One-nucleon Transfers to Resonances

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Traditional Role of (d,p) reactions

- Transfer neutron to $lj$ bound state $\phi_{nlj}(r_n)$
  - usually large momentum transfer
  - Shape of proton $\sigma_p(\theta)$ depends on $l$
  - Analyzing powers depend on $j$
  - Magnitude of $\sigma_p(\theta)$ depends on spectroscopic factor $S_{nlj}$

- Higher-order corrections calculable (CRC, CCBA)
Measuring resonances with (d,p)

- Do **not** probe spectroscopic factors
- Do **probe** partial widths and resonance energies
  - These parameters come from R-matrix energies and reduced width amplitudes
- Desirable to have (d,p) calculations from R-matrix parameters
  - Is this possible?
  - Can then predict n+target scattering not otherwise measurable
  - But R-matrix values are surface and external features!
- Note: $L$-dependence of $\sigma_p(\theta)$ decreases for less bound neutrons, and hence for continuum neutrons
  - Reduced magnitude of momentum transfers
Propose to use: Surface Transfer Operator

- Work of TORUS collaboration (www.reactiontheory.org)
- Proposed: Mukhamedzhyanov (PRC 84, 044616, 2011)
- Developed for 1-step transfers:
  - Escher et al., (PRC. 89, 054605, 2013)
- Now applied to transfers from entrance deuteron channels including breakup in CDCC basis.
Applicable Examples of Resonances

- Near single-particle resonances
- Structured p-shell resonances

$^{20}$O(d,p)$^{21}$O inverse-kinematics experiment

$^{15}$N(d,p)$^{16}$N

$^{15}$N(n,n)
Aim is to fit resonances in (d,p) cross sections in a region of the continuum. We see many wide and narrow resonances, often overlapping. Want to find neutron pole energies and partial widths, in entrance channel for (n,γ).
Post-prior equivalence in 1st order

- Post matrix element in 1st order
  \[ M_{dp}^{(post)} = \langle \phi_A^F \chi_{pF}(-) | \Delta V_p | \varphi_d \chi_{dA}^+(+) \rangle \]

- Prior matrix element in 1st order
  \[ M_{dp}^{(prior)} = \langle \phi_A^F \chi_{pF}(-) | \Delta V_d | \varphi_d \chi_{dA}^+(+) \rangle \]

- Equivalent: \( M_{dp}^{(post)} = M_{dp}^{(prior)} \) because
  - KE operators satisfy \( \langle \phi_A^F(r_n) | T_n + T_p | \varphi_d(r) \rangle = 0 \)
  - Because \( \phi_A^F(r_n) \to 0 \) at \( r_n = 0 \) and \( r_n \to \infty \)

- If the wave functions not zero, then get surface terms.
Splitting the Transfer Matrix Element

- Define \( M_{\text{post}}(a,b) \) & \( M_{\text{prior}}(a,b) \) with \( a < r_n < b \) limits
- General result:
  \[
  M_{\text{post}}(a,b) = M_{\text{surf}}(a) + M_{\text{prior}}(a,b) - M_{\text{surf}}(b)
  \]
where \( M_{\text{surf}}(\rho) = \langle f_p^{(-)} | \langle T - T^\dagger \rangle | f_d^{(+)} \rangle_{(r<\rho)} \)
- Previous slide used \( M_{\text{surf}}(0) = M_{\text{surf}}(\infty) = 0 \)

- So, for any surface \( \rho \): 
  from: \( M = M_{\text{post}}(0,\rho) + M_{\text{post}}(\rho,\infty) \)
  hence: \( M = M_{\text{post}}(0,\rho) + M_{\text{surf}}(\rho) + M_{\text{prior}}(\rho,\infty) \)

Mukhamedzhanov (2011):
Evaluating the Surface Matrix Element

With \( M_{\text{surf}}(\rho) = \langle f_p^{-} \phi_n \mid \hat{T} - \hat{T} \mid \phi_d f_d^{+} \rangle_{(r<\rho)} \)

- Need to calculate matrix elements like:

\[
\int_{r \leq R} \, dr \, f(r)[\hat{T} - \hat{T}]g(r)
\]

\[
= -\frac{1}{2\mu} \int_{r=R} \, dS[g(r)\nabla_r f(r) - f(r)\nabla_r g(r)]
\]

\[
= -\frac{1}{2\mu} R^2 \int \, d\Omega_r \left[ g(r) \frac{\partial f(r)}{\partial r} - f(r) \frac{\partial g(r)}{\partial r} \right]_{r=R}
\]

- That is, functions & derivatives on the surface \( r_n = \rho \)
- Do this for partial waves, in reaction code FRESCO
Preliminary Estimates of Magnitudes

- In DWBA (1st order), find surface term as:
  \[ M_{\text{surf}}(\rho) = M_{\text{post}}(0,\rho) - M_{\text{prior}}(0,\rho) \]

- Look at bound states and resonances.
  - See if convergence to breakup states is easier?

- Calculate all terms of
  \[ M = M_{\text{post}}(0,\rho) + M_{\text{surf}}(\rho) + M_{\text{prior}}(\rho,\infty) \]
Internal, surface, external contributions – $^{90}\text{Zr}(d,p)$ at $E_d=11$ MeV

\[ M = M^{(\text{post})}(0,a) + M^{(\text{surf})}(a) + M^{(\text{prior})}(a,\infty) \]

Observations
- Surface term dominant at 6-8 fm
- Small interior contributions
- Small exterior contributions
- Surface term does not produce the whole cross section

The surface term is dominant, but contributions from the interior and exterior terms remain.

Escher, Thompson, Mukhamedzhanov, JPCS (2012).
The surface contribution – $^{90}$Zr(d,p) at $E_d=11$ MeV

- Cross sections depend on surface radius
- The surface term is dominant, but corrections remain

Escher et al, PRC 89, 054605 (2014)
Numerical tests of the formalism (DWBA) – $^{48}$Ca(d,p) at $E_d=13, 19.3, 56$ MeV

Angular cross section – Surface term only

Calculations for $^{49}$Ca $1^{st}$ $(1/2^-)$ give similar results

Surface term approximation improves with decreasing energy

Peak cross section relative to full calculation

Escher et al, PRC (2014)
Surface formalism for DWBA – resonance states

\[ M^{\text{DW(post)}}(P, k_{dA}) = \langle \chi_{pF}^{(-)} \Psi_{bB}^{(\text{int})} | \Delta V_{pF} | \phi_d \phi_A \chi_{dA}^{(+)} \rangle, \quad (94) \]

Surface formulation:

\[ M = M^{(\text{post})}(0,a) + M_{\text{surf}}(a) + M^{(\text{prior})}(a, \infty) \]

Total post matrix element for \( b + B \neq n + A \) example:
The oxygen case - $^{20}\text{O}$ at $E_d=21$ MeV

Angular cross section – Surface term only

Peak cross section relative to full calculation

Surface at ~5 fm approximately reproduces measurement.

Escher et al, PRC 89, 054605 (2014)
Resonances – $^{90}$Zr at $E_d=11$ MeV

Angular cross section – Surface term only

Peak cross section relative to full calculation

- Results similar to bound-state cases
- Surface term dominant at larger radii
- Interior/exterior terms still contribute

Escher et al, PRC 89, 054605 (2014)
The surface formalism: can we save it? $^{20}$O at $E_d=21$ MeV

- reducing the surface radius
- adding prior-exterior contribution

Angular cross section at smaller radius

Angular cross section at peak radius

$^{21}$O $d_{3/2}$ resonance at 0.964 MeV

$^{21}$O $d_{3/2}$ resonance at 2.364 MeV
Extension of the formalism to include breakup

**DWBA matrix element**

\[ M^{(post)} = M^{(post)}(0,a) + M^{(surf)}(a) + M^{(prior)}(a,\infty) \]

**CDCC (Continuum-discretized coupled channels)**
- Approximate treatment of 3-body problem
- Describes breakup of deuteron

**CDCC extension of R-matrix formalism**
- Simultaneous calculation of breakup and transfer cross sections
- Exterior term included in breakup, convergence issues removed
- More peripheral, reduce interior contribution
- Surface term dominant

**CDCC matrix element**

\[ M^{(post)} = M^{(post)}(0,a) + M^{(surf)}(a) \]
\[ M^{(prior)}(a,\infty) = 0 \text{ (is included in breakup)} \]
Derivatives of products of 2 wfns

\[
\frac{\partial}{\partial r'} \langle \hat{R}, \hat{r} | \alpha \rangle \frac{1}{rR} \varphi\alpha(r) u\alpha(R) = \frac{1}{rR} \sum_{M_L m_L} C_{\alpha}^{M_L m_L : M} \\
Y_{LM}^{M_L} (\hat{R}) u\alpha(R) \frac{p}{r} \left\{ \sqrt{\frac{4\pi \ell(2\ell+1)}{3}} \sum_{\lambda=-1}^{1} \langle \ell-1 m-\lambda, 1\lambda|\ell m \rangle Y_{\ell-1}^{m-\lambda}(\hat{r}) Y_{1}^{\lambda}(\hat{r}') \frac{\varphi\alpha(r)}{r} \right. \\
+ Y_{\ell}^{m}(\hat{r}) \hat{r} \cdot \hat{r}' \left[ \varphi\alpha'(r) - \frac{\ell+1}{r} \varphi\alpha(r) \right] \right\} \\
+ Y_{\ell}^{m\ell}(\hat{r}) \varphi\alpha(r) \frac{P}{R} \left\{ \sqrt{\frac{4\pi L(2L+1)}{3}} \sum_{\Lambda=-1}^{1} \langle L-1 M_{L-\Lambda}, 1\Lambda|LM_{L} \rangle Y_{L-1}^{M_{L-\Lambda}}(\hat{R}) Y_{1}^{\Lambda}(\hat{r}') \frac{u\alpha(R)}{R} \right. \\
+ Y_{L}^{ML}(\hat{R}) \hat{R} \cdot \hat{r}' \left[ u\alpha'(R) - \frac{L+1}{R} u\alpha(R) \right] \right\} \\
\frac{r}{r'} = \frac{P r'}{R'} + \frac{Q R'}{R'} \\
R = P r' + Q R'
\]
Using Eq. (66), this is

\[ S_{\beta\alpha}^\text{surf}(R') = -\frac{\hbar^2 \rho^2}{2\mu_n} \sum_{M'_L m'_\ell M M_L m_\ell} F^M_{\beta'} m'_\ell M* C^M_{\alpha M L m_\ell} \langle Y^M_{M'_L}(\hat{R'}) Y^m_{m'_\ell}(\hat{r'}) \rangle_{r'=\rho \frac{1}{rR}} \]

\[
\begin{bmatrix}
\phi'_\beta(\rho) & Y^m_{\ell}(\hat{r}) Y^M_{L}(\hat{R}) & \varphi_\alpha(r) u_\alpha(R) \\
- \phi_\beta(\rho) \left( Y^M_{L}(\hat{R}) u_\alpha(R) \right) & & \\
+ Y^m_{\ell}(\hat{r}) \hat{r} \cdot \hat{r}' \left[ \varphi'_\alpha(r) - \frac{\ell+1}{r} \varphi_\alpha(r) \right] \\
+ Y^m_{\ell}(\hat{r}) \varphi_\alpha(r) \right) & & \\
+ Y^M_{L}(\hat{R}) \hat{R} \cdot \hat{r}' \left[ u'_\alpha(R) - \frac{L+1}{R} u_\alpha(R) \right] & & \\
\end{bmatrix} \] 

The \( F^M_{\beta'} m'_\ell M* C^M_{\alpha M L m_\ell} \) are the channel-defining Clebsch-Gordon coefs.
Implementation

- As \( R' \neq R' \), transfer couplings are still non-local

- With A, B, C as non-local operators, the transfer-channel exit equation is

\[
[H_\beta - E_\beta]u_\beta + \phi'_\beta(\rho) \ A_\beta \alpha u_\alpha + \phi_\beta(\rho) \ B_\beta \alpha u_\alpha + \phi_\beta(\rho) \ C_\beta \alpha \left[ u'_\alpha - \frac{L_\alpha + 1}{R} u_\alpha \right] = 0
\]

- More complicated than standard transfers, because of derivative \( u'_\alpha(R) \)
**R-matrix continuum parameterisation**

- **Definition**  
  \[ R(\epsilon_\beta) = \frac{1}{\rho \phi'(\rho; \epsilon_\beta)} \phi(\rho; \epsilon_\beta) \]

- **Parameterization:**  
  \[ R(\epsilon_\beta) = \sum_{p=1}^{N} \frac{\gamma_p^2}{\epsilon_p - \epsilon_\beta} \]  
  \( (N\text{-pole case}) \)

- **From**  \( R(\epsilon_\beta) \), get **S-matrix** \( S(\epsilon_\beta) \) and **wf**  \( \phi_\beta(\rho; \epsilon_\beta) \) by usual theory, for every energy  \( \epsilon_\beta = E_{\text{tot}} - E_\beta \)

- **Then exit channel eqn,** for continuous \( E_\beta \) is

  \[ [H_\beta - E_\beta]u_\beta + \phi_\beta(\rho; \epsilon_\beta) \left\{ \frac{1}{\rho R_\beta(\epsilon_\beta)} A_{\beta \alpha} u_\alpha + B_{\beta \alpha} u_\alpha + C_{\beta \alpha} \left[ u'_\alpha - \frac{L_\alpha + 1}{R} u_\alpha \right] \right\} = 0 \]

  Note that A, B, C and \( u_\alpha \) are independent of exit energy \( E_\beta \).
**Interior-post + Surface-term**

![Graph showing cross sections for different proton angles and neutron potentials.](image)

**Legend:**
- **Black:** interior post (depends on inside wf)
- **Red:** surface (depends on R-matrix parameters)

The 2 terms add as matrix elements: **Coherent sums.**

The sums (green) should be the same for any surface position. (should be outside the neutron potential! )
Conclusions

Surface formalism for studying resonances with (d,p):

- Uses successful R-matrix ideas to emphasize **asymptotic properties** of the wave function

- Separation into interior and exterior leads to a surface term which can be expressed in terms of familiar R-matrix parameters, thus providing **spectroscopic information**

- Our DWBA and CDCC studies show **surface term is dominant**; and dependence on model for nuclear interior is reduced.

- The surface term alone is **not sufficient** to describe transfer reactions, corrections are required

- Within a CDCC implementation (which includes breakup effects) the **exterior is already included**: not needed for transfer operator.