Inclusive deuteron–induced reactions

Grégory Potel Aguilar (FRIB/NSCL)

Seattle, March 28th 2017
(d, p) reactions: a probe for neutron–nucleus interactions

- Deuteron beam
- Proton detected
- Elastic scattering
- Inelastic excitation
- Direct transfer
- Compound nucleus
- B = A + n
General framework for inclusive experiments

this is detected $\rightarrow$ A $\quad$ B $\quad$ summed over states of this

$$\frac{d\sigma(E)}{dE_A} \sim \sum_n |\langle \phi_A \phi^n_B | V | \psi \rangle|^2$$

$$= \sum_n \langle \psi | V^\dagger | \phi_A \phi^n_B \rangle \delta(E - E_A - E^n_B) \langle \phi^n_B \phi_A | V | \psi \rangle$$

$$= \lim_{\epsilon \to 0} \text{Im} \langle \psi | V^\dagger | \phi_A \rangle \sum_n \frac{|\phi^n_B \rangle \langle \phi^n_B |}{E - E_A - H_B + i\epsilon} \langle \phi_A | V | \psi \rangle$$

Main challenges

- describe propagator $G_B = \lim_{\epsilon \to 0} \sum_n \frac{|\phi^n_B \rangle \langle \phi^n_B |}{E - E_A - H_B + i\epsilon}$
- describe wave function $\Psi$
Some applications

1. \((d, p)\):
   - \(A = p\)
   - \(B = F + 1\)

2. \((p, d)\):
   - \(A = d\)
   - \(B = F - 1\)

3. \((p, pn)\)/inelastic:
   - \(A = p\)
   - \(B = F^*\)
Implemented so far: Inclusive \((d, p)\) reaction

let’s concentrate in the reaction \(A+d\rightarrow B(=A+n)+p\)

we are interested in the inclusive cross section, \(\textit{i.e.},\) we will sum over all final states \(\phi_B^c\).
Derivation of the differential cross section

the double differential cross section with respect to the proton energy and angle for the population of a specific final $\phi^c_B$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d} \rho(E_p) \left| \left\langle \chi_p \phi^c_B | V | \psi(+) \right\rangle \right|^2.$$ 

Sum over all channels, with the approximation $\psi(+) \approx \chi_d \phi_d \phi_A$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2\pi}{\hbar v_d} \rho(E_p) \times \sum_c \left\langle \chi_d \phi_d \phi_A | V | \chi_p \phi^c_B \right\rangle \delta(E - E_p - E^c_B) \left\langle \phi^c_B \chi_p | V | \phi_A \chi_d \phi_d \right\rangle$$

$\chi_d \rightarrow$ deuteron incoming wave, $\phi_d \rightarrow$ deuteron wavefunction,
$\chi_p \rightarrow$ proton outgoing wave $\phi_A \rightarrow$ target core ground state.
the imaginary part of the Green’s function $G$ is an operator representation of the $\delta$–function,

$$\pi \delta (E - E_p - E_B^c) = \lim_{\epsilon \to 0} \Im \sum_c \frac{\langle \phi_B^c \rangle \langle \phi_B^c \rangle}{E - E_p - H_B + i \epsilon} = \Im G$$

$$\frac{d^2 \sigma}{d \Omega_p dE_p} = -\frac{2}{\hbar v_d} \rho(E_p) \Im \langle \chi_d \phi_d \phi_A \rangle \langle V \chi_p \rangle \langle V \phi_A \rangle \langle \chi_d \phi_d \rangle$$

- We got rid of the (infinite) sum over final states,
- but $G$ is an extremely complex object!
- We still need to deal with that.
Optical reduction of $G$

If the interaction $V$ do not act on $\phi_A$

\[
\langle \chi_d \phi_d \phi_A | V | \chi_p \rangle G \langle \chi_p | V | \phi_A \chi_d \phi_d \rangle
\]
\[
= \langle \chi_d \phi_d | V | \chi_p \rangle \langle \phi_A | G | \phi_A \rangle \langle \chi_p | V | \chi_d \phi_d \rangle
\]
\[
= \langle \chi_d \phi_d | V | \chi_p \rangle G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle,
\]

where $G_{opt}$ is the optical reduction of $G$

\[
G_{opt} = \lim_{\epsilon \to 0} \frac{1}{E - E_p - T_n - U_{An}(r_{An}) + i\epsilon},
\]

now $U_{An}(r_{An}) = V_{An}(r_{An}) + iW_{An}(r_{An})$ and thus $G_{opt}$ are single–particle, tractable operators.

The effective neutron–target interaction $U_{An}(r_{An})$, a.k.a. optical potential, a.k.a. self–energy can be provided by structure calculations.
Capture and elastic breakup cross sections

the imaginary part of $G_{opt}$ splits in two terms

$$\Im G_{opt} = -\pi \sum_{k_n} |\chi_n\rangle \delta \left( E - E_p - \frac{k_n^2}{2m_n} \right) \langle \chi_n | + G_{opt}^\dagger W_{An} G_{opt},$$

we define the neutron wavefunction $|\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle$

cross sections for non elastic breakup (NEB) and elastic breakup (EB)

$$\left[ \frac{d^2 \sigma}{d\Omega_p dE_p} \right]^{NEB} = -\frac{2}{\hbar \nu_d} \rho(E_p) \langle \psi_n | W_{An} | \psi_n \rangle,$$

$$\left[ \frac{d^2 \sigma}{d\Omega_p dE_p} \right]^{EB} = -\frac{2}{\hbar \nu_d} \rho(E_p) \rho(E_n) |\langle \chi_n \chi_p | V | \chi_d \phi_d \rangle|^2,$$
neutron transfer limit (isolated–resonance, first–order approximation)

Let’s consider the limit $W_{An} \to 0$ (single–particle width $\Gamma \to 0$). For an energy $E$ such that $|E - E_n| \ll D$, (isolated resonance)

$$G_{opt} \approx \lim_{W_{An} \to 0} \frac{|\phi_n\rangle \langle \phi_n|}{E - E_p - E_n - i\langle \phi_n|W_{An}|\phi_n\rangle};$$

with $|\phi_n\rangle$ eigenstate of $H_{An} = T_n + \Re(U_{An})$

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim \lim_{W_{An} \to 0} \langle \chi_d \phi_d | V | \chi_p \rangle \times \frac{|\phi_n\rangle \langle \phi_n|W_{An}|\phi_n\rangle \langle \phi_n|}{(E - E_p - E_n)^2 + \langle \phi_n|W_{An}|\phi_n\rangle^2} \langle \chi_p | V | \chi_d \phi_d \rangle,$$

we get the direct transfer cross section:

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim |\langle \chi_p \phi_n | V | \chi_d \phi_d \rangle|^2 \delta(E - E_p - E_n)$$
Validity of first order approximation

For $W_{An}$ small, we can apply first order perturbation theory,

$$\frac{d^2\sigma}{d\Omega_p dE_p} (E, \Omega) \left[N_{EB}\right] \approx \frac{1}{\pi} \frac{\langle \phi_n | W_{An} | \phi_n \rangle}{(E_n - E)^2 + \langle \phi_n | W_{An} | \phi_n \rangle^2} \frac{d\sigma_n}{d\Omega} (\Omega)$$

we compare the complete calculation with the isolated–resonance, first–order approximation for $W_{An} = 0.5$ MeV, $W_{An} = 3$ MeV and $W_{An} = 10$ MeV
Spectral function and absorption cross section

\[ \frac{d\sigma}{dE} \text{(mb/MeV)} \]

\[ W_{An} = 0.5 \text{ MeV} \]

\[ W_{An} = 3 \text{ MeV} \]

\[ L = 2 \]

\[ L = 4 \]

\[ L = 5 \]

\[ L = 0 \]
Choosing the potential/propagator

Konig–Delaroche (KD)
- Phenomenological fit of elastic scattering. Local.
- Available for a wide range of stable nuclei and energies.
- Not defined for negative energies. Good reproduction of scattering observables.

Dispersive optical model (DOM)
- Elastic scattering fit $\rightarrow$ dispersion $\rightarrow$ negative energies. Local and non–local versions.

Coupled cluster (CC)
- Ab–initio calculation $\rightarrow$ predictive power. Non–local.
- Accuracy of reproduction of observables ?.
Neutron states in nuclei

![Diagram showing neutron states in nuclei]

- Imaginary part of optical potential
- Neutron states

Mahaux, Bortignon, Broglia and Dasso Phys. Rep. 120 (1985) 1
Desired reaction: neutron induced fission, gamma emission and neutron emission.

The surrogate method consists in producing the same compound nucleus $B^*$ by bombarding a deuteron target with a radioactive beam of the nuclear species $A$.

A theoretical reaction formalism that describes the production of all open channels $B^*$ is needed.
We obtain spin–parity distributions for the compound nucleus. Contributions from elastic and non elastic breakup disentangled.
Using the DOM: Calcium isotopes

\[ ^{40}\text{Ca}(d,p) \]
\[ E_d=20 \text{ MeV} \]

\[ ^{48}\text{Ca}(d,p) \]
\[ E_d=20 \text{ MeV} \]

\[ ^{60}\text{Ca}(d,p) \]
\[ E_d=20 \text{ MeV} \]

\[ ^{40}\text{Ca}(d,p) \]
\[ E_d=40 \text{ MeV} \]

\[ ^{48}\text{Ca}(d,p) \]
\[ E_d=40 \text{ MeV} \]

\[ ^{60}\text{Ca}(d,p) \]
\[ E_d=40 \text{ MeV} \]

\[ \text{d}\sigma/dE \text{ (mb/MeV)} \]

\[ \text{E}_n \text{ (MeV)} \]
Using the DOM: angular momenta

\[ E_d = 40 \text{ MeV} \]

\[ l = 0, 1, 2, 3, 4 \]

\[ 40\text{Ca}(d,p) \]

\[ 48\text{Ca}(d,p) \]

\[ 60\text{Ca}(d,p) \]

\[ E_n \text{ (MeV)} \]

\[ d\sigma/dE \text{ (mb/MeV)} \]

\[ \theta_{lab} = 11^\circ \]

\[ \theta_{lab} = 39^\circ \]

\[ \theta_{lab} = 69^\circ \]

\[ \theta_{lab} = 89^\circ \]

\[ Uozimi et al., NPA 576 (1994) 123 \]
Using the DOM: comparing with data

\[ ^{40}\text{Ca}(d,p) \]

\[ ^{48}\text{Ca}(d,p), E_d=56 \text{ MeV} \]
\[ g_{9/2} \] resonance

\[ ^{60}\text{Ca}(d,p), E_d=40 \text{ MeV} \]
\[ g_{9/2} \] ground state resonance

48Ca(d,p), Ed=56 MeV

f7/2 (gs) data

Seattle, March 28th 2017 slide 7/9
first results using a propagator and self–energy generated within the coupled–cluster formalism (J. Rotureau, MSU and G. Hagen, ORNL)
Open questions (personal selection)

- **Separate** different channels in the NEB (possibly with M. Dupuis and G. Blanchon?).
- Better description of deuteron channel (adiabatic approximation? CDCC? Coupled cluster?)
- **Implement** \((p, pn)\). Could shed light on the problem of spectroscopic factors as a function of asymmetry. Two–particle Green’s function? Final neutron as a spectator?

**Gade et al. PRC 77, 044306 (2008)**

**Flavigny et al. PRL 110, 122503 (2013)**
Inclusive cross section

Single–particle Green’s function of $B$ system:

$$G_B = \lim_{\eta \to 0} (E - T - U_B + i\eta)^{-1}$$

Strength at given $E, r_n, r'_n, J, \pi \rightarrow \text{Im} G_B(E, r_n, r'_n, J, \pi)$

inclusive cross section $\rightarrow$ folding of strength with ”breakup probability density” $|\rho_{bu}(r_n, k_i, k_f)|^2$:

$$\rho_{bu}(r_n, k_i, k_f) = (\chi_p(r_p, k_f)| V |\chi_d(r_d, k_i)\phi_d(r_{pn})\rangle$$

$$\frac{d\sigma}{dEd\Omega} \sim \int \rho^*_{bu}(r_n, k_i, k_f)\text{Im}G_B(E, r_n, r'_n, J, \Pi)\rho_{bu}(r'_n, k_i, k_f)dr_n dr'_n$$

$$\text{Im}G_B = \text{elastic breakup spectrum} + G_B^\dagger \text{Im}U_B G_B$$

How to write $U_B = U_{\text{compound}} + U_{\text{rest}}$?
Inclusive deuteron–induced reactions

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Filomena Nunes (NSCL)
Ian Thompson (LLNL)

East Lansing, July 19th 2016
Inclusive \((d, p)\) reaction

let’s concentrate in the reaction \(A+d \rightarrow B(=A+n)+p\)

we are interested in the inclusive cross section, \(i.e.,\) we will sum over all final states \(\phi^c_B\).
the neutron wavefunctions

\[ |\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle \]

can be computed for **ANY** neutron energy, positive or negative

\[ |\psi_n\rangle \] are the solutions of an inhomogeneous Schrödinger equation

\[ (H_{An} - E_{An}) |\psi_n\rangle = \langle \chi_p | V | \chi_d \phi_d \rangle \]
Breakup above neutron–emission threshold

proton angular differential cross section

$^9\text{Nb} \,(d,p), \, E_d=15 \, \text{MeV}$

$E_p=9 \, \text{MeV} \quad E_n=3.8 \, \text{MeV}$

East Lansing, July 19th 2016
We can also transfer charged clusters
The interaction $V$ can be taken either in the \textit{prior} or the \textit{post} representation,

- Austern (post) $\rightarrow V \equiv V_{\text{post}} \sim V_{pn}(r_{pn})$ (recently revived by Moro and Lei, from Sevilla and Carlson from São Paulo)
- Udagawa (prior) $\rightarrow V \equiv V_{\text{prior}} \sim V_{An}(r_{An}, \xi_{An})$ (used in calculations showed here)

in the prior representation, $V$ can act on $\phi_A \rightarrow$ the optical reduction gives rise to new terms:

$$\frac{d^2\sigma}{d\Omega_p dE_p}^{\text{post}} = -\frac{2}{\hbar V_d} \rho(E_p) \left[ \Im \left\langle \psi_{n}^{\text{prior}} | W_{An} | \psi_{n}^{\text{prior}} \right\rangle + 2\Re \left\langle \psi_{n}^{\text{NON}} | W_{An} | \psi_{n}^{\text{prior}} \right\rangle + \left\langle \psi_{n}^{\text{NON}} | W_{An} | \psi_{n}^{\text{NON}} \right\rangle \right],$$

where $\psi_{n}^{\text{NON}} = \langle \chi_p | \chi_d \phi_d \rangle$.

The nature of the 2–step process \textbf{depends on the representation}
We have presented a reaction formalism for inclusive deuteron–induced reactions. Valid for final neutron states from Fermi energy → to scattering states. Disentangles elastic and non elastic breakup contributions to the proton singles. Probe of nuclear structure in the continuum. Provides spin–parity distributions. Useful for surrogate reactions. Need for optical potentials. Need to address non–locality. Can be generalized to other three–body problems. Can be extended for \((p, d)\) reactions (hole states).
From $H$ to $H_{3B}$

- $H = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + V_{An}(r_{An}, \xi_A) + V_{Ap}(r_{Ap}, \xi_A)$

- $H_{3B} = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + U_{An}(r_{An}) + U_{Ap}(r_{Ap})$
Observables: angular differential cross sections (neutron bound states)

- capture at resonant energies compared with direct transfer (FRESCO) calculations,
- capture cross sections rescaled by a factor \( \langle \phi_n | W_{An} | \phi_n \rangle \pi \).

\[
\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d} \rho(E_p) \sum_{l,m,l_p} \int \left| \varphi_{lml_p}(r_{Bn}; k_p) Y_{-m}^{l_p}(\theta_p) \right|^2 W(r_{An}) \, dr_{Bn}.
\]
Observables: elastic breakup and capture cross sections

elastic breakup and capture cross sections as a function of the proton energy. The Koning–Delaroche global optical potential has been used as the $U_{An}$ interaction (Koning and Delaroche, Nucl. Phys. A 713 (2003) 231).
Non-orthogonality term

$^{93}$Nb(d,p) $E_d=25.5$ MeV
$\theta_p=10^\circ$

$\frac{d\sigma}{dE}$ (mb/MeV)$\theta_{CM}$

$E_n=-3$ MeV
$E_n=5$ MeV

$\frac{d^2\sigma}{dEd\Omega}$ (mb/MeV sr)

- NEB with non orthogonality
- NEB without non orthogonality

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Obtaining spin distributions

\[
\frac{d\sigma}{dE_p} = \frac{2\pi}{\hbar \nu_d} \rho(E_p) \sum_{l_p, m} \int |\varphi_{lml_p}(r_{Bn}; k_p)|^2 W(r_{An}) \, dr_{Bn}.
\]
Getting rid of Weisskopf–Ewing approximation

Weisskopf–Ewing approximation:

\[ P(d, nx) = \sigma(E)G(E, x) \]

inaccurate for \( x = \gamma \) and for \( x = f \) in the low–energy regime

can be replaced by

\[ P(d, nx) = \sum_{J, \pi} \sigma(E, J, \pi)G(E, J, \pi, x) \]

if \( \sigma(E, J, \pi) \) can be predicted.

Younes and Britt, PRC 68(2003)034610
We present a formalism for inclusive deuteron–induced reactions. We thus want to describe within the same framework:

- Direct neutron transfer: should be compatible with existing theories.
- Elastic deuteron breakup: “transfer” to continuum states.
- Non elastic breakup (direct transfer, inelastic excitation and compound nucleus formation): absorption above and below neutron emission threshold.
- Important application in surrogate reactions: obtain spin–parity distributions, get rid of Weisskopf–Ewing approximation.
Historical background

breakup-fusion reactions

Kerman and McVoy, Ann. Phys. \textbf{122} (1979) 197


Controversy between Udagawa and Austern formalism left somehow unresolved.

Britt and Quinton, Phys. Rev. \textbf{124} (1961) 877

protons and α yields

bombarding $^{209}$Bi with $^{12}$C and $^{16}$O

East Lansing, July 19th 2016 slide 24/27
2–step process (post representation)

**Step 1**: Breakup

- Non-elastic breakup
- Elastic breakup

**Step 2**: Propagation of n in the field of A

- to detector
- Non-elastic breakup
- Elastic breakup
Weisskopf–Ewing approximation

\[ \sigma_{a \chi}(E_a) = \sum_{J, \pi} \sigma_{a}^{CN}(E_{ex}, J, \pi) G_{\chi}^{CN}(E_{ex}, J, \pi) \]

W-E approximation

\[ \sigma_{a \chi}^{WE}(E_a) = \sigma_{a}^{CN}(E_{ex}) G_{\chi}^{CN}(E_{ex}) \]

Weisskopf-Ewing approximation: probability of $\gamma$ decay independent of $J, \pi$

Escher and Dietrich, PRC 81 024612 (2010)

Different $J, \pi$ Different cross section for $\gamma$ emission

Weisskopf–Ewing is inaccurate for $(n, \gamma)$
Weisskopf–Ewing approximation

\[ \sigma_{\alpha \chi}(E_a) = \sum_{J, \pi} \sigma_{\alpha}^{CN}(E_{ex}, J, \pi) G_{\chi}^{CN}(E_{ex}, J, \pi) \]

Weisskopf-Ewing approximation: probability of \( \gamma \) decay independent of \( J, \pi \)

W-E approximation

\[ \sigma_{\alpha \chi}^{WE}(E_a) = \sigma_{\alpha}^{CN}(E_{ex}) G_{\chi}^{CN}(E_{ex}) \]

Escher and Dietrich, PRC 81 024612 (2010)

Different \( J, \pi \)  Different cross section for \( \gamma \) emission

We need theory to predict \( J, \pi \) distributions
Inclusive three–body cross sections

January 31, 2017
General description of experiments in which we measure the energy of final product $A$ and do not measure the energy of final product $B$:

\[
\frac{d\sigma}{dE_A}(E) \sim \sum_n \left| \langle \phi_A \phi_B^n | V | \psi \rangle \right|^2
\]

\[
= \sum_n \langle \psi | V^\dagger | \phi_A \phi_B^n \rangle \delta(E - E_A - E_B^n) \langle \phi_B^n \phi_A | V | \psi \rangle
\]

\[
= \mathcal{S} \lim_{\epsilon \to 0} \langle \psi | V^\dagger | \phi_A \rangle \sum_n \frac{|\phi_B^n \rangle \langle \phi_B^n|}{E - E_A - H_B} \langle \phi_A | V | \psi \rangle
\]

Main challenge: get the $B$–system propagator $G_B = \sum_n \frac{|\phi_B^n \rangle \langle \phi_B^n|}{E - E_A - H_B}$
ICNT Workshop "Deuteron–induced reactions and beyond: Inclusive breakup fragment cross sections" (July 2016)

structure
W. Dickhoff (St. Louis)
J. Rotureau (MSU)
J. Escher (LLNL)

reactions
B. Carlson (Sao Paulo)
A. Moro (Sevilla)
F. Nunes (MSU)
M. Husein (Sao Paulo)
P. Capel (Bruxelles)
G. Potel (MSU)

experiment
G. Perdikakis (CMU, NSCL)
A. Macchiavelli (LBNL)
S. Pain (ORNL)
Recent results (I): Ca isotopes

- $^{40}\text{Ca}(d,p)$
  - $E_d=20\text{ MeV}$

- $^{48}\text{Ca}(d,p)$
  - $E_d=20\text{ MeV}$
  - $E_d=40\text{ MeV}$

- $^{60}\text{Ca}(d,p)$
  - $E_d=20\text{ MeV}$
  - $E_d=40\text{ MeV}$

$d\sigma/dE$ (mb/MeV) vs. $E_n$ (MeV)
Recent results (II): Ca isotopes

$^{40}\text{Ca}(d,p)$, $E_d=10$ MeV

$^{48}\text{Ca}(d,p)$, $E_d=56$ MeV $g_{9/2}$ resonance

$^{60}\text{Ca}(d,p)$, $E_d=40$ MeV $g_{9/2}$ ground state resonance
- Implement $(p, d)$ (with DOM optical potential from Wim Dickhoff and Mack Atkinson).
- Include non–locality in the propagator (with Weichuan).
- Use Coupled–Clusters propagator (with Jimmy).
the double differential cross section with respect to the proton energy and angle for the population of a specific final $\phi^c_B$

\[
\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d} \rho(E_p) \left| \langle \chi_p \phi^c_B | V | \psi(+) \rangle \right|^2.
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Sum over all channels, with the approximation $\psi(+) \approx \chi_d \phi_d \phi_A$

\[
\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2\pi}{\hbar v_d} \rho(E_p) \times \sum_c \langle \chi_d \phi_d \phi_A | V | \chi_p \phi^c_B \rangle \delta(E - E_p - E^c_B) \langle \phi^c_B \chi_p | V | \phi_A \chi_d \phi_d \rangle
\]

$\chi_d \to$ deuteron incoming wave, $\phi_d \to$ deuteron wavefunction, $\chi_p \to$ proton outgoing wave $\phi_A \to$ target core ground state.
the imaginary part of the Green’s function $G$ is an operator representation of the $\delta$–function,

$$\pi \delta(E - E_p - E^c_B) = \lim_{\epsilon \to 0} \Im \sum_c \frac{\langle \phi^c_B | \phi^c_B \rangle}{E - E_p - H_B + i\epsilon} = \Im G$$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2}{\hbar v_d} \rho(E_p) \Im \langle \chi_d \phi_d \phi_A | V | \chi_p \rangle G \langle \chi_p | V | \phi_A \chi_d \phi_d \rangle$$

- We got rid of the (infinite) sum over final states,
- but $G$ is an extremely complex object!
- We still need to deal with that.
Optical reduction of $G$

If the interaction $V$ do not act on $\phi_A$

$$\langle \chi_d \phi_d \phi_A | V | \chi_p \rangle G \langle \chi_p | V | \phi_A \chi_d \phi_d \rangle$$

$$= \langle \chi_d \phi_d | V | \chi_p \rangle \langle \phi_A | G | \phi_A \rangle \langle \chi_p | V | \chi_d \phi_d \rangle$$

$$= \langle \chi_d \phi_d | V | \chi_p \rangle G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle,$$

where $G_{opt}$ is the optical reduction of $G$

$$G_{opt} = \lim_{\epsilon \to 0} \frac{1}{E - E_p - T_n - U_{An}(r_{An}) + i\epsilon},$$

now $U_{An}(r_{An}) = V_{An}(r_{An}) + iW_{An}(r_{An})$ and thus $G_{opt}$ are single–particle, tractable operators.

The effective neutron–target interaction $U_{An}(r_{An})$, a.k.a. optical potential, a.k.a. self–energy can be provided by structure calculations.
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the imaginary part of $G_{opt}$ splits in two terms

$$\Im G_{opt} = -\pi \sum_{k_n} |\chi_n\rangle \delta \left( E - E_p - \frac{k_n^2}{2m_n} \right) \langle \chi_n | + G_{opt} \dagger W_{An} G_{opt} ,$$

we define the neutron wavefunction $|\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle$

cross sections for non elastic breakup (NEB) and elastic breakup (EB)

$$\frac{d^2\sigma}{d\Omega_p dE_p} \bigg|_{NEB} = -\frac{2}{\hbar \nu_d} \rho(E_p) \langle \psi_n | W_{An} | \psi_n \rangle ,$$

$$\frac{d^2\sigma}{d\Omega_p dE_p} \bigg|_{EB} = -\frac{2}{\hbar \nu_d} \rho(E_p) \rho(E_n) |\langle \chi_n \chi_p | V | \chi_d \phi_d \rangle|^2 ,$$
2–step process (post representation)

**Step 1:**
- Breakup
  - $\langle \chi_p | V | \phi_A \chi d \phi_d \rangle$

**Step 2:**
- Propagation of $n$ in the field of $A$
  - $B^*$
  - $G$
  - $n$
  - $p$
  - Elastic breakup
  - Non-elastic breakup
  - To detector
Austern (post)–Udagawa (prior) controversy

The interaction $V$ can be taken either in the *prior* or the *post* representation,

- Austern (post) $\rightarrow V \equiv V_{post} \sim V_{pn}(r_{pn})$ (recently revived by Moro and Lei, University of Sevilla)
- Udagawa (prior) $\rightarrow V \equiv V_{prior} \sim V_{An}(r_{An}, \xi_{An})$

in the prior representation, $V$ can act on $\phi_A \rightarrow$ the optical reduction gives rise to new terms:

$$\frac{d^2 \sigma}{d\Omega_p dE_p} \bigg|_{post} = -\frac{2}{\hbar \nu_d} \rho(E_p) \left[ \Im \left\langle \psi_{prior}^n | W_{An} | \psi_{prior}^n \right\rangle + 2 \Re \left\langle \psi_{NON}^n | W_{An} | \psi_{prior}^n \right\rangle + \left\langle \psi_{NON}^n | W_{An} | \psi_{NON}^n \right\rangle \right],$$

where $\psi_{NON}^n = \langle \chi_p | \chi_d \phi_d \rangle$.

The nature of the 2–step process depends on the representation.
the neutron wavefunctions

\[ |\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle \]

can be computed for any neutron energy

these wavefunctions are not eigenfunctions of the Hamiltonian

\[ H_{An} = T_n + \Re(U_{An}) \]
Breakup above neutron–emission threshold

proton angular differential cross section

\[ \frac{d\sigma}{d\Omega}(\text{mb/sr, MeV}) \]

\[ \text{Ep}=9, \text{MeV} \]
\[ \text{En}=3.8, \text{MeV} \]

\[ ^{93}\text{Nb} \ (\text{d,p}), \ E_d=15, \text{MeV} \]

\[ \theta \]

\[ \text{total} \]

L=1
L=2
L=0
L=4
L=5
neutron transfer limit (isolated–resonance, first–order approximation)

Let’s consider the limit $W_{An} \rightarrow 0$ (single–particle width $\Gamma \rightarrow 0$). For an energy $E$ such that $|E - E_n| \ll D$, (isolated resonance)

$$G_{opt} \approx \lim_{W_{An} \rightarrow 0} \frac{\langle \phi_n | \phi_n \rangle}{E - E_p - E_n - i \langle \phi_n | W_{An} | \phi_n \rangle};$$

with $|\phi_n\rangle$ eigenstate of $H_{An} = T_n + \Re(U_{An})$

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim \lim_{W_{An} \rightarrow 0} \langle \chi_d \phi_d | V | \chi_p \rangle \times \frac{\langle \phi_n | W_{An} | \phi_n \rangle \langle \phi_n | \rangle}{(E - E_p - E_n)^2 + \langle \phi_n | W_{An} | \phi_n \rangle^2} \langle \chi_p | V | \chi_d \phi_d \rangle,$$

we get the direct transfer cross section:

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim |\langle \chi_p \phi_n | V | \chi_d \phi_d \rangle|^2 \delta(E - E_p - E_n)$$
Validity of first order approximation

For $W_{An}$ small, we can apply first order perturbation theory,

$$
\frac{d^2\sigma}{d\Omega_p dE_p}(E, \Omega) \bigg|_{N_{\text{EB}}} \approx \frac{1}{\pi} \frac{\langle \phi_n | W_{An} | \phi_n \rangle}{(E_n - E)^2 + \langle \phi_n | W_{An} | \phi_n \rangle^2} \frac{d\sigma_n}{d\Omega}(\Omega)
$$

we compare the complete calculation with the isolated–resonance, first–order approximation for $W_{An} = 0.5$ MeV, $W_{An} = 3$ MeV and $W_{An} = 10$ MeV.
Desired reaction: neutron induced fission, gamma emission and neutron emission.

The surrogate method consists in producing the same compound nucleus $B^*$ by bombarding a deuteron target with a radioactive beam of the nuclear species $A$.

A theoretical reaction formalism that describes the production of all open channels $B^*$ is needed.
Weisskopf–Ewing approximation

\[ \sigma_{a\alpha}(E_\alpha) = \sum_{J,\pi} \sigma_{a}^{CN}(E_{cx}, J, \pi) G_{X}^{CN}(E_{cx}, J, \pi) \]

\[ \sigma_{a\alpha}^{WE}(E_\alpha) = \sigma_{a}^{CN}(E_{cx}) G_{X}^{CN}(E_{cx}) \]

Weisskopf-Ewing approximation: probability of \( \gamma \) decay independent of \( J, \pi \)

Different \( J, \pi \) Different cross section for \( \gamma \) emission

Weisskopf–Ewing is inaccurate for \( (n, \gamma) \)

Escher and Dietrich, PRC 81 024612 (2010)
Weisskopf–Ewing approximation

Weisskopf–Ewing approximation: probability of $\gamma$ decay independent of $J, \pi$

Escher and Dietrich, PRC 81 024612 (2010)

Different $J, \pi$  Different cross section for $\gamma$ emission

We need theory to predict $J, \pi$ distributions
We obtain spin–parity distributions for the compound nucleus.

Contributions from elastic and non elastic breakup disentangled.

\[ ^{93}\text{Nb}(d, p) \] (Mastroleo et al., Phys. Rev. C 42 (1990) 683) \[ ^{93}\text{Nb}(d,p) E_g=15 \text{ MeV} \]
Extending the formalism

We can also transfer charged clusters.
We have presented a reaction formalism for inclusive deuteron–induced reactions.

Valid for final neutron states from Fermi energy → to scattering states

Disentangles elastic and non elastic breakup contributions to the proton singles.

Probe of nuclear structure in the continuum.

Provides spin–parity distributions.

Useful for surrogate reactions.

Need for optical potentials.

Can easily be generalized to other three–body problems.

Can be extended for \((p, d)\) reactions (hole states).
The 3-body model

From $H$ to $H_{3B}$

- $H = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + V_{An}(r_{An}, \xi_A) + V_{Ap}(r_{Ap}, \xi_A)$
- $H_{3B} = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + U_{An}(r_{An}) + U_{Ap}(r_{Ap})$
Observables: angular differential cross sections (neutron bound states)

- capture at resonant energies compared with direct transfer (FRESCO) calculations,
- capture cross sections rescaled by a factor $\langle \phi_n | W_{An} | \phi_n \rangle \pi$.

Double proton differential cross section

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{h\nu_d} \rho(E_p) \sum_{l,m,l_p} \int \left| \varphi_{lml_p}(r_{Bn} ; k_p) Y_{l-m}^{l_p}(\theta_p) \right|^2 W(r_{An}) \, dr_{Bn}.$$
elastic breakup and capture cross sections as a function of the proton energy. The Koning–Delaroche global optical potential has been used as the $U_{An}$ interaction (Koning and Delaroche, Nucl. Phys. A 713 (2003) 231).
Sub–threshold capture

\[ \frac{d\sigma}{dE} \text{ (mb/MeV)} \]

\( W_{An} = 0.5 \text{ MeV} \)

\( W_{An} = 3 \text{ MeV} \)

\( L = 0 \)
\( L = 2 \)
\( L = 4 \)
\( L = 5 \)
Non-orthogonality term

\[ {^{93}\text{Nb}(d,p)} \]
\[ E_d = 25.5 \text{ MeV} \]
\[ \theta_p = 10^\circ \]

\[ \frac{d\sigma}{dE} \text{ (mb/MeV)} \]

\[ \frac{d\sigma}{dE} \text{ (mb/MeV sr)} \]

\[ E_n = -3 \text{ MeV} \]
\[ E_n = 5 \text{ MeV} \]
Obtaining spin distributions

\[ \frac{d\sigma}{dE} = \frac{2\pi}{\hbar \nu_d} \rho(E_p) \sum_{l_p, m}\int \left| \varphi_{lml_p}(r_{Bn}; k_p) \right|^2 W(r_{An}) \, dr_{Bn}. \]
Weisskopf–Ewing approximation:
\[ P(d, nx) = \sigma(E) G(E, x) \]

- inaccurate for \( x = \gamma \) and for \( x = f \) in the low–energy regime
- can be replaced by \( P(d, nx) = \sum_{J, \pi} \sigma(E, J, \pi) G(E, J, \pi, x) \) if \( \sigma(E, J, \pi) \) can be predicted.