Matrix element for $0\nu\beta\beta$ decay of $^{130}\text{Te}$ and $^{136}\text{Xe}$ within the shell-model framework

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Neutrinoless Double-beta Decay
June 14th, 2017
Institute for Nuclear Theory, Seattle
Outline

- The neutrinoless double-$\beta$ decay
- The problematics of the calculation of the nuclear matrix element (NME) of $0\nu\beta\beta$ decay
- Shell-model calculations of the NME
- The realistic nuclear shell model
- Testing the theoretical framework: calculation of the GT strengths and the nuclear matrix element of $2\nu\beta\beta$ decay
- Outlook
The detection of the $0\nu\beta\beta$ decay is nowadays one of the main targets in many laboratories all around the world, triggered by the search of "new physics" beyond the Standard Model.

- Its detection
  - would correspond to a violation of the conservation of the **leptonic number**,
  - may provide more informations on the nature of the neutrinos (the neutrino as a **Majorana particle**, determination of its **effective mass**, ..).
The double $\beta$-decay

The semiempirical mass formula provides two different parabolas for even-mass isobars:

- Maria Goeppert-Mayer (1935) suggested the possibility to detect
  \[(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e\]

- Historically, Giulio Racah was the first one, to test the neutrino as a Majorana particle, to consider the process:
  \[(A, Z) \rightarrow (A, Z+2) + e^- + e^-\]
The neutrinoless double $\beta$-decay

The inverse of the $0\nu\beta\beta$-decay half-life is proportional to the squared nuclear matrix element (NME).
This evidences the relevance to calculate the NME

\[
\left[ T_{1/2}^{0\nu} \right]^{-1} = G^{0\nu} \left| M^{0\nu} \right|^2 \langle m_\nu \rangle^2,
\]

- $G^{0\nu}$ is the so-called phase-space factor, obtained by integrating over the single electron energies and angles, and summing over the final-state spins;
- $\langle m_\nu \rangle = | \sum_k m_k U_{ek}^2 |$ effective mass of the Majorana neutrino, $U_{ek}$ being the lepton mixing matrix.
The detection of the $0\nu\beta\beta$-decay

It is necessary to locate the nuclei that are the best candidates to detect the $0\nu\beta\beta$-decay

- The main factors to be taken into account are:
  - the $Q$-value of the reaction;
  - the phase-space factor $G^{0\nu}$;
  - the isotopic abundance
The detection of the $0\nu\beta\beta$-decay

- **First group:** $^{76}$Ge, $^{130}$Te, and $^{136}$Xe.
- **Second group:** $^{82}$Se, $^{100}$Mo, and $^{116}$Cd.
- **Third group:** $^{48}$Ca, $^{96}$Zr, and $^{150}$Nd.
TeO₂ crystals used as low heat capacity bolometers, arranged into towers and cooled in a large cryostat to approximately 10 m°K with a dilution refrigerator.

The detectors are isolated from backgrounds by ultrapure low-radioactivity shielding.

Temperature spikes from electrons emitted in Te $^{0}\beta\beta$ are collected for spectrum analysis.
The KamLAND detector’s outer layer consists of an 18 meter-diameter stainless steel containment vessel with an inner lining of 1879 photo-multiplier tubes. The second, inner layer consists of a 13 m-diameter nylon balloon filled with a liquid scintillator composed of 1000 metric tons of mineral oil, benzene, and fluorescent chemicals.

As part of the KamLAND-Zen $0\nu\beta\beta$ decay search, a balloon of scintillator with 320 kg of dissolved xenon was suspended in the center of the detector in 2011.

KamLAND-Zen uses the detector to study beta decay of $^{136}$Xe from a balloon placed in the scintillator in summer 2011. Observations set a limit for neutrinoless double-beta decay half-life of $1.9 \times 10^{25}$ yr.
The prototype EXO-200 uses a copper cylindrical time projection chamber filled with 150 kg of pure liquid xenon. Xenon is a scintillator, so decay particles produce prompt light which is detected by avalanche photodiodes, providing the event time.

EXO-200 was designed with a goal of less than 40 events per year within two standard deviations of expected decay energy. This background was achieved by selecting and screening all materials for radiopurity. Originally the vessel was to be made of Teflon, but the final design of the vessel uses thin, ultra-pure copper.

EXO-200 has observed double-beta decay of $^{136}\text{Xe}$, with a half life of $2.165 \pm 0.016\text{(stat)} \pm 0.059\text{(sys)} \times 10^{21}\text{ yr}$. EXO set a limit on neutrinoless beta decay of $1.1 \times 10^{25}$ and mass to 450 meV.
The calculation of the NME

The NME is given by

\[ M^{0\nu} = M_{GT}^{0\nu} - \left( \frac{g_V}{g_A} \right)^2 M_{F}^{0\nu} - M_{T}^{0\nu}, \]

where the matrix elements are defined as follows:

\[ M_{\alpha}^{0\nu} = \sum_{m,n} \langle 0_f^+ | \tau_m \tau_n^- O_{mn}^\alpha | 0_i^+ \rangle, \]

with \( \alpha = (GT, F, T) \).

Since the transition operator is a two-body one, we may write it as:

\[ M_{\alpha}^{0\nu} = \sum_{j,j',j_n,j_n',J_\pi} \text{TBTD} (j_p j_{p'}, j_n j_{n'}; J_\pi) \langle j_p j_{p'}; J_\pi T | \tau_1^- \tau_2^- O_{12}^\alpha | j_n j_{n'}; J_\pi T \rangle \]
The calculation of the NME

where the two-body transition-density matrix elements are defined as

$$TBTD (j_p j_p', j_n j_n'; J_\pi) = \langle 0^+_f | (a_{j_p}^{\dagger} a_{j_p'}^{\dagger})^{J_\pi} (a_{j_n'} a_{j_n})^{J_\pi} | 0^+_i \rangle$$

and the Gamow-Teller ($GT$), Fermi ($F$), and tensor ($T$) operators as

$$O_{12}^{GT} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 H_{GT}(r) ,$$
$$O_{12}^{F} = H_{F}(r) ,$$
$$O_{12}^{T} = [3 (\vec{\sigma}_1 \cdot \hat{r}) (\vec{\sigma}_1 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2] H_{T}(r) .$$

These operators should be regularized consistently with the two-body $NN$ potential

So, short-range correlations (SRC) need to be taken into account (Jastrow functions, UCOM, CCM,...)
The calculation of the NME

To describe the nuclear properties detected in the experiments, one needs to resort to nuclear structure models.

- Every model is characterized by a certain number of parameters.
- The calculated value of the NME may depend upon the chosen nuclear structure model.

All models may present advantages and/or shortcomings to calculate the NME.
The spread of nuclear structure calculations evidences inconsistencies among results obtained with different models.
The nuclear shell model

The nucleons are subjected to the action of a mean field, that takes into account most of the interaction of the nucleus constituents.

Only valence nucleons interact by way of a residual two-body potential, within a reduced model space.

\[
\begin{align*}
\text{protons} & : & p_{3/2}, p_{1/2}, s_{1/2} \\
\text{neutrons} & : & s_{1/2}, d_{5/2}, d_{3/2}, s_{1/2}, p_{3/2}, p_{1/2}, s_{1/2}
\end{align*}
\]

\[{}^{19}\text{F}\]

- **Advantage** → It is a microscopic model, the degrees of freedom of the valence nucleons are explicitly taken into account.
- **Shortcoming** → High-degree computational complexity.

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Disassembling the nuclear matrix elements of the neutrinoless $\beta\beta$ decay

J. Menéndez, A. Poves, E. Caurier, F. Nowacki

- Shell-model hamiltonian GCN5082, empirically constructed
- Light-neutrino exchange considered
- Study of seniority truncation of the SM wave functions
- Jastrow and UCOM short-range correlations (SRC)
- Comparison with RPA calculation
Shell-model analysis of the $^{136}\text{Xe}$ double beta decay nuclear matrix elements

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B. A. Brown
Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824, USA

- Shell-model hamiltonian hamiltonian derived from $N^3LO$ potential by way of the many-body perturbation theory (MBT), plus empirical adjustments
- Both light-neutrino and heavy-neutrino exchange considered
- Two different model space considered, $jj77$ and $jj55$
- Jastrow SRC from CD-Bonn and AV18 potential

<table>
<thead>
<tr>
<th>$n=0$</th>
<th>$M_{0\nu}^0$</th>
<th>$M_N^{0\nu}$</th>
<th>$M_{\Lambda'}^{0\nu}$</th>
<th>$M_{\tilde{q}}^{0\nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRC1</td>
<td>2.21</td>
<td>143.0</td>
<td>1106</td>
<td>206.8</td>
</tr>
<tr>
<td>SRC2</td>
<td>2.06</td>
<td>98.79</td>
<td>849.0</td>
<td>197.2</td>
</tr>
</tbody>
</table>

| $n=1$ | $|\eta_j^{up}|$ [9] | $|\eta_j^{up}|$ [35] |
|-------|-----------------|-----------------|
| SRC1  | 1.46            | 9.02            |
| SRC2  | 1.12            | 0.103           |
Chiral Two-Body Currents in Nuclei: Gamow-Teller Transitions and Neutrinoless Double-Beta Decay

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3Racah Institute of Physics, The Hebrew University, 91904 Jerusalem, Israel

• Shell-model hamiltonian GCN5082, empirically constructed
• Contributions of the chiral EFT two-body currents to the quenching of the GT transitions
• Normal-ordered one-body approximation

![Graph showing contributions of different currents to the decay matrix elements](image)
The derivation of the shell-model hamiltonian using the many-body theory may provide a reliable approach.

The model space may be “shaped” according to the computational needs of the diagonalization of the shell-model hamiltonian.

In such a case, the effects of the neglected degrees of freedom are taken into account by the effective hamiltonian $H_{\text{eff}}$ theoretically.
An example: $^{19}\text{F}$

- 9 protons & 10 neutrons interacting
- spherically symmetric mean field (e.g. harmonic oscillator)
- 1 valence proton & 2 valence neutrons interacting in a truncated model space

The degrees of freedom of the core nucleons and the excitations of the valence ones above the model space are not considered explicitly.
Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered.

Two alternative approaches
- phenomenological
- microscopic

\[ V_{NN} ( + V_{NNN} ) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}} \]

Definition
The eigenvalues of \( H_{\text{eff}} \) belong to the set of eigenvalues of the full nuclear hamiltonian.
Workflow for a realistic shell-model calculation

1. Choose a realistic $NN$ potential ($NNN$)
2. Determine the model space better tailored to study the system under investigation
3. Derive the effective shell-model hamiltonian by way of the many-body theory
4. Calculate the physical observables (energies, e.m. transition probabilities, ...)

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Realistic nucleon-nucleon potential: $V_{NN}$

Several realistic potentials $\chi^2/datum \simeq 1$: CD-Bonn, Argonne V18, Nijmegen, ...

Strong short-range repulsion

How to handle the short-range repulsion?

- Brueckner $G$ matrix
- EFT inspired approaches
Realistic nucleon-nucleon potential: $V_{NN}$

Several realistic potentials $\chi^2/\text{datum} \simeq 1$: CD-Bonn, Argonne V18, Nijmegen, ...

How to handle the short-range repulsion?

- Brueckner $G$ matrix
- EFT inspired approaches
- $V_{\text{low-k}}$, SRG
- Chiral potentials
Realistic nucleon-nucleon potential: $V_{NN}$

Several realistic potentials $\chi^2/datum \simeq 1$: CD-Bonn, Argonne V18, Nijmegen, ...

How to handle the short-range repulsion?

- Brueckner $G$ matrix
- EFT inspired approaches
  - $V_{\text{low-}k}$, SRG
  - chiral potentials

Strong short-range repulsion

![Graph showing the short-range repulsion profile](image-url)
Several realistic potentials $\chi^2/\text{datum} \approx 1$: CD-Bonn, Argonne V18, Nijmegen, ...

How to handle the short-range repulsion?

- Brueckner $G$ matrix
- EFT inspired approaches
  - $V_{\text{low-}} - k$, SRG
  - Chiral potentials

Strong short-range repulsion
Realistic nucleon-nucleon potential: $V_{NN}$

Several realistic potentials $\chi^2/datum \simeq 1$: CD-Bonn, Argonne V18, Nijmegen, ...

How to handle the short-range repulsion?

- Brueckner $G$ matrix
- EFT inspired approaches
  - $V_{\text{low-}k}$, SRG
  - chiral potentials

Strong short-range repulsion

\[ k' \quad k \]

\[ \lambda_0, \lambda_1, \lambda_2 \]
Realistic nucleon-nucleon potential: $V_{NN}$

Several realistic potentials $\chi^2/datum \simeq 1$: CD-Bonn, Argonne V18, Nijmegen, ...

How to handle the short-range repulsion?

- Brueckner $G$ matrix
- EFT inspired approaches
  - $V_{\text{low-}k}$, SRG
  - chiral potentials

Strong short-range repulsion
The shell-model effective hamiltonian

A-nucleon system Schrödinger equation

\[ H|\psi_\nu\rangle = E_\nu|\psi_\nu\rangle \]

with

\[ H = H_0 + H_1 = \sum_{i=1}^{A} (T_i + U_i) + \sum_{i<j} (V_{NN}^{ij} - U_i) \]

Model space

\[ |\Phi_i\rangle = [a_1^\dagger a_2^\dagger \ldots a_n^\dagger]_i |c\rangle \Rightarrow P = \sum_{i=1}^{d} |\Phi_i\rangle \langle \Phi_i| \]

Model-space eigenvalue problem

\[ H_{\text{eff}} P |\psi_\alpha\rangle = E_\alpha P |\psi_\alpha\rangle \]
The shell-model effective hamiltonian

\[
\begin{pmatrix}
    PHP & PHQ \\
    QHP & QHQ
\end{pmatrix}
\begin{align*}
    \mathcal{H} &= X^{-1}HX \\
    \implies Q\mathcal{H}P &= 0
\end{align*}
\Rightarrow

\begin{pmatrix}
    P\mathcal{H}P & P\mathcal{H}Q \\
    0 & Q\mathcal{H}Q
\end{pmatrix}

H_{\text{eff}} = P\mathcal{H}P

Suzuki & Lee \Rightarrow X = e^\omega \text{ with } \omega = \begin{pmatrix} 0 & 0 \\ Q\omega P & 0 \end{pmatrix}

H_{1\text{eff}}^{\text{eff}}(\omega) = P\mathcal{H}_1P + P\mathcal{H}_1Q\frac{1}{\epsilon - QHQ}Q\mathcal{H}_1P - P\mathcal{H}_1Q\frac{1}{\epsilon - QHQ}\omega H_{1\text{eff}}^{\text{eff}}(\omega)
The shell-model effective hamiltonian

Folded-diagram expansion

\( \hat{Q} \)-box vertex function

\[
\hat{Q}(\epsilon) = PH_1 P + PH_1 Q \frac{1}{\epsilon - QHQ} QH_1 P
\]

\( \Rightarrow \) Recursive equation for \( H_{\text{eff}} \) \( \Rightarrow \) iterative techniques

(Krenciglowa-Kuo, Lee-Suzuki, ...)

\[
H_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \ldots,
\]
The perturbative approach to the shell-model $H_{\text{eff}}$

$$\hat{Q}(\epsilon) = PH_1 P + PH_1 Q \frac{1}{\epsilon - QHQ} QH_1 P$$

The $\hat{Q}$-box can be calculated perturbatively

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1 Q)^n}{(\epsilon - QH_0 Q)^{n+1}}$$

The diagrammatic expansion of the $\hat{Q}$-box

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Q-box perturbative expansion: 1-body diagrams
Q-box perturbative expansion: 2-body diagrams
Q-box perturbative expansion: 2-body diagrams
$Q$-box perturbative expansion: 2-body diagrams
Consistently, any shell-model effective operator may be calculated.

It has been demonstrated that, for any bare operator $\Theta$, a non-Hermitian effective operator $\Theta_{\text{eff}}$ can be written in the following form:

$$
\Theta_{\text{eff}} = (P + \hat{Q}_1 + \hat{Q}_1 \hat{Q}_1 + \hat{Q}_2 \hat{Q} + \hat{Q} \hat{Q}_2 + \cdots) (\chi_0 + \chi_1 + \chi_2 + \cdots),
$$

where

$$
\hat{Q}_m = \frac{1}{m!} \left. \frac{d^m \hat{Q}(\epsilon)}{d\epsilon^m} \right|_{\epsilon=\epsilon_0},
$$

$\epsilon_0$ being the model-space eigenvalue of the unperturbed hamiltonian $H_0$.

The $\chi_n$ operators are defined as follows:

\[
\chi_0 = (\hat{\Theta}_0 + h.c.) + \Theta_{00}, \\
\chi_1 = (\hat{\Theta}_1 \hat{Q} + h.c.) + (\hat{\Theta}_{01} \hat{Q} + h.c.), \\
\chi_2 = (\hat{\Theta}_1 \hat{Q}_1 \hat{Q} + h.c.) + (\hat{\Theta}_2 \hat{Q}\hat{Q} + h.c.) + \\
(\hat{\Theta}_{02} \hat{Q}\hat{Q} + h.c.) + \hat{Q}\hat{\Theta}_{11}\hat{Q}, \\
\vdots
\]

and

\[
\hat{\Theta}(\epsilon) = P\Theta P + P\Theta Q \frac{1}{\epsilon - QHQ} QH_1 P, \\
\hat{\Theta}(\epsilon_1; \epsilon_2) = P\Theta P + PH_1 Q \frac{1}{\epsilon_1 - QHQ} \times \\
Q\Theta Q \frac{1}{\epsilon_2 - QHQ} QH_1 P, \\
\hat{\Theta}_m = \frac{1}{m!} \left. \frac{d^m \hat{\Theta}(\epsilon)}{d\epsilon^m} \right|_{\epsilon = \epsilon_0}, \\
\hat{\Theta}_{nm} = \frac{1}{n! m!} \left. \frac{d^n}{d\epsilon_1^n} \frac{d^m}{d\epsilon_2^m} \hat{\Theta}(\epsilon_1; \epsilon_2) \right|_{\epsilon_1 = \epsilon_0, \epsilon_2 = \epsilon_0}
\]
The shell-model effective operators

We arrest the $\chi$ series at $\chi_0$, and expand it perturbatively:

One-body operator

\[
\begin{align*}
\begin{array}{c}
\begin{array}{c}
\chi_0
\end{array}
\end{array}
\end{align*}
\]

Two-body operator

\[
\begin{align*}
\begin{array}{c}
\begin{array}{c}
\chi_0
\end{array}
\end{array}
\end{align*}
\]

see also J. D. Holt and J. Engel, Phys. Rev. C 87, 064315 (2013)
Our recipe for realistic shell model

- Input $V_{NN}$: $V_{\text{low-k}}$ derived from the high-precision $NN$ CD-Bonn potential with a cutoff: $\Lambda = 2.6 \, \text{fm}^{-1}$.

- $H_{\text{eff}}$ obtained calculating the $Q$-box up to the 3rd order in perturbation theory.

- Effective operators are consistently derived by way of the MBPT.
Realistic shell-model calculations for $^{130}$Te and $^{136}$Xe

Check this approach calculating observables related to the GT strengths and $2\nu\beta\beta$ decay and compare the results with data.

$$\left[ T_{1/2}^{2\nu} \right]^{-1} = G^{2\nu} |M_{2\nu}^{GT}|^2$$
Search for Neutrinoless Double-Beta Decay of $^{130}$Te with CUORE-0


(CUORE Collaboration)

Search for Neutrinoless Double-Beta Decay in $^{136}$Xe with EXO-200


((EXO Collaboration)

**Shell-model calculations for $^{130}$Te, $^{136}$Xe**

- **five proton and neutron orbitals outside double-closed $^{100}$Sn**
  
  $0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2}$

- **1292 two-body matrix elements and 10 SP energies:** (I) theoretical SP energies, (II) empirical SP energies fitted to the observed low-lying states in $^{133}$Sb and $^{131}$Sn

<table>
<thead>
<tr>
<th>Proton SP spacings</th>
<th>Neutron SP spacings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0g_{7/2}$ I</td>
<td>$0.0$ I</td>
</tr>
<tr>
<td>$0g_{7/2}$ II</td>
<td>$0.0$ II</td>
</tr>
<tr>
<td>$1d_{5/2}$</td>
<td>$0.3$</td>
</tr>
<tr>
<td>$1d_{3/2}$</td>
<td>$1.2$</td>
</tr>
<tr>
<td>$2s_{1/2}$</td>
<td>$1.1$</td>
</tr>
<tr>
<td>$0h_{11/2}$</td>
<td>$1.9$</td>
</tr>
</tbody>
</table>

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Spectroscopy of $^{130}\text{Te}$ and $^{130}\text{Xe}$

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$J_i \rightarrow J_f$</th>
<th>$B(E2)_{\text{Expt}}$</th>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{130}\text{Te}$</td>
<td>$2^+ \rightarrow 0^+$</td>
<td>580 ± 20</td>
<td>430</td>
<td>420</td>
</tr>
<tr>
<td></td>
<td>$6^+ \rightarrow 4^+$</td>
<td>240 ± 10</td>
<td>220</td>
<td>200</td>
</tr>
<tr>
<td>$^{130}\text{Xe}$</td>
<td>$2^+ \rightarrow 0^+$</td>
<td>$1170^{+20}_{-10}$</td>
<td>954</td>
<td>876</td>
</tr>
</tbody>
</table>
Spectroscopy of $^{136}$Xe and $^{136}$Ba

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$J_i \rightarrow J_f$</th>
<th>$B(E2)_{Expt}$</th>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{136}$Xe</td>
<td>$2^+ \rightarrow 0^+$</td>
<td>$420 \pm 20$</td>
<td>$300$</td>
<td>$300$</td>
</tr>
<tr>
<td></td>
<td>$4^+ \rightarrow 2^+$</td>
<td>$53 \pm 1$</td>
<td>$9$</td>
<td>$11$</td>
</tr>
<tr>
<td></td>
<td>$6^+ \rightarrow 4^+$</td>
<td>$0.55 \pm 0.02$</td>
<td>$1.58$</td>
<td>$2.42$</td>
</tr>
<tr>
<td>$^{136}$Ba</td>
<td>$2^+ \rightarrow 0^+$</td>
<td>$800^{+80}_{-40}$</td>
<td>$590$</td>
<td>$520$</td>
</tr>
</tbody>
</table>
Proton/neutron occupancies/vacancies for $^{130}\text{Te} \rightarrow ^{130}\text{Xe}$

Data from the cross sections of the $(d,^3\text{He})$ and $(\alpha,^3\text{He})$ reactions.

\begin{itemize}
  \item Protons
  \begin{itemize}
    \item Experiment (Exp)
    \item Theory (Th)
    \item States: $0g_{7/2}$, $0h_{11/2}$, $1d$, $2s_{1/2}$
  \end{itemize}

  \item Neutrons
  \begin{itemize}
    \item Experiment (Exp)
    \item Theory (Th)
    \item States: $0g_{7/2}$, $0h_{11/2}$, $1d$, $2s_{1/2}$
  \end{itemize}
\end{itemize}
Proton/neutron occupancies/vacancies for $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$

Data from the cross sections of the $(d,^3\text{He})$ and $(\alpha,^3\text{He})$
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$ GT$^-$ running sums

$^{130}\text{Te}$ GT strength

bare

exp ($^3\text{He,t}$)

effective
$^{130}$Te → $^{130}$Xe nuclear matrix element

$$M_{2
u}^{GT} = \sum_n \frac{\langle 0^+_f | \vec{\sigma}^\tau^- | 1^+_n \rangle \langle 1^+_n | \vec{\sigma}^\tau^- | 0^+_i \rangle}{E_n + E_0}$$

<table>
<thead>
<tr>
<th>1st order</th>
<th>2nd order</th>
<th>3rd order</th>
<th>Expt</th>
</tr>
</thead>
</table>
| $M_{2
u}^{GT}$ (in MeV$^{-1}$) | 0.142     | 0.050     | 0.044      | 0.034 ± 0.003 |
$^{136}$Xe $\rightarrow ^{136}$ Ba GT$^-$ running sums

$^{136}$Xe GT strength

bare

exp ($^3$He,t)

effective

$\Sigma B(GT)$ vs $E_x$ (keV)
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$ nuclear matrix element

$$M_{2\nu}^{\text{GT}} = \sum_n \frac{\langle 0_f^+ || \vec{\sigma}^- || 1_n^+ \rangle \langle 1_n^+ || \vec{\sigma}^- || 0_i^+ \rangle}{E_n + E_0}$$

<table>
<thead>
<tr>
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<th>2nd order</th>
<th>3rd order</th>
<th>Expt</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{2\nu}^{\text{GT}}$ (in MeV$^{-1}$)</td>
<td>0.0975</td>
<td>0.0272</td>
<td>0.0285</td>
</tr>
</tbody>
</table>
Outlook

- $2\nu\beta\beta$
  - Role of real three-body forces and two-body currents (present collaboration with Pisa group)
  - Evaluation of the contribution of three-body correlations (blocking effect)

- $0\nu\beta\beta$
  - Derivation of the two-body effective operator
  - Calculation of the two-body transition-density matrix elements (in collaboration with Frederic Nowacki)
  - SRC calculated consistently with $V_{\text{low--k}}$
Acknowledgements

- L. De Angelis (INFN-NA)
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- N. Itaco (SUN and INFN-NA)
- F. Nowacki (IPHC Strasbourg and SUN)
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