</tbody>
</table>

Calculation of LO triton charge radius in unitary limit gives

$$mE_{3B} \langle r_C^2 \rangle = 0.224...$$

Using analytical techniques in (Braaten and Hammer (2006)) cond-mat/0410417 it can be shown that

$$mE_{3B} \langle r_C^2 \rangle = (1 + s_0^2)/9 = 0.224...$$ in the unitary limit.
Tritium $\beta$-decay

Half life $t_{1/2}$ of tritium given by

$$
(1 + \delta_R) f_V \frac{K}{G_V^2} t_{1/2} = \frac{1}{\langle F \rangle^2 + f_A/f_V g_A^2 \langle GT \rangle^2}
$$

The Gamow-Teller matrix element is

$$
\frac{\langle GT \rangle_{\text{Exp}}}{\sqrt{3}} = 0.9551, \quad \frac{\langle GT \rangle_0}{\sqrt{3}} = 0.9807, \quad \frac{\langle GT \rangle_{0+1}}{\sqrt{3}} = 0.9935
$$

Fitting $L_{1A}$ to the GT-matrix element gives

$$
L_{1A} = 3.46 \pm 1.19 \text{ fm}^3
$$

Compares well to lattice prediction

$$
L_{1A} = 3.9(0.1)(1.0)(0.3)(0.9) \text{ fm}^3
$$
Gamow-Teller and Wigner-symmetry

The GT-matrix element is given by

$$\langle GT \rangle \simeq \sqrt{3}(P_S + P_D/3 - P_S'/3)$$

In Wigner-SU(4) limit $P_S' = 0$ and $P_S = 1$ hence $\langle GT \rangle = \sqrt{3}$ and can also be seen by

$$\langle GT \rangle = \langle 3\text{He} \mid \sum_i \sigma^{(i)} \tau^{(i)}_+ \mid 3\text{H} \rangle$$

$$= \langle 3\text{He} \mid \sum_i \sigma^{(i)} \mid 3\text{He} \rangle$$

$$= \langle 3\text{He} \mid \sigma \mid 3\text{He} \rangle$$

$$= \sqrt{3}$$
Fermi matrix element

Half life $t_{1/2}$ of tritium given by

$$
\frac{(1 + \delta_R)f_V}{K/G_V^2} t_{1/2} = \frac{1}{\langle F \rangle^2 + f_A/f_V g_A^2 \langle GT \rangle^2}
$$

In the isospin limit the Fermi matrix element reduces to

$$
\langle F \rangle = \langle ^3\text{He} \mid \sum_i \tau_+^{(i)} \mid ^3\text{H} \rangle
$$

$$
= \langle ^3\text{He} \mid 1 \mid ^3\text{He} \rangle
$$

$$
= 1
$$

Indeed, we find $\langle F \rangle = 1$. 
Consequences of Wigner-symmetry and Unitarity

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Table: \(^3\text{H}/^3\text{He} \) charge radius in unitary and Wigner-limit ([Vanasse and Phillips (2016)](arXiv:1607.08585))

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\mu(^3\text{H}) = \mu_p = 2.79 \frac{e}{2M_N}, \quad \mu(^3\text{He}) = \mu_n = -1.91 \frac{e}{2M_N}
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Conclusions and Future directions

- Charge radii of $^3\text{H}$ and $^3\text{He}$ reproduced well at NNLO in EFT($\frac{\alpha}{\pi}$).
- Magnetic moments and radii reproduced within errors at NLO in EFT($\frac{\alpha}{\pi}$).
- $L_{1A}$ prediction agrees with LQCD prediction. Better prediction for $L_{1A}$ will further constrain EFT($\frac{\alpha}{\pi}$) prediction for $pp$ fusion.
- Wigner-symmetry gives good expansion for charge radii and is interesting limit for three-nucleon magnetic moments and GT-matrix element. Results should be used as benchmark.
- Reproduce analytical results in unitary limit for charge radii. Should be used as benchmark for all such calculations.