Correlations and two-body currents in (electro)weak processes

Saori Pastore
INT Program INT-17-2a - Neutrinoless Double-beta Decay
Seattle WA - June 2017

WITH
Carlson & Gandolfi (LANL) - Schiavilla & Baroni (ODU/JLAB) - Wiringa & Piarulli & Pieper (ANL)
Mereghetti & Dekens & Cirigliano (LANL)

REFERENCES
Fundamental Physics Quests: Double Beta Decay

Observation of $0\nu\beta\beta$-decay

$\rightarrow$

Lepton number $L = l - \bar{l}$ not conserved

$\rightarrow$

Implications in matter-antimatter imbalance

* Detectors’ active material $^{76}\text{Ge}$ *

$0\nu\beta\beta$-decay $\tau_{1/2} \gtrsim 10^{25}$ years (age of the universe $1.4 \times 10^{10}$ years)

1 ton of material to see (if any) $\sim 5$ decays per year

* Also, if nuclear m.e.’s are known, absolute $\nu$-masses can be extracted *

2015 Long Range Plane for Nuclear Physics
The question

- What are the present uncertainties in nuclear matrix elements relevant for neutrinoless double beta decay, and how can they be improved?

**OUTLINE**

Role of correlations and many-body currents in
- Single beta-decay in $A \leq 10$ nuclei
- Neutrinoless double beta-decay in $A \leq 12$ nuclei
Nuclear Interactions

The nucleus is made of a non-relativistic interacting nucleons and its energy is

\[ H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} \nu_{ij} + \sum_{i<j<k} V_{ijk} + \ldots \]

where \( \nu_{ij} \) and \( V_{ijk} \) are two- and three-nucleon operators based on EXPT data fitting and fitted parameters subsume underlying QCD

Carlson et al. Rev.Mod.Phys.87(2015)1067
Correlations in our formalism

Minimize expectation value of $H = T + AV18 + IL7$

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using trial function

$$|\Psi_V \rangle = \left[ S \prod_{i<j}(1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}) \right] \left[ \prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3) \rangle$$

* single-particle $\Phi_A(JMTT_3)$ is fully antisymmetric and translationally invariant
* central pair correlations $f_c(r)$ keep nucleons at favorable pair separation
* pair correlation operators $U_{ij}$ reflect influence of $v_{ij}$ ($AV18$)
* triple correlation operators $U_{ijk}$ reflect the influence of $V_{ijk}$ ($IL7$)

In an uncorrelated wave function
1) $U_{ij}$ and $U_{ijk}$ are turned off, and
2) only the dominant spatial symmetry is kept

Lomnitz-Adler, Pandharipande, and Smith NPA361(1981)399
Wiringa, PRC43(1991)1585
Electroweak Reactions

* $\omega \sim 10^2$ MeV: Accelerator neutrinos
* $\omega \sim 10^1$ MeV: EM decay, $\beta$-decay
* $\omega \lesssim 10^1$ MeV: Nuclear Rates for Astrophysics
Nuclear Currents

\[
\begin{align*}
\rho & = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots , \\
\mathbf{j} & = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots 
\end{align*}
\]

* In Impulse Approximation IA nuclear currents are expressed in terms of those associated with individual protons and nucleons, i.e., \( \rho_i \) and \( \mathbf{j}_i \), 1b-operators

* Two-body 2b currents essential to satisfy current conservation

\[
\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + v_{ij} + V_{ijk}, \rho]
\]
Electromagnetic Currents from Nuclear Interactions (SNPA currents)

\[
q \cdot j = [H, \rho] = [t_i + \nu_{ij} + V_{ijk}, \rho]
\]

1) Longitudinal component fixed by current conservation
2) Plus transverse “phenomenological” terms

\[
j = j^{(1)} + j^{(2)}(\nu) + j^{(3)}(V)
\]

Villars, Myiazawa (40-ies), Chemtob, Riska, Schiavilla . . .
see, e.g., Marcucci et al. PRC72(2005)014001 and references therein
Currents from nuclear interactions

Satisfactory description of a variety of nuclear em properties in $A \leq 12$

$^2\text{H}(p,\gamma)^3\text{He}$ capture

![Graph showing $S(E)$ vs. $E_{\text{CM}}$](image)

Marcucci et al. PRC72, 014001 (2005)
Electromagnetic Currents from Chiral Effective Field Theory

\[ \text{LO : } j^{(-2)} \sim eQ^{-2} \]

\[ \text{NLO : } j^{(-1)} \sim eQ^{-1} \]

\[ \text{N}^2\text{LO : } j^{(-0)} \sim eQ^0 \]

* 3 unknown Low Energy Constants:
  fixed so as to reproduce \(d\), \(^3H\), and \(^3\text{He}\) magnetic moments

\[ \text{N}^3\text{LO : } j^{(1)} \sim eQ \]

* analogue expansion exists for the Axial nuclear current - Baroni et al. PRC93 (2016)015501 *

Electromagnetic LECs

\[ d^S, d_1^V, d_2^V \]

\[ c^S, c^V \]

\( d^S, d_1^V, \) and \( d_2^V \) could be determined by \( \pi \gamma \)-production data on the nucleon

\[ d_2^V = 4\mu^* h_A / 9m_N (m_\Delta - m_N) \]

\[ d_1^V = 0.25 \times d_2^V \]

assuming \( \Delta \)-resonance saturation

Left with 3 LECs: Fixed in the \( A = 2 - 3 \) nucleons’ sector

* Isoscalar sector:
  * \( d^S \) and \( c^S \) from EXPT \( \mu_d \) and \( \mu_s(3H/3He) \)

* Isovector sector:
  * \( c^V \) from EXPT npd\( \gamma \) xsec.
  
  or
  
  * \( c^V \) from EXPT \( \mu_V(3H/3He) \) m.m.

* Regulator \( C(\Lambda) = \exp\left(-\left(p/\Lambda\right)^4\right) \) with \( \Lambda = 500 - 600 \) MeV

<table>
<thead>
<tr>
<th>( \Lambda )</th>
<th>NN/NNN</th>
<th>( 10 \times d^S )</th>
<th>( c^S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>AV18/UIX (N3LO/N2LO)</td>
<td>-1.731 (2.190)</td>
<td>2.522 (4.072)</td>
</tr>
<tr>
<td>600</td>
<td>AV18/UIX (N3LO/N2LO)</td>
<td>-2.033 (3.231)</td>
<td>5.238 (11.38)</td>
</tr>
</tbody>
</table>
Electromagnetic LECs

\[ d^S, d^V_1, d^V_2 \]
\[ c^S, c^V \]

\[ d^S, d^V_1, \text{ and } d^V_2 \] could be determined by \( \pi \gamma \)-production data on the nucleon

\[ d^V_2 = 4 \mu^* h_A / 9 m_N (m_\Delta - m_N) \] and
\[ d^V_1 = 0.25 \times d^V_2 \]
assuming \( \Delta \)-resonance saturation

Left with 3 LECs: Fixed in the \( A = 2 - 3 \) nucleons’ sector

* Isoscalar sector:
  * \( d^S \) and \( c^S \) from EXPT \( \mu_d \) and \( \mu_S (^3\text{H}/^3\text{He}) \)

* Isovector sector:
  * \( c^V \) from EXPT \( npd\gamma \) xsec.
  * \( c^V \) from EXPT \( \mu_V (^3\text{H}/^3\text{He}) \) m.m.

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<table>
<thead>
<tr>
<th>( \Lambda )</th>
<th>NN/NNN</th>
<th>Current</th>
<th>( d^V_1 )</th>
<th>( c^V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>AV18/UIX</td>
<td>I</td>
<td>4.98</td>
<td>-11.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>II</td>
<td>4.98</td>
<td>-1.025</td>
</tr>
</tbody>
</table>
Convergence and cutoff dependence

$np$ capture x-section/ $\mu_V$ of $A = 3$ nuclei
bands represent nuclear model dependence [NN(N3LO)+3N(N2LO) – AV18+UIX]

Piarulli et al. PRC(2013)014006
Calculations with EM Currents from $\chi$EFT with $\pi$’s and N’s

► Park, Min, and Rho et al. (1996)
  applications to A=2–4 systems by Song, Lazauskas, Park at al. (2009-2011)
  within the hybrid approach
  ....
  * Based on EM $\chi$EFT currents from NPA596(1996)515

► Meissner and Walzl (2001);
  Kölling, Epelbaum, Krebs, and Meissner (2009–2011)
  applications to:
  $d$ and $^3$He photodisintegration by Rozpedzik et al. (2011); $e$-scattering (2014);
  $d$ magnetic f.f. by Kölling, Epelbaum, Phillips (2012);
  radiative $N - d$ capture by Skibinski et al. (2014)
  ....
  * Based on EM $\chi$EFT currents from PRC80(2009)045502 &
    PRC84(2011)054008 and consistent $\chi$EFT potentials from UT method

► Phillips (2003-2007)
  applications to deuteron static properties and f.f.’s
  ....
Magnetic Moments and M1 Transitions

* 2b electromagnetic currents bring the THEORY in agreement with the EXPT
* $\sim 40\%$ 2b-current contribution found in $^9\text{C}$ m.m.
* $\sim 60 - 70\%$ of total 2b-current component is due to one-pion-exchange currents
* $\sim 20-30\%$ 2b found in M1 transitions in $^8\text{Be}$

**Error Estimate**

EE *et al.* error algorithm  
Epelbaum, Krebs, and Meissner EPJA51(2015)53

$$\delta^{N3LO} = \max \left[ Q^4 \mu^{LO}, Q^3|\mu^{LO} - \mu^{NLO}|, \\ Q^2|\mu^{NLO} - \mu^{N2LO}|, \\ Q^1|\mu^{N2LO} - \mu^{N3LO}| \right]$$

$$Q = \max \left[ \frac{m_\pi}{\Lambda}, \frac{p}{\Lambda} \right]$$

<table>
<thead>
<tr>
<th>m.m.</th>
<th>THEO</th>
<th>EXP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^9$C</td>
<td>-1.35(4)(7)</td>
<td>-1.3914(5)</td>
</tr>
<tr>
<td>$^9$Li</td>
<td>3.36(4)(8)</td>
<td>3.4391(6)</td>
</tr>
</tbody>
</table>

* ‘N3LO-\(\Delta\)’ corrections can be ‘large’ *
* SNPA and \(\chi\)EFT currents qualitatively in agreement, \(\chi\)EFT isoscalar currents provide better description exp data *

Pastore *et al.* PRC87(2013)035503
Two-body M1 transitions densities

\[
8 \text{Be}(1^+; 1\ 2^+, 0) \rightarrow 8 \text{Be}(1^+; 0\ 2^+, 0)
\]

\[
8 \text{Be}(1^+; 0\ 2^+, 0) \rightarrow 8 \text{Be}(1^+; 0\ 2^+, 0)
\]

Pastore et al. PRC90(2014)024321

<table>
<thead>
<tr>
<th>( (J_i, T_i) \rightarrow (J_f, T_f) )</th>
<th>IA</th>
<th>NLO-OPE</th>
<th>N2LO-RC</th>
<th>N3LO-TPE</th>
<th>N3LO-CT</th>
<th>N3LO-\Delta</th>
<th>MEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (1^+; 1) \rightarrow (2^+_2; 0) )</td>
<td>2.461 (13)</td>
<td>0.457 (3)</td>
<td>-0.058 (1)</td>
<td>0.095 (2)</td>
<td>-0.035 (3)</td>
<td>0.161 (21)</td>
<td>0.620 (5)</td>
</tr>
</tbody>
</table>
The "$g_A$ problem" and the role of two-nucleon correlations and two-body currents
Theory vs Experiment: The “$g_A$ problem”

\[ g_{A}^{\text{eff}} \simeq 0.70 \, g_A \]

Fig. from Chou et al. PRC47(1993)163
Correlations in our formalism

Minimize expectation value of $H = T + AV18 + IL7$

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using trial function

$$|\Psi_V\rangle = \left[ \mathcal{S} \prod_{i < j} (1 + U_{ij} + \sum_{k \neq i, j} U_{ijk}) \right] \left[ \prod_{i < j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

* single-particle $\Phi_A(JMTT_3)$ is fully antisymmetric and translationally invariant
* central pair correlations $f_c(r)$ keep nucleons at favorable pair separation
* pair correlation operators $U_{ij}$ reflect influence of $\nu_{ij}$ (AV18)
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In an uncorrelated wave function

1) $U_{ij}$ and $U_{ijk}$ are turned off, and
2) only the dominant spatial symmetry is kept

Lomnitz-Adler, Pandharipande, and Smith NPA361(1981)399
Wiringa, PRC43(1991)1585
Role of correlations in beta-decay m.e.’s

\[ \frac{10}{10} C \rightarrow \frac{10}{10} B ; \quad q = 0.76 \]

\[ \frac{7}{7} Be \rightarrow \frac{7}{7} Li ; \quad q = 0.87 \]

\[ \frac{6}{6} He \rightarrow \frac{6}{6} Li ; \quad q = 0.82 \]

\[ \frac{3}{3} H \rightarrow \frac{3}{3} He ; \quad q = 0.97 \]

\[ q = \text{quenching from correlations} \]


* Preliminary *
1) One body has GT, relativistic corrections, PS from pion-pole diagrams

2) Two-body currents
   2.a) Major contribution from $\Delta$-excitation current
   2.b) Negligible contributions from $A\pi, A\rho, A\pi\rho$

3) $AN\Delta$ coupling fixed to tritium beta-decay

4) $\sim 3\%$ additive correction from $\Delta$-current

Chemtob, Rho, Towner, Riska, Schiavilla, Marcucci …

see, e.g., Marcucci et al. PRC63(2001)015801 and references therein
Two-body Axial Currents from $\chi$EFT

$c_3$ and $c_4$

* are saturated by the $\Delta$ and $\rho\pi$ d.o.f.
* enter also the $\chi$EFT two- and three-nucleon $\chi$EFT potential
* are taken from Entem and Machleidt $c_3 = -3.2 \text{ GeV}^{-1}$, $c_4 = 5.4 \text{ GeV}^{-1}$


A. Baroni et al. PRC93(2016)015501 & PRC94(2016)024003
Two-body Axial Currents from $\chi$EFT

* fitted to GT m.e. of tritium beta-decay
* for both $\chi$EFT potentials and AV18+UIX
* because of N4LO two-body currents $c_D$ value changes

<table>
<thead>
<tr>
<th>$\Lambda$</th>
<th>N3LO 500</th>
<th>N3LO 600</th>
<th>N4LO 500</th>
<th>N4LO 600</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_D$</td>
<td>-0.353</td>
<td>-0.443</td>
<td>-1.847</td>
<td>-2.030</td>
</tr>
</tbody>
</table>

A. Baroni et al. PRC93(2016)015501 & PRC94(2016)024003
Three-body Axial Currents from $\chi$EFT

A. Baroni et al. PRC93(2016)015501 & PRC94(2016)024003
Tritium $\beta$-decay

* $\sim 2\%$ additive contribution from two-body currents

A. Baroni et al. PRC93(2016)015501 & PRC94(2016)024003
Calculations with EW Currents from $\chi$EFT with $\pi$’s and N’s

Incomplete history

- Park, Min, and Rho et al. (90-ies)
  applications to $A=2–4$ systems including $\mu$-capture, $pp$-fusion, $hep$ ·
- Krebs and Epelbaum et al. (2016)
- Klos et al. (2015)

......
Role of two-body currents in beta-decay m.e.’s

SNPA currents
VMC Calculations

χEFT currents
GFMC calculations

*Preliminary*

* SNPA and χEFT two-body currents are qualitatively in agreement (both are fitted to the tritium β-decay)*

* Two-body currents are found to provide a small (negligible) contribution to the quenching, limited to the light systems we studied*
\( \chi \)EFT currents: a closer look

### \( A = 7 \) Captures

<table>
<thead>
<tr>
<th></th>
<th>gs</th>
<th>ex</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>2.334</td>
<td>2.150</td>
</tr>
<tr>
<td>N2LO</td>
<td>(-3.18 \times 10^{-2})</td>
<td>(-2.79 \times 10^{-2})</td>
</tr>
<tr>
<td>N3LO(OPE)</td>
<td>(-2.99 \times 10^{-2})</td>
<td>(-2.44 \times 10^{-2})</td>
</tr>
<tr>
<td><strong>N3LO(CT)</strong></td>
<td><strong>2.79 \times 10^{-1})</strong></td>
<td><strong>2.36 \times 10^{-1})</strong></td>
</tr>
<tr>
<td>N4LO(2b)</td>
<td>(-1.61 \times 10^{-1})</td>
<td>(-1.33 \times 10^{-1})</td>
</tr>
<tr>
<td>N4LO(3b)</td>
<td>(-6.59 \times 10^{-3})</td>
<td>(-4.86 \times 10^{-3})</td>
</tr>
<tr>
<td>TOT(2b+3b)</td>
<td><strong>0.050)</strong></td>
<td><strong>0.046)</strong></td>
</tr>
</tbody>
</table>

* Large cancellations due to positive CT at N3LO with \( c_D \) fixed to GT m.e. of tritium

In preparation
$\beta\beta$–decay

The “$g_A$ problem”
and
the role of two-nucleon correlations and two-body currents

Berna U.

*Preliminary results*
Double beta-decay m.e.’s: Correlations

*Preliminary*

$$^8\text{He}(0^+;2) - ^8\text{Be}(0^+;0) - \text{AV18+UX}$$

$$\rho_V = \tau^+_1 \tau^+_2 / r_{12}$$

$$J_f \rho_V \, J_i = .0406$$

$$J_f \rho_V \, J_i = .0227$$

$$\rho_A = 1 \times \tau^+_1 \tau^+_2 / r_{12}$$

$$J_f \rho_A \, J_i = -.0368$$

$$J_f \rho_A \, J_i = -.122$$

$$< \rho_V >_{\text{corr}} \sim 0.56 < \rho_V >_{\text{uncorr}}, \quad q_V = 0.75$$

$$< \rho_A >_{\text{corr}} \sim 0.30 < \rho_A >_{\text{uncorr}}, \quad q_A = 0.55$$

Bob Wiringa et al.
Double beta-decay m.e.’s: Correlations

*Preliminary*

\[ \rho_V = \frac{\tau_1^+ \tau_2^+}{r_{12}} \]
\[ \rho_A = \frac{\tau_1^+ \tau_2^+}{r_{12}} \]

\[ J_f \rho_V \ J_i = 0.063 \]
\[ J_f \rho_A \ J_i = -0.137 \]

\[ < \rho_V >_{\text{corr}} \sim 0.86 < \rho_V >_{\text{uncorr}}, \quad q_V = 0.93 \]
\[ * \quad < \rho_A >_{\text{corr}} \sim 0.72 < \rho_A >_{\text{uncorr}}, \quad q_A = 0.85 \]

Bob Wiringa et al.
Double beta-decay m.e.'s: Two-body currents

\[
\begin{align*}
\nu_{\pi} & = L_{\pi} \tau_{1,+} \tau_{2,+} \frac{\sigma_1 \cdot \sigma_2}{m_\pi q^2} \\
\nu_{\pi\pi} & = L_{\pi\pi} \tau_{1,+} \tau_{2,+} \frac{\sigma_1 \cdot q \sigma_2 \cdot q}{m_\pi (q^2 + m_\pi^2)^2} \\
\nu_{\pi} & = L_{\pi} \tau_{1,+} \tau_{2,+} \frac{\sigma_1 \cdot q \sigma_2 \cdot q}{m_\pi (q^2 + m_\pi^2)} \\
\nu_{CT} & = L_{CT} \tau_{1,+} \tau_{2,+} \frac{\sigma_1 \cdot \sigma_2}{m_\pi^3}
\end{align*}
\]

\(L_{\pi\pi}, L_{\pi}, L_{CT}\) are model dependent

WITH

Emanuele Mereghetti & Dekens & Cirigliano & Graesser & Wiringa \textit{et al.}
Double beta-decay m.e.’s in $^6$He$(0^+;2) \rightarrow ^6$Be$(0^+;0)$: A test case I

Axial $\propto \tau_1^+ \tau_2^+ \sigma_1 \cdot \sigma_2$

Tensor $\propto \tau_1^+ \tau_2^+ S_{12}$

* Preliminary *

WITH

Emanuele Mereghetti & Dekens & Cirigliano & Graesser & Wiringa et al.
Double beta-decay m.e.’s in $^6$He(0$^+;2$) $\rightarrow$ $^6$Be(0$^+;0$): A test case I

Axial $\propto \tau_1^+ \tau_2^+ \sigma_1 \cdot \sigma_2$

Tensor $\propto \tau_1^+ \tau_2^+ S_{12}$

$C(r) = \frac{e^{-(r/R)^2}}{(\pi)^{3/2} R^3}$

* Preliminary *

WITH

Emanuele Mereghetti & Dekens & Cirigliano & Graesser & Wiringa et al.
Double beta-decay m.e.’s in $^{8}\text{He}(0^{+};2) \rightarrow ^{8}\text{Be}(0^{+};0)$: A test case II

Axial $\propto \tau_1^+ \tau_2^+ \sigma_1 \cdot \sigma_2$

Tensor $\propto \tau_1^+ \tau_2^+ S_{12}$

* Preliminary *

WITH

Emanuele Mereghetti & Dekens & Cirigliano & Graesser & Wiringa et al.
Double beta-decay m.e.'s in $^8$He($0^+;2$) → $^8$Be($0^+;0$): A test case II

Axial $\propto \tau_1^+ \tau_2^+ \sigma_1 \cdot \sigma_2$

Tensor $\propto \tau_1^+ \tau_2^+ S_{12}$

$$C(r) = \frac{e^{-r/R^2}}{(\pi)^{3/2} R^3}$$

* Preliminary *

WITH

Emanuele Mereghetti & Dekens & Cirigliano & Graesser & Wiringa et al.
Summary and Outlook

We discussed the role played by correlations and many-body currents in $\beta$- and $\nu0\beta\beta$-decay m.e.’s of $A \leq 12$ nuclei

* Two-body currents (both SNPA and $\chi$EFT) provide negligible quenching in the $\beta$-decay m.e.’s we studied

* Correlations provide a quenching $q \sim 0.95$ in $A = 3$ and $q \sim 0.76$ in $A = 10$ $\beta$-decay m.e.’s

* Correlations affect $\nu0\beta\beta$-decay m.e.’s leading to a quenching $q \sim 0.55$ in Standard Axial $A = 8$ and $q \sim 0.93$ in Standard Fermi $A = 12$

* A cancellation in the Axial Standard two-body current in $A = 8$ $\nu0\beta\beta$-decay m.e.’s could enhance contributions from Non-Standard two-body currents

Outlook

* Benchmark both single- and double-beta decay m.e.’s
* Characterize two-body currents entering double-beta decay m.e.’s
* Calculate more single- and double-beta decay m.e.’s and study model dependence using AV18+IL7 and $\Delta$-full chiral potential by Piarulli et al.