$0\nu\beta\beta$ decay nuclear matrix elements with the generator coordinate method

Tomás R. Rodríguez

INT-Program 17-2a

Seattle, June 13-14, 2017
$0\nu\beta\beta$ decay nuclear matrix elements with energy density functional (EDF) methods

Tomás R. Rodríguez

INT-Program 17-2a

Seattle, June 13-14, 2017
J. Menéndez (University of Tokio)
G. Martínez-Pinedo (GSI-Darmstadt and TU-Darmstadt)
A. Poves (UAM-Madrid)
F. Nowacki (IPHC-Strasbourg)
J. Engel (UNC-Chapel Hill)
N. Hinohara (University of Tsukuba)
N. López-Vaquero (UAM-Madrid)
J. L. Egido (UAM-Madrid)
1. Introduction

2. EDF applications

3. GCM-EDF vs. Shell Model

4. Summary and open questions
Different many-body methods provide different $0\nu\beta\beta$ NMEs

Where the differences come from?

- Correlations are not the same.
- Interactions are different.
- Valence spaces are different.
- Transition operator is (sometimes) different.
• Leading lepton number violating process contributing to $0\nu\beta\beta$ decay
  - Exchange of light Majorana neutrino.
  - Exchange of heavy Majorana neutrino.
  - Leptoquarks.
  - Supersymmetric particles.
  - ...

• Transition operator connecting initial and final states
  - Relativistic/Non-relativistic.
  - Nucleon size effects.
  - Two-body weak currents.
  - Form factors.
  - Short-range correlations.
  - Closure approximation.
  - ...

• Nuclear structure method (fully consistent or not with the operator) for calculating these NME.
  - Correlations.
  - Symmetry conservation.
  - Valence space.
  - ...
This is a general method based on the concept of configuration mixing. The wave function that describes the system in this framework can be expressed as:

\[ |\Psi^\sigma\rangle = \int f^\sigma(\vec{q})|\Phi(\vec{q})\rangle d\vec{q} \]

\( \{ |\Phi(\vec{q})\rangle \} \) is a set of (in general) non-orthonormal many-body wave functions that depends parametrically on the collective variables \( \vec{q} \), called generating coordinates.

\( f^\sigma(\vec{q}) \) are found by minimizing the energy:

\[ E[|\Psi^\sigma\rangle] = \frac{\langle \Psi^\sigma | \hat{H} | \Psi^\sigma \rangle}{\langle \Psi^\sigma | \Psi^\sigma \rangle} \]
This is a general method based on the concept of configuration mixing. The wave function that describes the system in this framework can be expressed as:

\[ |\Psi^\sigma\rangle = \int f^\sigma (\vec{q}) |\Phi (\vec{q})\rangle d\vec{q} \]

→ \( f^\sigma (\vec{q}) \) are found by minimizing the energy:

\[ E [|\Psi^\sigma\rangle] = \frac{\langle \Psi^\sigma | \hat{H} |\Psi^\sigma\rangle}{\langle \Psi^\sigma | \Psi^\sigma\rangle} \]

\[ \delta E [|\Psi^\sigma\rangle] = 0 \Rightarrow \int \mathcal{H} (\vec{q}, \vec{q}') f^\sigma (\vec{q}') d\vec{q}' = E^\sigma \int \mathcal{N} (\vec{q}, \vec{q}') f^\sigma (\vec{q}') d\vec{q}' \]

Hill-Wheeler-Griffin (HWG) equations
This is a general method based on the concept of configuration mixing. The wave function that describes the system in this framework can be expressed as:

$$|\Psi^\sigma\rangle = \int f^\sigma(\vec{q})|\Phi(\vec{q})\rangle d\vec{q}$$

→ $f^\sigma(\vec{q})$ are found by minimizing the energy:

$$E[|\Psi^\sigma\rangle] = \frac{\langle\Psi^\sigma|\hat{H}|\Psi^\sigma\rangle}{\langle\Psi^\sigma|\Psi^\sigma\rangle}$$

$$\delta E[|\Psi^\sigma\rangle] = 0 \Rightarrow \int \mathcal{H}(\vec{q}, \vec{q}^\prime)f^\sigma(\vec{q}^\prime)d\vec{q}^\prime = E^\sigma \int \mathcal{N}(\vec{q}, \vec{q}^\prime)f^\sigma(\vec{q}^\prime)d\vec{q}^\prime$$

Hill-Wheeler-Griffin (HWG) equations

$$\mathcal{N}(\vec{q}, \vec{q}^\prime) = \langle\Phi(\vec{q})|\Phi(\vec{q}^\prime)\rangle$$ norm overlap matrix

$$\mathcal{H}(\vec{q}, \vec{q}^\prime) = \langle\Phi(\vec{q})|\hat{H}|\Phi(\vec{q}^\prime)\rangle$$ hamiltonian overlap matrix
How to solve the HGW equations

\[ \int \mathcal{H}(\vec{q}, \vec{q}') f^\sigma(\vec{q}') d\vec{q}' = E^\sigma \int \mathcal{N}(\vec{q}, \vec{q}') f^\sigma(\vec{q}') d\vec{q}' \]
Generator Coordinate Method

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

How to solve the HGW equations

\[
\int \mathcal{H}(\vec{q}, \vec{q}') f^{\sigma}(\vec{q}') d\vec{q}' = E^{\sigma} \int \mathcal{N}(\vec{q}, \vec{q}') f^{\sigma}(\vec{q}') d\vec{q}'
\]

1. Find the eigenvalues and eigenvectors of the norm overlap matrix:

\[
\int \mathcal{N}(\vec{q}, \vec{q}') u_\Lambda(\vec{q}') d\vec{q}' = n_\Lambda u_\Lambda(\vec{q})
\]
Generator Coordinate Method

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

How to solve the HGW equations

\[
\int \mathcal{H}(\vec{q}, \vec{q}') f^\sigma(\vec{q}') d\vec{q}' = E^\sigma \int \mathcal{N}(\vec{q}, \vec{q}') f^\sigma(\vec{q}') d\vec{q}'
\]

1. Find the eigenvalues and eigenvectors of the norm overlap matrix:

\[
\int \mathcal{N}(\vec{q}, \vec{q}') u_\Lambda(\vec{q}') d\vec{q}' = n_\Lambda u_\Lambda(\vec{q})
\]

2. From the eigenvalues and eigenvectors of the norm overlap matrix, build an orthonormal basis removing the linear dependencies (natural basis):

\[
|\Lambda\rangle = \frac{1}{\sqrt{n_\Lambda}} \int u_\Lambda(\vec{q})|\Phi(\vec{q})\rangle d\vec{q} \quad n_\Lambda > 0 \quad \langle \Lambda|\Lambda'\rangle = \delta_{\Lambda\Lambda'}
\]
How to solve the HGW equations

\[ \int \mathcal{H}(\vec{q}, \vec{q}') f^\sigma(\vec{q}') d\vec{q}' = E^\sigma \int \mathcal{N}(\vec{q}, \vec{q}') f^\sigma(\vec{q}') d\vec{q}' \]

1. Find the eigenvalues and eigenvectors of the norm overlap matrix:

\[ \int \mathcal{N}(\vec{q}, \vec{q}') u_\Lambda(\vec{q}') d\vec{q}' = n_\Lambda u_\Lambda(\vec{q}) \]

2. From the eigenvalues and eigenvectors of the norm overlap matrix, build an orthonormal basis removing the linear dependencies (natural basis):

\[ |\Lambda\rangle = \frac{1}{\sqrt{n_\Lambda}} \int u_\Lambda(\vec{q})|\Phi(\vec{q})\rangle d\vec{q} ; \quad n_\Lambda > 0 \quad \langle \Lambda|\Lambda'\rangle = \delta_{\Lambda\Lambda'} \]

3. Re-write the GCM wave functions and the HWG equation in the natural basis:

\[ |\Psi^\sigma\rangle = \sum_\Lambda g_\Lambda^\sigma |\Lambda\rangle = \int |\Phi(\vec{q})\rangle \sum_\Lambda \frac{1}{\sqrt{n_\Lambda}} g_\Lambda^\sigma u_\Lambda(\vec{q}) d\vec{q} \]
1. Find the eigenvalues and eigenvectors of the norm overlap matrix:

\[ \int \mathcal{N}(\vec{q}, \vec{q}') u_\Lambda(\vec{q}') d\vec{q}' = n_\Lambda u_\Lambda(\vec{q}) \]

2. From the eigenvalues and eigenvectors of the norm overlap matrix, build an orthonormal basis removing the linear dependencies (natural basis):

\[ |\Lambda\rangle = \frac{1}{\sqrt{n_\Lambda}} \int u_\Lambda(\vec{q}) |\Phi(\vec{q})\rangle d\vec{q} \quad ; \quad n_\Lambda > 0 \]

\[ \langle \Lambda | \Lambda' \rangle = \delta_{\Lambda\Lambda'} \]

3. Re-write the GCM wave functions and the HWG equation in the natural basis:

\[ |\Psi^\sigma\rangle = \sum_\Lambda g_\Lambda^\sigma |\Lambda\rangle = \int |\Phi(\vec{q})\rangle \sum_\Lambda \frac{1}{\sqrt{n_\Lambda}} g_\Lambda^\sigma u_\Lambda(\vec{q}) d\vec{q} \]

\[ f^\sigma(\vec{q}) \]
How to solve the HGW equations

1. Find the eigenvalues and eigenvectors of the norm overlap matrix:

\[ \int \mathcal{N}(\vec{q}, \vec{q}') u_\Lambda(\vec{q}') d\vec{q}' = n_\Lambda u_\Lambda(\vec{q}) \]

2. From the eigenvalues and eigenvectors of the norm overlap matrix, build an orthonormal basis removing the linear dependencies (natural basis):

\[ |\Lambda\rangle = \frac{1}{\sqrt{n_\Lambda}} \int u_\Lambda(\vec{q}) |\Phi(\vec{q})\rangle d\vec{q}; \quad n_\Lambda > 0 \quad \langle \Lambda | \Lambda' \rangle = \delta_{\Lambda\Lambda'} \]

3. Re-write the GCM wave functions and the HWG equation in the natural basis:

\[ |\Psi^\sigma\rangle = \sum_\Lambda g_\Lambda^\sigma |\Lambda\rangle = \int |\Phi(\vec{q})\rangle \sum_\Lambda \frac{1}{\sqrt{n_\Lambda}} g_\Lambda^\sigma u_\Lambda(\vec{q}) d\vec{q} \quad \Rightarrow \sum_\Lambda \langle\Lambda|\hat{H}|\Lambda'\rangle g_\Lambda^\sigma = E^\sigma g_\Lambda^\sigma \]

HGW equations are now regular eigenvalue problems.
1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

How to solve the HGW equations

\[ \sum_{\Lambda'} \langle \Lambda | \hat{H} | \Lambda' \rangle g_{\Lambda'}^{\sigma} = E^{\sigma} g_{\Lambda}^{\sigma} \]

4. Hamiltonian (or any other operator \( \hat{O} \)) matrix elements in the natural basis:

\[ \langle \Lambda | \hat{O} | \Lambda' \rangle = \int \left( \frac{u_{\Lambda}(\tilde{q})}{\sqrt{n_{\Lambda}}} \right)^* \langle \Phi(\tilde{q}) | \hat{O} | \Phi(\tilde{q}') \rangle \left( \frac{u_{\Lambda'}(\tilde{q}')}{\sqrt{n_{\Lambda'}}} \right) d\tilde{q} d\tilde{q}' \]
Generator Coordinate Method

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

How to solve the HGW equations

\[ \sum_{\Lambda'} \langle \Lambda | \hat{H} | \Lambda' \rangle g_{\Lambda'}^\sigma = E^\sigma g_{\Lambda}^\sigma \]

4. Hamiltonian (or any other operator \( \hat{O} \)) matrix elements in the natural basis:

\[ \langle \Lambda | \hat{O} | \Lambda' \rangle = \int \left( \frac{u_{\Lambda}(\vec{q})}{\sqrt{n_{\Lambda}}} \right)^* \langle \Phi(\vec{q}) | \hat{O} | \Phi(\vec{q}') \rangle \left( \frac{u_{\Lambda'}(\vec{q}')}{\sqrt{n_{\Lambda'}}} \right) d\vec{q} d\vec{q}' \]

matrix elements between different "deformations"
1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

How to solve the HGW equations

\[ \Rightarrow \sum_{\Lambda'} \langle \Lambda | \hat{H} | \Lambda' \rangle g^\sigma_{\Lambda'} = E^\sigma g^\sigma_{\Lambda} \]

4. Hamiltonian (or any other operator \( \hat{O} \)) matrix elements in the natural basis:

\[ \langle \Lambda | \hat{O} | \Lambda' \rangle = \int \left( \frac{u_\Lambda(q)}{\sqrt{n_\Lambda}} \right)^* \langle \Phi(q') | \hat{O} | \Phi(q) \rangle \left( \frac{u_{\Lambda'}(q')}{\sqrt{n_{\Lambda'}}} \right) dq dq' \]

5. Hamiltonian (or any other operator \( \hat{O} \)) expectation values in the GCM states (and transitions):

\[ \langle \Psi^\sigma | \hat{O} | \Psi^\sigma \rangle = \sum_{\Lambda \Lambda'} g^\sigma_{\Lambda}^* \langle \Lambda | \hat{O} | \Lambda' \rangle g^\sigma_{\Lambda'} \]
1. Introduction

2. EDF applications

3. GCM vs Shell Model

4. Summary and open questions

How to solve the HGW equations

\[ \sum_{\Lambda'} \langle \Lambda | \hat{H} | \Lambda' \rangle g_{\Lambda}^{\sigma} = E_{\Lambda}^{\sigma} g_{\Lambda}^{\sigma} \]

4. Hamiltonian (or any other operator \( \hat{O} \)) matrix elements in the natural basis:

\[ \langle \Lambda | \hat{O} | \Lambda' \rangle = \int \left( \frac{u_{\Lambda}(\vec{q})}{\sqrt{n_{\Lambda}}} \right)^{\ast} \langle \Phi(\vec{q}) | \hat{O} | \Phi(\vec{q}') \rangle \left( \frac{u_{\Lambda'}(\vec{q}')}{\sqrt{n_{\Lambda'}}} \right) d\vec{q} d\vec{q}' \]

5. Hamiltonian (or any other operator \( \hat{O} \)) expectation values in the GCM states (and transitions):

\[ \langle \Psi_{\sigma} | \hat{O} | \Psi_{\sigma} \rangle = \sum_{\Lambda \Lambda'} g_{\Lambda}^{\sigma \ast} \langle \Lambda | \hat{O} | \Lambda' \rangle g_{\Lambda'}^{\sigma} \]

6. Collective wave functions: Weight of the different \( \vec{q} \) in the GCM wave function:

\[ F_{\sigma}(\vec{q}) = \sum_{\Lambda} g_{\Lambda}^{\sigma} u_{\Lambda}(\vec{q}) \]
How to solve the HGW equations

\[ \sum_{\Lambda'} \langle \Lambda | \hat{H} | \Lambda' \rangle g_{\Lambda}^{\sigma} = E^{\sigma} g_{\Lambda}^{\sigma} \]

7. Transition matrix elements in the natural basis:

\[ \langle \Lambda_f | \hat{T}^i \rightarrow f | \Lambda_i \rangle = \int \left( \frac{u_{\Lambda_f}(\vec{q}_f)}{\sqrt{n_{\Lambda_f}}} \right)^* \langle \Phi(\vec{q}_f) | \hat{T}^i \rightarrow f | \Phi(\vec{q}_i) \rangle \left( \frac{u_{\Lambda_i}(\vec{q}_i)}{\sqrt{n_{\Lambda_i}}} \right) d\vec{q}_f d\vec{q}_i \]

8. Transition matrix elements between GCM states:

\[ \langle \Psi_f^{\sigma} | \hat{T}^i \rightarrow f | \Psi_i^{\sigma} \rangle = \sum_{\Lambda_f \Lambda_i} g_{\Lambda_f}^{\sigma} * \langle \Lambda_f | \hat{T}^i \rightarrow f | \Lambda_i \rangle g_{\Lambda_i}^{\sigma} \]
Generator Coordinate Method

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

How to solve the HGW equations

\[ \Rightarrow \sum_{\Lambda'} \langle \Lambda | \hat{H} | \Lambda' \rangle g_{\Lambda'}^\sigma = E^\sigma g_{\Lambda}^\sigma \]

7. Transition matrix elements in the natural basis:

\[ \langle \Lambda_f | \hat{T}^{i \rightarrow f} | \Lambda_i \rangle = \int \left( \frac{u_{\Lambda_f}(\vec{q}_f)}{\sqrt{n_{\Lambda_f}}} \right)^* \langle \Phi(\vec{q}_f) | \hat{T}^{i \rightarrow f} | \Phi(\vec{q}_i) \rangle \left( \frac{u_{\Lambda_i}(\vec{q}_i)}{\sqrt{n_{\Lambda_i}}} \right) d\vec{q}_f d\vec{q}_i \]

transition matrix elements between different “deformations”

8. Transition matrix elements between GCM states:

\[ \langle \Psi_{\sigma f}^* | \hat{T}^{i \rightarrow f} | \Psi_{\sigma i} \rangle = \sum_{\Lambda_f \Lambda_i} g_{\Lambda_f}^{\sigma *} \langle \Lambda_f | \hat{T}^{i \rightarrow f} | \Lambda_i \rangle g_{\Lambda_i}^\sigma \]
How to solve the HGW equations

\[ \Rightarrow \sum_{\Lambda'} \langle \Lambda | \hat{H} | \Lambda' \rangle g_{\Lambda'}^\sigma = E^\sigma g_{\Lambda}^\sigma \]

7. Transition matrix elements in the natural basis:

\[ \langle \Lambda_f | \hat{T}^i \rightarrow f | \Lambda_i \rangle = \int \left( \frac{u_{\Lambda_f} (\vec{q}_f)}{\sqrt{n_{\Lambda_f}}} \right)^* \langle \Phi (\vec{q}_f) | \hat{T}^i \rightarrow f | \Phi (\vec{q}_i) \rangle \left( \frac{u_{\Lambda_i} (\vec{q}_i)}{\sqrt{n_{\Lambda_i}}} \right) d\vec{q}_f d\vec{q}_i \]

8. Transition matrix elements between GCM states:

\[ \langle \Psi_{\sigma_f}^f | \hat{T}^i \rightarrow f | \Psi_{\sigma_i}^i \rangle = \sum_{\Lambda_f \Lambda_i} g_{\Lambda_f}^{\sigma_f} \langle \Lambda_f | \hat{T}^i \rightarrow f | \Lambda_i \rangle g_{\Lambda_i}^{\sigma_i} \]
REMARKS

- GCM ground states are variational approaches to the exact ground state wave functions.

- The quality of the approximation depends on the sensitivity of the collective coordinates to the nuclear Hamiltonian and/or transition operators.

- Very intuitive physical insight about the role of collective degrees of freedom on $0\nu\beta\beta$ NMEs.

IMPLEMENTATIONS

- Non-relativistic Gogny and Relativistic energy density functionals (EDF).

- $SO(8)$ and Pairing (isoscalar and isovector) plus quadrupole Hamiltonians.

- Shell Model interactions in reduced valence spaces (in progress).
Gogny interaction

Effective nucleon-nucleon interaction: Gogny force (D1S/D1M)

\[
V(1, 2) = \sum_{i=1}^{2} e^{-\frac{(\vec{r}_1 - \vec{r}_2)^2}{\mu_i^2}} \left( W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau \right) \\
+ iW_0 (\sigma_1 + \sigma_2) \vec{k} \times \delta(\vec{r}_1 - \vec{r}_2) \vec{k} + V_{\text{Coulomb}}(\vec{r}_1, \vec{r}_2) \\
+ t_3 (1 + x_0 P^\sigma) \delta(\vec{r}_1 - \vec{r}_2) \rho^\alpha ((\vec{r}_1 + \vec{r}_2)/2)
\]
**Effective nucleon-nucleon interaction: Gogny force (D1S/D1M)**

\[
V(1, 2) = \sum_{i=1}^{2} e^{-\left(\vec{r}_1 - \vec{r}_2\right)^2 / \mu_i^2} \left(W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau\right)
+iW_0(\sigma_1 + \sigma_2)\vec{k} \times \delta(\vec{r}_1 - \vec{r}_2)\vec{k} + V_{\text{Coulomb}}(\vec{r}_1, \vec{r}_2)
\]

\[
+ t_3 (1 + x_0 P^\sigma) \delta(\vec{r}_1 - \vec{r}_2) \rho^\alpha ((\vec{r}_1 + \vec{r}_2)/2)
\]
Effective nucleon-nucleon interaction: Gogny force (D1S/D1M)

\[
V(1, 2) = \sum_{i=1}^{2} e^{-\frac{(\vec{r}_1 - \vec{r}_2)^2}{\mu_i^2}} (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau) \\
+ iW_0 (\sigma_1 + \sigma_2) \vec{k} \times \delta(\vec{r}_1 - \vec{r}_2) \vec{k} + V_{\text{Coulomb}}(\vec{r}_1, \vec{r}_2)
\]

\[
+ t_3 (1 + x_0 P^\sigma) \delta(\vec{r}_1 - \vec{r}_2) \rho^\alpha ((\vec{r}_1 + \vec{r}_2)/2)
\]
Effective nucleon-nucleon interaction: Gogny force (D1S/D1M)

\[ V(1, 2) = \sum_{i=1}^{2} e^{-\frac{(\vec{r}_1 - \vec{r}_2)^2}{\mu_i^2}} \left( W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau \right) \]

\[ + i W_0 (\sigma_1 + \sigma_2) \vec{k} \times \delta(\vec{r}_1 - \vec{r}_2) \vec{k} + V_{\text{Coulomb}}(\vec{r}_1, \vec{r}_2) \]

\[ + t_3 (1 + x_0 P^\sigma) \delta(\vec{r}_1 - \vec{r}_2) \rho^\alpha \left( (\vec{r}_1 + \vec{r}_2)/2 \right) \]

Other alternatives: Skyrme, relativistic Lagrangians, BCPM, ...
1. Introduction

2. EDF applications

3. GCM vs Shell Model

4. Summary and open questions

**Initial intrinsic states: PN-VAP**

\[
E^{NZ} [|\Phi(q)\rangle] = \frac{\langle \Phi(q) | \hat{H} \hat{P}_N \hat{P}_Z | \Phi(q) \rangle}{\langle \Phi(q) | \hat{P}_N \hat{P}_Z | \Phi(q) \rangle} - \tilde{\lambda}_q \left( \langle \Phi(q) | \tilde{Q} | \Phi(q) \rangle - \bar{q} \right)
\]
EDF axial

1. Introduction  
2. EDF applications  
3. GCM vs Shell Model  
4. Summary and open questions

• Initial intrinsic states: PN-VAP

\[ E^{NJZ} [\Phi(\vec{q})] = \frac{\langle \Phi(\vec{q}) | \hat{H} \hat{P}^N \hat{P}^Z | \Phi(\vec{q}) \rangle}{\langle \Phi(\vec{q}) | \hat{P}^N \hat{P}^Z | \Phi(\vec{q}) \rangle} - \vec{\lambda} \cdot \langle \Phi(\vec{q}) | \hat{Q} | \Phi(\vec{q}) \rangle - \vec{q} \]

• Intermediate Particle Number and Angular Momentum Projected states

\[ | J; N Z ; \vec{q} \rangle = \frac{2J + 1}{2} \int_0^\pi d\beta^* \beta \hat{P}^N \hat{P}^Z | \Phi(\vec{q}) \rangle d\beta \]
EDF axial

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

- **Initial intrinsic states: PN-VAP**
  \[ E^{NZ} \left[ \Phi(q) \right] = \frac{\langle \Phi(q) | \hat{H} \hat{P}^N \hat{P}^Z | \Phi(q) \rangle}{\langle \Phi(q) | \hat{P}^N \hat{P}^Z | \Phi(q) \rangle} - q \left( \langle \Phi(q) | \hat{Q} | \Phi(q) \rangle - q \right) \]

- **Intermediate Particle Number and Angular Momentum Projected states**
  \[ | J; NZ; q \rangle = \frac{2J + 1}{2} \int_0^\pi d^J_0(\beta) e^{-i\beta J_y} \hat{P}^N \hat{P}^Z | \Phi(q) \rangle d\beta \]

- **Final GCM states**
  \[ | J; NZ; \sigma \rangle = \int f^{J;NZ;\sigma} (q) | J; NZ; q \rangle d\tilde{q} \]
1. Introduction

2. EDF applications

3. GCM vs Shell Model

4. Summary and open questions

- Initial intrinsic states: PN-VAP

\[ E^{NZ} [\langle \Phi(q) \rangle] = \frac{\langle \Phi(q) | \hat{H} \hat{P}^N \hat{P}^Z | \Phi(q) \rangle}{\langle \Phi(q) | \hat{P}^N \hat{P}^Z | \Phi(q) \rangle} - \lambda q \left( \langle \Phi(q) \langle \hat{Q} \rangle | \Phi(q) \rangle - q \right) \]

- Intermediate Particle Number and Angular Momentum Projected states

\[ |J; NZ; \tilde{q}\rangle = \frac{2J + 1}{2} \int_0^\pi d_0^* \beta e^{-i\beta J_y} \hat{P}^N \hat{P}^Z |\Phi(q)\rangle d\beta \]

- Final GCM states

\[ |J; NZ; \sigma\rangle = \int f^{J; NZ; \sigma} (q) |J; NZ; \tilde{q}\rangle d\tilde{q} \]

\[ \int \left( \mathcal{H}^{J; NZ} (\tilde{q}, \tilde{q}') - E^{J; NZ; \sigma} \mathcal{N}^{J; NZ} (\tilde{q}, \tilde{q}') \right) f^{J; NZ; \sigma} (\tilde{q}') d\tilde{q}' = 0 \]

\[ \mathcal{H}^{J; NZ} (\tilde{q}, \tilde{q}') = \langle J; NZ; \tilde{q} | \hat{H} | J; NZ; \tilde{q}' \rangle \]

\[ \mathcal{N}^{J; NZ} (\tilde{q}, \tilde{q}') = \langle J; NZ; \tilde{q} | J; NZ; \tilde{q}' \rangle \]
1. Introduction

2. EDF applications

3. GCM vs Shell Model

4. Summary and open questions

---

**Initial intrinsic states: PN/VAP**

\[
E^{NZ}_{\{\Phi(\vec{q})\}} = \frac{\langle \Phi(\vec{q}) | \hat{H} \hat{P}^N \hat{P}^Z | \Phi(\vec{q}) \rangle}{\langle \Phi(\vec{q}) | \hat{P}^N \hat{P}^Z | \Phi(\vec{q}) \rangle} - \vec{q} \left( \frac{\langle \Phi(\vec{q}) | \hat{Q} | \Phi(\vec{q}) \rangle}{\langle \Phi(\vec{q}) | \Phi(\vec{q}) \rangle} - \vec{q} \right)
\]

**Intermediate Particle Number and Angular Momentum Projected states**

\[
| J; NZ; \vec{q} \rangle = \frac{2J + 1}{2} \int_0^{\pi} d_0^*(\beta) e^{-i\beta j_y} \hat{P}^N \hat{P}^Z | \Phi(\vec{q}) \rangle d\beta
\]

**Final GCM states**

\[
| J; NZ; \sigma \rangle = \int f^{J;NZ;\sigma}(\vec{q}) | J; NZ; \vec{q} \rangle d\vec{q}
\]

\[
\int \left( \mathcal{H}^{J;NZ}(\vec{q}, \vec{q}') - E^{J;NZ;\sigma} \mathcal{N}^{J;NZ}(\vec{q}, \vec{q}') \right) f^{J;NZ;\sigma}(\vec{q}') d\vec{q}' = 0
\]

\[
\mathcal{H}^{J;NZ}(\vec{q}, \vec{q}') = \langle J; NZ; \vec{q} | \hat{H} | J; NZ; \vec{q}' \rangle
\]

\[
\mathcal{N}^{J;NZ}(\vec{q}, \vec{q}') = \langle J; NZ; \vec{q} | J; NZ; \vec{q}' \rangle
\]

---

**plain HFB**
1. Axial states  \( K = 0 \)
2. Angular momentum  \( J = 0 \)
3. Quadrupole deformations  \( q = q_{20} \)
4. Quadrupole and pairing pp/nn correlations  \( q = (q_{20}, \delta) \)
5. Quadrupole and pn correlations  \( q = (q_{20}, p_0) \)
6. Quadrupole and octupole deformations  \( q = (q_{20}, q_{30}) \)

\[
|0; N_i Z_i; \sigma\rangle = \sum_{\Lambda_i} G_{\Lambda_i}^{0; N_i Z_i; \sigma} |\Lambda_i^{0; N_i Z_i}\rangle
\]

\[
|0; N_f Z_f; \sigma\rangle = \sum_{\Lambda_f} G_{\Lambda_f}^{0; N_f Z_f; \sigma} |\Lambda_f^{0; N_f Z_f}\rangle
\]
1. Axial states \( K = 0 \)
2. Angular momentum \( J = 0 \)
3. Quadrupole deformations \( q = q_{20} \)
4. Quadrupole and pairing \( pp/nn \) correlations \( q = (q_{20}, \delta) \)
5. Quadrupole and \( pn \) correlations \( q = (q_{20}, p_0) \)
6. Quadrupole and octupole deformations \( q = (q_{20}, q_{30}) \)

\[
M_{\xi}^{0 \nu \beta \beta} = \langle 0_f^+ | \hat{O}_{\xi}^{0 \nu \beta \beta} | 0_i^+ \rangle = \langle 0; N_f Z_f | \hat{O}_{\xi}^{0 \nu \beta \beta} | 0; N_i Z_i \rangle = \\
\sum_{\Lambda_f \Lambda_i} \left( G_{\Lambda_f}^{0; N_f Z_f} \right)^* \left( \Lambda_f^{0; N_f Z_f} | \Lambda_i^{0; N_i Z_i} \right) G_{\Lambda_i}^{0; N_i Z_i} = \sum_{q_i q_f : \Lambda_f \Lambda_i} \left( \frac{u_{q_f, \Lambda_f}^{0; N_f Z_f}}{\sqrt{n_{\Lambda_f}^{0; N_f Z_f}}} \right)^* \left( \frac{u_{q_i, \Lambda_i}^{0; N_i Z_i}}{\sqrt{n_{\Lambda_i}^{0; N_i Z_i}}} \right) \hat{O}_{\xi}^{0 \nu \beta \beta} \langle 0; N_f Z_f; q_f | 0; N_i Z_i; q_i \rangle \left( \frac{u_{q_i, \Lambda_i}^{0; N_i Z_i}}{\sqrt{n_{\Lambda_i}^{0; N_i Z_i}}} \right)
\]

TRANSITIONS:
1. **Axial states** \( K = 0 \)
2. **Angular momentum** \( J = 0 \)
3. **Quadrupole deformations** \( q = q_{20} \)
4. **Quadrupole and pairing pp/nn correlations** \( q = (q_{20}, \delta) \)
5. **Quadrupole and pn correlations** \( q = (q_{20}, p_0) \)
6. **Quadrupole and octupole deformations** \( q = (q_{20}, q_{30}) \)

**TRANSITIONS:**

\[
M^0_{\xi} = \langle 0^+_f | \hat{O}^0_{\xi} | 0^+_i \rangle = \langle 0; N_f Z_f | \hat{O}^0_{\xi} | 0; N_i Z_i \rangle = \sum_{\Lambda_i} G_{\Lambda_i}^{0;N_i Z_i;\sigma} |\Lambda_i^{0;N_i Z_i} \rangle \]

\[
= \sum_{\Lambda_f} \left( G_{\Lambda_f}^{0;N_f Z_f} \right)^* \langle \Lambda_f^{0;N_f Z_f} | \hat{O}^0_{\xi} | \Lambda_i^{0;N_i Z_i} \rangle G_{\Lambda_i}^{0;N_i Z_i} \]

\[
= \sum_{q_i q_f;\Lambda_f \Lambda_i} \left( \begin{array}{c}
0; N_f Z_f \\
\frac{u_{q_f,\Lambda_f}}{n_{\Lambda_f}^{0;N_f Z_f}}
\end{array} \right)^* \left( G_{\Lambda_f}^{0;N_f Z_f} \right)^* \langle 0; N_f Z_f; q_f | \hat{O}^0_{\xi} | 0; N_i Z_i; q_i \rangle \left( G_{\Lambda_i}^{0;N_i Z_i} \right) \left( \begin{array}{c}
\frac{u_{q_i,\Lambda_i}}{n_{\Lambda_i}^{0;N_i Z_i}}
\end{array} \right)
\]

Matrix elements of the double beta transition operators between particle number and angular momentum projected states.
Determination of initial and final states (I)

![Graphs showing E_{norm} distribution for 150Nd and 150Sm](image)

- E_{norm} (MeV)
- PN-VAP
- 150Nd
- 150Sm

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions
Determination of initial and final states (II)

\[ E_{\text{norm}} \text{ (MeV)} \]

\[ \beta \]

\[ {^150}\text{Nd} \]

\[ {^150}\text{Sm} \]
Determination of initial and final states (and III)

$E_{\text{norm}}$ (MeV)

$\beta$

$150\text{Nd}$

$150\text{Sm}$

Determination of initial and final states (and III)
Neutrinoless double beta decay candidates

Table 1
Masses, rms charge radii and total Gamow–Teller strengths $S_{+,-}$ for mother (granddaughter) calculated with Gogny D1S GCM+PNAMP functional compared to experimental values. Theoretical values for $S_{+,-}$ are quenched by a factor $(0.74)^2$.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>$BE\text{ (MeV)}$</th>
<th>$BE^{\text{exp}}\text{ (MeV)}$ [27]</th>
<th>$R_{\text{th}}\text{ (fm)}$</th>
<th>$R^{\text{exp}}\text{ (fm)}$ [28]</th>
<th>$S_{\text{theo}}^{+}$</th>
<th>$S_{\text{exp}}^{+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}\text{Ca}$</td>
<td>420.623</td>
<td>415.991</td>
<td>3.465</td>
<td>3.473</td>
<td>13.55</td>
<td>$(14.4 \pm 2.2)$ [29]</td>
</tr>
<tr>
<td>$^{48}\text{Ti}$</td>
<td>423.597</td>
<td>418.699</td>
<td>3.557</td>
<td>3.591</td>
<td>1.99</td>
<td>$(1.9 \pm 0.5)$ [29]</td>
</tr>
<tr>
<td>$^{76}\text{Ge}$</td>
<td>664.204</td>
<td>661.598</td>
<td>4.024</td>
<td>4.081</td>
<td>20.97</td>
<td>$(19.89)$ [30]</td>
</tr>
<tr>
<td>$^{76}\text{Se}$</td>
<td>664.949</td>
<td>662.072</td>
<td>4.074</td>
<td>4.139</td>
<td>1.49</td>
<td>$(1.45 \pm 0.07)$ [31]</td>
</tr>
<tr>
<td>$^{82}\text{Se}$</td>
<td>716.794</td>
<td>712.842</td>
<td>4.100</td>
<td>4.139</td>
<td>23.56</td>
<td>$(21.91)$ [30]</td>
</tr>
<tr>
<td>$^{82}\text{Kr}$</td>
<td>717.859</td>
<td>714.273</td>
<td>4.130</td>
<td>4.192</td>
<td>1.24</td>
<td></td>
</tr>
<tr>
<td>$^{90}\text{Zr}$</td>
<td>829.432</td>
<td>828.995</td>
<td>4.298</td>
<td>4.349</td>
<td>27.63</td>
<td></td>
</tr>
<tr>
<td>$^{96}\text{Mo}$</td>
<td>833.793</td>
<td>830.778</td>
<td>4.319</td>
<td>4.384</td>
<td>2.56</td>
<td>$(0.29 \pm 0.08)$ [32]</td>
</tr>
<tr>
<td>$^{100}\text{Mo}$</td>
<td>861.526</td>
<td>860.457</td>
<td>4.372</td>
<td>4.445</td>
<td>27.87</td>
<td>$(26.69)$ [30]</td>
</tr>
<tr>
<td>$^{100}\text{Ru}$</td>
<td>864.875</td>
<td>861.927</td>
<td>4.388</td>
<td>4.453</td>
<td>2.48</td>
<td></td>
</tr>
<tr>
<td>$^{116}\text{Cd}$</td>
<td>988.469</td>
<td>987.440</td>
<td>4.556</td>
<td>4.628</td>
<td>34.30</td>
<td>$(32.70)$ [30]</td>
</tr>
<tr>
<td>$^{116}\text{Sn}$</td>
<td>991.079</td>
<td>988.684</td>
<td>4.567</td>
<td>4.626</td>
<td>2.61</td>
<td>$(1.09^{+0.55}_{-0.33})$ [33]</td>
</tr>
<tr>
<td>$^{124}\text{Sn}$</td>
<td>1051.668</td>
<td>1049.96</td>
<td>4.622</td>
<td>4.675</td>
<td>40.65</td>
<td></td>
</tr>
<tr>
<td>$^{124}\text{Te}$</td>
<td>1051.562</td>
<td>1050.69</td>
<td>4.664</td>
<td>4.717</td>
<td>1.63</td>
<td></td>
</tr>
<tr>
<td>$^{128}\text{Te}$</td>
<td>1082.257</td>
<td>1081.44</td>
<td>4.686</td>
<td>4.735</td>
<td>40.48</td>
<td>$(40.08)$ [30]</td>
</tr>
<tr>
<td>$^{128}\text{Xe}$</td>
<td>1080.996</td>
<td>1080.74</td>
<td>4.723</td>
<td>4.775</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>$^{130}\text{Te}$</td>
<td>1096.627</td>
<td>1095.94</td>
<td>4.695</td>
<td>4.742</td>
<td>43.57</td>
<td>$(45.90)$ [30]</td>
</tr>
<tr>
<td>$^{130}\text{Xe}$</td>
<td>1097.245</td>
<td>1096.91</td>
<td>4.732</td>
<td>4.783</td>
<td>1.19</td>
<td></td>
</tr>
<tr>
<td>$^{136}\text{Xe}$</td>
<td>1143.333</td>
<td>1141.88</td>
<td>4.756</td>
<td>4.799</td>
<td>46.71</td>
<td></td>
</tr>
<tr>
<td>$^{138}\text{Ba}$</td>
<td>1143.202</td>
<td>1142.77</td>
<td>4.786</td>
<td>4.832</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>$^{150}\text{Nd}$</td>
<td>1234.512</td>
<td>1237.45</td>
<td>5.034</td>
<td>5.041</td>
<td>50.32</td>
<td></td>
</tr>
<tr>
<td>$^{150}\text{Sm}$</td>
<td>1235.936</td>
<td>1239.25</td>
<td>5.041</td>
<td>5.040</td>
<td>1.45</td>
<td></td>
</tr>
</tbody>
</table>

Good agreement between experimental and theoretical Q-values, radii and total strength (quenched)
1. Introduction

2. EDF applications

3. GCM vs Shell Model

4. Summary and open questions

- GT strength greater than Fermi.
- Similar deformation between mother and granddaughter is favored by the transition operators
- Maxima are found close to sphericity although some other local maxima are found

NME: axial quadrupole deformation

Tomás R. Rodríguez

0νββ decay nuclear matrix elements with the GCM

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

- GT strength greater than Fermi.
- Similar deformation between mother and granddaughter is favored by the transition operators
- Maxima are found close to sphericity although some other local maxima are found

\[ \frac{\langle 0; N_f Z_f; q_f | G_{K=0}^{\nu \beta \beta} | 0; N_i Z_i; q_i \rangle}{\sqrt{\langle 0; N_f Z_f; q_f | 0; N_i Z_i; q_i \rangle \langle 0; N_f Z_f; q_f | 0; N_i Z_i; q_i \rangle}} \]

\[ |F(\beta)|^2 \]

- Final result depends on the distribution of probability of the corresponding initial and final collective states within this plot

A=150

NME: axial quadrupole deformation

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

\[ \frac{\langle 0;N_f Z_f; q_f | \mathcal{O}_{\alpha}^{\text{reac}} | 0;N_i Z_i; q_i \rangle}{\sqrt{\langle 0;N_f Z_f; q_f | 0;N_f Z_f; q_f \rangle \langle 0;N_i Z_i; q_i | 0;N_i Z_i; q_i \rangle}} \]

- GT strength greater than Fermi.
- Similar deformation between mother and granddaughter is favored by the transition operators
- Maxima are found close to sphericity although some other local maxima are found
- Final result depends on the distribution of probability of the corresponding initial and final collective states within this plot
NME: axial quadrupole deformation

- GT strength greater than Fermi.
- Similar deformation between mother and granddaughter is favored by the transition operators.
- Maxima are found close to sphericity although some other local maxima are found.
- Final result depends on the distribution of probability of the corresponding initial and final collective states within this plot.
NME: axial quadrupole plus octupole deformation

1. Introduction

2. EDF applications

3. GCM vs Shell Model

4. Summary and open questions

FIG. 5: (Color online) The final matrix element $M_{0\gamma}^o$ from the GCM calculation with and without [46] octupole shape fluctuations (REDF) and those of the QRPA (“QRPA_F” [66], “QRPA_M” [45], “QRPA_T” [47]), the IMB-2 [67], and the non-relativistic GCM, based on the Gogny D1S interaction, with [68] and without [44] pairing fluctuations.
NME: triaxial quadrupole deformation

A=76

NME: triaxial quadrupole deformation

A=76


PES J=0

Theory

Experiment

NME??
(work in progress...)

PES J=0

Theory

Experiment

INT double-beta decay workshop

0νββ decay nuclear matrix elements with the GCM

Tomás R. Rodríguez
NME: Shape and pp/nn pairing fluctuations

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

Angular momentum projected potential energy surfaces

Collective ground state wave functions

NME: Shape and pp/nn pairing fluctuations

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

Dependence on deformation

Dependence on pp/nn pairing

### 2. EDF applications

<table>
<thead>
<tr>
<th>Isotope</th>
<th>$\Delta Q(\beta_2)$</th>
<th>$\Delta Q(\beta_2, \delta)$</th>
<th>$M^{0\nu}(\beta_2)$</th>
<th>$M^{0\nu}(\beta_2, \delta)$</th>
<th>Var (%)</th>
<th>$T_{1/2}(\beta_2, \delta)$</th>
<th>$T_{1/2}(\beta_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}$Ca</td>
<td>0.265</td>
<td>0.131</td>
<td>$2.370^{1.914}_{0.456}$</td>
<td>$2.229^{1.797}_{0.431}$</td>
<td>-6</td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>$^{76}$Ge</td>
<td>0.271</td>
<td>0.190</td>
<td>$4.601^{3.715}_{0.886}$</td>
<td>$5.551^{4.470}_{1.082}$</td>
<td>21</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>$^{82}$Se</td>
<td>-0.366</td>
<td>-0.246</td>
<td>$4.218^{3.381}_{0.837}$</td>
<td>$4.674^{3.743}_{0.931}$</td>
<td>11</td>
<td>0.81</td>
<td></td>
</tr>
<tr>
<td>$^{96}$Zr</td>
<td>2.580</td>
<td>2.628</td>
<td>$5.650^{4.618}_{1.032}$</td>
<td>$6.498^{5.296}_{1.202}$</td>
<td>15</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>$^{100}$Mo</td>
<td>1.879</td>
<td>1.757</td>
<td>$5.084^{4.149}_{0.935}$</td>
<td>$6.588^{5.361}_{1.227}$</td>
<td>30</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>$^{116}$Cd</td>
<td>1.365</td>
<td>1.337</td>
<td>$4.795^{3.931}_{0.864}$</td>
<td>$5.348^{4.372}_{0.976}$</td>
<td>12</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>$^{124}$Sn</td>
<td>-0.830</td>
<td>-0.687</td>
<td>$4.808^{3.893}_{0.916}$</td>
<td>$5.787^{4.680}_{1.107}$</td>
<td>20</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>$^{128}$Te</td>
<td>-0.564</td>
<td>-0.594</td>
<td>$4.107^{3.079}_{1.027}$</td>
<td>$5.687^{4.255}_{1.432}$</td>
<td>38</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>$^{130}$Te</td>
<td>-0.348</td>
<td>-0.628</td>
<td>$5.130^{4.141}_{0.989}$</td>
<td>$6.405^{5.161}_{1.244}$</td>
<td>25</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>$^{136}$Xe</td>
<td>-1.027</td>
<td>-0.787</td>
<td>$4.199^{3.673}_{0.526}$</td>
<td>$4.773^{4.170}_{0.604}$</td>
<td>14</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>$^{150}$Nd</td>
<td>-0.380</td>
<td>-0.282</td>
<td>$1.707^{1.278}_{0.429}$</td>
<td>$2.190^{1.639}_{0.551}$</td>
<td>29</td>
<td>0.61</td>
<td></td>
</tr>
</tbody>
</table>
NME: Shape and pn pairing fluctuations

\[ H = h_0 - \sum_{\mu=-1}^{1} g_{\mu}^{T=1} S_{\mu} S_{\mu} - \frac{\chi}{2} \sum_{K=-2}^{2} Q_{2K}^{+} Q_{2K} \]
\[ - g_{T=0}^{0} \sum_{\nu=-1}^{1} P_{\nu}^{+} P_{\nu} + g_{ph} \sum_{\mu,\nu=-1}^{1} F_{\mu}^{\nu} F_{\nu}^{\mu}, \tag{2} \]

where \( h_0 \) contains spherical single particle energies, \( Q_{2K} \) are the components of a quadrupole operator defined in Ref. [15], and

\[ S_{\mu}^{\dagger} = \frac{1}{\sqrt{2}} \sum_{l} \hat{l}[c_{l}^{\dagger} c_{l}^{\dagger}]_{00\mu} \]
\[ P_{\mu}^{\dagger} = \frac{1}{\sqrt{2}} \sum_{l} \hat{l}[c_{l}^{\dagger} c_{l}^{\dagger}]_{01\mu} \]
\[ F_{\mu}^{\nu} = \frac{1}{2} \sum_{i} \sigma_{i}^{\mu} \tau_{i}^{\nu} = \sum_{l} \hat{l}[c_{l}^{\dagger} c_{l}^{\dagger}]_{011\nu}. \tag{3} \]

\[ H' = H - \lambda_{Z} N_{Z} - \lambda_{N} N_{N} - \lambda_{Q} Q_{20} - \frac{\lambda_{P}}{2} \left( P_{0} + P_{0}^{+} \right), \tag{6} \]

N. Hinohara and J. Engel, PRC 031031(R) (2014)
Very difficult (perhaps impossible) to implement with current EDFs!
We want to study the role of

- Pairing pp/nn correlations.
- Deformation.
- Shell effects.
- Spatial dependence of the neutrino potentials.

in the nuclear matrix elements in a whole isotopic chain using state-of-the-art energy density functional methods.
Collective wave functions for Cd and Sn

- Sn isotopes are spherical and Cd slightly prolate deformed when beyond mean field correlations are included.

Collective wave functions for Cd and Sn

- Sn isotopes are spherical and Cd slightly prolate deformed when beyond mean field correlations are included.
- Good agreement between experimental and theoretical Q-values within the accuracy of the force (Gogny D1S).

A=116 (possible candidate for detection)
A=116 (possible candidate for detection)

- Reduction of the NME with respect to the spherical value when shape mixing is included
**A=116 (possible candidate for detection)**

- Reduction of the NME with respect to the spherical value when shape mixing is included

- Larger pairing correlations in mother/daughter nuclei produces larger NMEs.
**NME: \(^{116}\text{Cd} \rightarrow ^{116}\text{Sn}\)**

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

**A=116 (possible candidate for detection)**

- Reduction of the NME with respect to the spherical value when shape mixing is included
- NMEs almost proportional to the ones found with using constant neutrino potentials.
- Larger pairing correlations in mother/daughter nuclei produces larger NMEs.
**A=116 (possible candidate for detection)**

- Reduction of the NME with respect to the spherical value when shape mixing is included
- NMEs almost proportional to the ones found with using constant neutrino potentials.
- Larger pairing correlations in mother/daughter nuclei produces larger NMEs.
- GT component is always larger than Fermi.

NME: $^{A}\text{Cd} \rightarrow ^{A}\text{Sn}$ Shell Effects

- GT component is always larger than Fermi.
- Large enhancement of the NME for the mirror decay $A=98$.

NME: $^A$Cd$\rightarrow^A$Sn Shell Effects

- GT component is always larger than Fermi.
- Large enhancement of the NME for the mirror decay $A=98$.
- Shell effects associated to the filling of neutrons in the corresponding sub-shells. Consistent with seniority model.
NME: $^A$Cd$\rightarrow^A$Sn Shell Effects

- GT component is always larger than Fermi.
- Large enhancement of the NME for the mirror decay $A=98$.
- Shell effects associated to the filling of neutrons in the corresponding sub-shells. Consistent with seniority model.

**NME: $^ACd \rightarrow ^ASn$**

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

- Reduction of the NME with respect to the spherical value when shape mixing is included
- Larger reduction when the difference in deformation is larger

![Graphs showing NME reduction](image)

$\beta^2_{12}$ vs Mass number $A$

- Reduction of the NME with respect to the spherical value when shape mixing is included
- Larger reduction when the difference in deformation is larger

- Larger pairing correlations in mother/daughter nuclei produces larger NMEs.
- Closely related to shell effects
1. Introduction

2. EDF applications

3. GCM vs Shell Model

4. Summary and open questions

- Reduction of the NME with respect to the spherical value when shape mixing is included.
- Larger reduction when the difference in deformation is larger.

- NMEs almost proportional to the ones found with using constant neutrino potentials.
- The agreement is worse when $1h\frac{11}{2}$ starts to be filled in. Parity? Multipoles?

- Larger pairing correlations in mother/daughter nuclei produce larger NMEs.
- Closely related to shell effects.

NME: $pf$-shell

Where do the differences between SM and GCM come from?

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

INT double-beta decay workshop
0νββ decay nuclear matrix elements with the GCM
Tomás R. Rodríguez
Where do the differences between SM and GCM come from?

- Same pattern in spherical EDF, seniority 0 Shell Model, and Generalized Seniority model (overall scale?)

- What is the effect of including more correlations?

INT double-beta decay workshop

Tomás R. Rodríguez

0νββ decay nuclear matrix elements with the GCM

- NMEs are reduced with respect to the spherical value when correlations are included.

- The biggest reduction is produced by angular momentum restoration and configuration mixing produces an increase of the NME.

- Cross-check nuclei: $^{42}$Ca, $^{50}$Ca, $^{56}$Fe

NME: pf-shell

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

---

**NME: pf-shell**

---

**3. GCM vs Shell Model**

- **Figure 48**
  - (Color online) Gamow-Teller $\nu^\beta\beta$ decay nuclear matrix elements with the GCM
  - Shell model (SM) results are shown as a function of the seniority.
  - The right-hand panels in Fig. 48 compare SM calculations of NMEs obtained from collective deformation and shape mixing. These final states and the full EDF calculation use self-consistent shape mixing of quasiparticle excitations on top of each intrinsic state. A step further, beyond the scope of this work, would include on-shell particles.

---

The biggest reduction (in Shell model calculations) is produced by including higher seniority components in the nuclear wave functions.

Isospin projection is relevant for the Fermi part of the NME and less important for the Gamow-Teller part.

EDF does not include properly those higher seniority components, specially in spherical nuclei.

p-n pairing effects could also be important in the reduction of the NME.
The biggest reduction (in Shell model calculations) is produced by including higher seniority components in the nuclear wave functions.

Isospin projection is relevant for the Fermi part of the NME and less important for the Gamow-Teller part.

EDF does not include properly those higher seniority components, specially in spherical nuclei.

More comparisons: see Nobuo’s talk!

- Pairing effects could also be important in the reduction of the NME.
### 3. GCM vs Shell Model

<table>
<thead>
<tr>
<th></th>
<th>$^{48}$Ca</th>
<th>$^{48}$Ti</th>
<th>NME (F/GT/T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>spherical</td>
<td>-7.558</td>
<td>-20.497</td>
<td>-2.276/4.736/0.116</td>
</tr>
<tr>
<td>GCM:Q$_{20}$</td>
<td>-7.670</td>
<td>-23.556</td>
<td>in progress</td>
</tr>
<tr>
<td>GCM:Q$_{20}$+T=1</td>
<td>-7.855</td>
<td>-24.198</td>
<td>in progress</td>
</tr>
<tr>
<td>GCM:Q$_{20}$+T=1+T=0</td>
<td>-</td>
<td>-24.467</td>
<td>in progress</td>
</tr>
<tr>
<td>SM seniority 0</td>
<td>-7.578</td>
<td>-20.507</td>
<td>-2.287/4.783/0.116</td>
</tr>
<tr>
<td>SM full</td>
<td>-7.959</td>
<td>-24.896</td>
<td>-0.234/0.886/0.057</td>
</tr>
</tbody>
</table>

- GCM and Shell Model calculations have been performed in the $pf$-shell with KB3G interactions both!
- Variational approach to SM results with GCM approaches is evident.
- Almost perfect agreement between SM seniority 0 and PN-VAP spherical calculations both for energies and NMEs!

T. R. R., J. Menéndez, ... in progress
### NME: $pf$-shell

<table>
<thead>
<tr>
<th></th>
<th>$^{48}\text{Ca}$</th>
<th>$^{48}\text{Ti}$</th>
<th>NME ($F/GT/T$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>spherical</td>
<td>-7.558</td>
<td>-20.497</td>
<td>-2.276/4.736/0.116</td>
</tr>
<tr>
<td>GCM:$Q_{20}$</td>
<td>-7.670</td>
<td>-23.556</td>
<td>in progress</td>
</tr>
<tr>
<td>GCM:$Q_{20}+T=1$</td>
<td>-7.855</td>
<td>-24.198</td>
<td>in progress</td>
</tr>
<tr>
<td>GCM:$Q_{20}+T=1+T=0$</td>
<td>-24.467</td>
<td></td>
<td>in progress</td>
</tr>
<tr>
<td>SM seniority 0</td>
<td>-7.578</td>
<td>-20.507</td>
<td>-2.287/4.783/0.116</td>
</tr>
<tr>
<td>SM full</td>
<td>-7.959</td>
<td>-24.896</td>
<td>-0.234/0.886/0.057</td>
</tr>
</tbody>
</table>

- GCM and Shell Model calculations have been performed in the $pf$-shell with KB3G interactions both!
- Variational approach to SM results with GCM approaches is evident.
- Almost perfect agreement between SM seniority 0 and PN-VAP spherical calculations both for energies and NMEs!
Summary

1. Introduction
2. EDF applications
3. GCM vs Shell Model
4. Summary and open questions

- NMEs with EDF methods have been implemented exploring many degrees of freedom so far (axial quadrupole and octupole deformations, axial pp/nn pairing). Transitions between spherical and superfluid nuclei are the most favored ones.

- Inclusion of proton-neutron pairing reduces the NMEs but it is difficult to implement in actual EDF applications.

- Relativistic effects and tensor terms are small in the EDF framework

- Systematic comparisons between ISM/EDF methods have been performed. Striking similarity between EDF spherical and SM seniority zero calculations is found. Is it confirmed by GCM calculations with SM interactions?
Some open questions

- Isospin mixing has to be done in the future. However, it is very involved (perhaps impossible) with the current Gogny EDFs?

- Triaxiality has to be taken into account in $A=76$ decay (at least).

- How relevant is the proper description of the spectra in $0\nu\beta\beta$ NMEs?

- Odd-odd nuclei is still a major challenge for GCM calculations.

- Computational time?!?