THE TROUBLE WITH THE LATTICE AXIAL CHARGE
Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories. Physical observables are calculated from the path integral

\[
\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi \, e^{iS(\bar{\psi}, \psi, A)} O(\bar{\psi}, \psi, A)
\]

in **Euclidian** space.

- Quark mass parameter (described by \( m_\pi \))
- Impose a UV cutoff
discretize spacetime
- Impose an infrared cutoff
**finite volume**

Recover physical limit

\( m_\pi \rightarrow m_\pi^{\text{phys}}, \, a \rightarrow 0, \, L \rightarrow \infty \)

Huey-Wen Lin — INT, Seattle
Wide-Scale Applications

§ What can we learn from it?

Parton distribution functions
Properties for new-physics searches
H WL et al, 1402.1462, 1506.06411; 1506.04196 ...

Nuclei and why we exist
1409.3556, 1206.5219, 1109.2889, 1012.3812 ...

Neutron matter
Neutron-star evolution
1204.3606

HWL et al, 1402.1462, 1506.06411; 1506.04196 ...

10^{-15} m

10^4 m
Lattice gauge theory was proposed in the 1970s by Wilson.

Why haven’t we solved QCD yet?

Progress is limited by computational resources.

Greatly assisted by advances in algorithms.

Physical pion-mass ensembles are not uncommon!
Lattice flavor physics provides precise inputs from the SM
A. El-Khadra, Sep. 2015, INT workshop “QCD for New Physics at the Precision Frontier”

Very precise results in many meson systems

errors (in %) (preliminary) FLAG-3 averages

We are beginning to do precision calculations in nucleons
Origin of Proton Spin

§ What is the makeup of the nucleon?

 Decomp. using Ji’s GPD moment connection

 Preliminary result from $\chi$QCD (2+1f ov/DWF 400 MeV)

 $J^q = \frac{1}{2} (A_{20}^q + B_{20}^q)$

 $\Delta \Sigma$: quark spin

 $L = J - \Delta \Sigma$: orbital angular momentum

 ETMC (2f TMF 130 MeV) $M_\pi L = 3$ Preliminary

 $\Delta \Sigma_{u+d+s} = 0.214(61), L_{u+d+s} = 0.168(60), J^g = 0.118(57)$

 M. Constantinou, Spin 16
Better determined strange form factors

- **LHPC (2+1f):** clover $M_\pi = 317$ MeV, $a = 0.11$ fm
- **χQCD (2+1f):** ov/DWF $M_\pi = 207,140$ MeV, $a = 0.11$ fm
§ First time in LQCD history to study antiquark distribution!

\[ M_\pi \approx 310 \text{ MeV} \]

HWL et al. 1402.1462

\[ \bar{q}(x) = -q(-x) \]

Lost resolution in small-\(x\) region

Future improvement: larger lattice volume

\[ \int dx \left( \bar{u}(x) - \bar{d}(x) \right) \approx -0.16(7) \]

<table>
<thead>
<tr>
<th>Experiment</th>
<th>( x ) range</th>
<th>( \int_{x}^{1} [\bar{d}(x) - \bar{u}(x)] dx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E866</td>
<td>0.015 &lt; (x) &lt; 0.35</td>
<td>0.118 ± 0.012</td>
</tr>
<tr>
<td>NMC</td>
<td>0.004 &lt; (x) &lt; 0.80</td>
<td>0.148 ± 0.039</td>
</tr>
<tr>
<td>HERMES</td>
<td>0.020 &lt; (x) &lt; 0.30</td>
<td>0.16 ± 0.03</td>
</tr>
</tbody>
</table>

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
Nucleons and BSM

Many opportunities to probe BSM with nucleon inputs

§ Parton distribution functions for SM background 1402.1462
  ✔ Especially less known intrinsic strange/charm contribution

§ Dark matter detection 1306.6939
  ✔ Popular candidates (e.g. SuSy neutralinos) exchange Higgs

§ Electric dipole moment 1506.04196
  ✔ CP-violating effect, extremely small: in SM ≈ 10^{-30} e\cdot cm

§ Neutron beta decay 1110.6448; 1506.06411
  ✔ Non-$V-A$ interactions to probe the existence of new particles
    (mediating new forces) with masses in the multi-TeV range

§ Nucleon (transition) axial form factor 0803.3020, 1003.3387
  ✔ First-principles inputs into Monte Carlo event generators for
    precision neutrino physics

Many of these are supported by P5 recommendations
Many opportunities to probe BSM with nucleon inputs

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Many of these are supported by P5 recommendations
Nucleon Axial Charge

§ Summary

$g_A$

\begin{align*}
N_f = 2+1 & \quad N_f = 2+1+1 \\
\text{Other} & \\
\text{Neutron Expts} & \\
\end{align*}

\begin{align*}
1.195(33)(20) & \quad \text{PNDME '16} \\
& \quad \text{LHPC '12} \\
& \quad \text{LHPC '10} \\
& \quad \text{RBC/UKQCD '08} \\
& \quad \text{Lin/Orginos '07} \\
& \quad \text{RQCD '14} \\
& \quad \text{QCDSF/UKQCD '13} \\
& \quad \text{ETMC '15} \\
& \quad \text{CLS '12} \\
& \quad \text{RBC '08} \\
1609.01350 & \quad \text{Adler-Weisberger SR '16} \\
& \quad \text{Mund '13} \\
& \quad \text{Mendenhall '12} \\
& \quad \text{Liu '10} \\
& \quad \text{Abele '02} \\
& \quad \text{Mostovoi '01} \\
& \quad \text{Liaud '97} \\
& \quad \text{Yerozolimsky '97} \\
& \quad \text{Bopp '86} \\
\end{align*}
Nucleon Axial Charge

§ Summary

$g_A$

N$_f=2+1$

1.195(33)(20) PNDME ’16

LHPC ’12
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RQCD ’14
QCDSF/UKQCD ’13
ETMC ’15
CLS ’12
RBC ’08

1609.01350 Adler-Weisberger SR ’16

N$_f=2$

Lattice 2016 Prelim.

RBC* 2+1f 1.15(4)
PACS* 2+1f 1.18(4)

§ Implications?

$p \approx 2\sigma$ might go away with greater statistics

§ What’s going on?

Mund ’13
Mendenhall ’12
Liu ’10
Abele ’02
Mostovoi ’01
Liaud ’97
Yerozolimsky ’97
Bopp ’86

Huey-Wen Lin — INT, Seattle
§ What do we really know about axial charge?
  ☞ Revisit the experiment

§ Does LQCD calculation control ALL systematics?
  ☞ Issues and problems
  ☞ The tale of a 6-year quest

§ Conclusions(?)
Nucleon Axial Charge

§ A fundamental measure of nucleon structure

§ Axial-vector–current matrix element

\[ g_A = G_A^{u-d}(Q^2=0) \]

§ Important to many nuclear processes

☞ The rate of \( pp \) fusion (as in Sun-like stars)

☞ \( 0νββ \) searches, “quenching” \( g_A^4 \)

☞ \( V_{ud} \) values through \( n \)-lifetime measurements

☞ New-physics searches such as right-handed neutrinos

§ In lattice QCD, it was long called

“A benchmark for nucleon structure”
### Nucleon Axial Charge

A fundamental measure of nucleon structure

- Axial-vector current matrix element

\[ g_A = G_A u - d \quad (Q^2 = 0) \]

Important to many nuclear processes:
- The rate of pp fusion (as in Sun-like stars)
- \( V_{ud} \) values through \( n \)-lifetime measurements
- New-physics searches such as right-handed neutrinos
- \( 0\nu\beta\beta \) searches, “quenching” \( g_A^4 \)

In lattice QCD, it was long called “A benchmark for nucleon structure”

Huey-Wen Lin — INT, Seattle

---

### Processes governed by lifetime and \( g_A/g_V \)

<table>
<thead>
<tr>
<th>Process</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primordial element formation</td>
<td>( n + e^+ \rightarrow p + \nu_e ) \quad ( \sigma_v \sim 1/\tau )</td>
</tr>
<tr>
<td>( ^2\text{H}, ^3\text{He}, ^4\text{He}, ^7\text{Li}, \ldots )</td>
<td>( p + e^- \rightarrow n + \nu_e ) \quad ( \sigma_v \sim 1/\tau )</td>
</tr>
<tr>
<td>Neutron star formation</td>
<td>( n \rightarrow p + e^- + \nu_e ) \quad ( \tau )</td>
</tr>
<tr>
<td>Solar cycle</td>
<td>( p + p \rightarrow ^2\text{H} + e^+ + \nu_e )</td>
</tr>
<tr>
<td></td>
<td>( p + p + e^- \rightarrow ^2\text{H} + \nu_e ) etc. \quad \sim (g_A/g_V)^5</td>
</tr>
<tr>
<td>Neutron star formation</td>
<td>( p + e^- \rightarrow n + \nu_e )</td>
</tr>
<tr>
<td>Pion decay</td>
<td>( \pi^- \rightarrow \pi^0 + e^- + \nu_e' )</td>
</tr>
<tr>
<td>Neutrino detectors</td>
<td>( \nu_e' + p \rightarrow e^+ + n )</td>
</tr>
<tr>
<td>Neutrino forward scattering</td>
<td>( \nu_e + n \rightarrow e^- + p ) etc.</td>
</tr>
<tr>
<td>W and Z production</td>
<td>( u' + d \rightarrow W^- \rightarrow e^- + \nu_e' ) etc.</td>
</tr>
</tbody>
</table>

from D. Dubbers
Ask somebody what they know about the axial charge...

The PDG number has errorbars so tiny, we just drop the error!
Ask somebody what they know about the axial charge...

The PDG number has error bars so tiny, we just drop the error!

If you look closer, it's changed over the years.
§ Ask somebody what they know about the axial charge...
☞ The PDG number has errorbars so tiny, we just drop the error!
§ If you look closer, it’s changed over the years
Let us look closely at how $g_A$ is determined experimentally.

Two main types of experimental input:

- Asymmetry in neutron differential decay rate (by UCN)

$$d\Gamma \propto F(E_e) \left( 1 + a \frac{p_e \cdot p_{\nu}}{E_e E_{\nu}} + A \frac{\sigma_n \cdot p_e}{E_e} + \cdots \right)$$

$$A_0 = \frac{-2(\lambda^2 - |\lambda|)}{1+3\lambda^2}$$

$$\lambda = \frac{G_A}{G_V} = 1.2755(30)$$

UCNA 13
§ Let us look closely at how $g_A$ is determined experimentally

§ Two main types of experimental input

❓ Asymmetry in neutron differential decay rate (by UCN)
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\]
\[
A_0 = \frac{-2(\lambda^2 - |\lambda|)}{1 + 3\lambda^2}
\]
\[
\lambda = G_A / G_V = 1.2755(30) \quad \text{UCNA 13}
\]

❓ $n$-lifetime decay (requires additional input $V_{ud}$)
\[
\tau_n^{\text{ave}} = 880.2(1.0) \text{ sec} \quad |V_{ud}|^2 = \frac{4908.7(1.9) \text{ sec}}{\tau_n(1 + 3 g_A^2)}
\]

❓ $V_{ud}$ from...

❓ nuclear $0^+ \to 0^+$ superallowed: $0.97417(21) \Rightarrow g_A = 1.2749(10)$

❓ BR($0^+ \to e^+ \nu_e (\gamma)$): $0.9728(30) \Rightarrow g_A = 1.2771(44)$
Let us look closely at how $g_A$ is determined experimentally.

Two main types:

1. Asymmetry in neutron differential decay rate (by UCN)

$$\frac{d\Gamma}{dE} \propto F(E_e) \left(1 + \lambda \right)$$

2. $n$-lifetime decay (requires additional input)

$$\tau_n^{\text{ave}} = 880.2_{-1.0}^{+1.2} \text{sec}$$

$V_{ud}$ from...

- Nuclear $0^+ \rightarrow 0^+$ superallowed

$$\text{BR}(0^+ \rightarrow e^+\nu_e\gamma) = 0.19 \pm 0.20(\text{th})$$

$$g_A = 1.27 \pm 0.02(\text{th})$$
Weak Experiments

§ Let us look closely at how $g_A$ is determined experimentally

§ Two main types

☞ Asymmetry in neutron differential decay rate (by UCN)

$$d\Gamma \propto F(E_e) \left( 1 + \lambda \right)$$

☞ $n$-lifetime decay

$$\tau_{n}^{\text{ave}} = 880.1 \pm 0.0 \text{ sec}$$

☞ $V_{ud}$ from...

☞ nuclear $0^+ \rightarrow 0^+$

☞ $\text{BR}(0^+ \rightarrow e^+\nu\gamma)$

Correction at $10^{-4}$

Nuclear correction not fully resolved: errorbar bigger

Courtesy Geoff Greene

A. Yue, et al. PRL 111, 222501 (2013)
Let us look closely at how $g_A$ is determined experimentally.

Two main types of experimental input:

- Asymmetry in neutron differential decay rate (by UCN)
  \[ d\Gamma \propto F(E_e) \left(1 + \frac{\lambda}{3}\right) \]

- $n$-lifetime decay
  \[ \tau_{\text{ave}} = 880.21 \text{ sec} \]

$V_{ud}$ from...

- Nuclear $0^+$ transition
  \[ \text{BR}(0^+ \rightarrow e^+ \nu) \]

- $g_A \approx 1.258$

\[ \tau_n \approx 900 \text{ sec} \]

\[ V_{ud} = 0.974 \]

\[ g_A \approx 1.258 \]
§ What can we infer about $g_A$ from other observables?
§ Constraints from $V_{ud}$ experiments (must be $\leq 1$)
☞ The allowed region is $g_A \geq 1.23524(98)$
How about QCD experiments?

- With a polarized target or polarized beam, one can find the helicity distribution and get $g_A$
- Global analysis? $g_A$ is used as a constraint

LQCD currently is the only reliable QCD source for $g_A$

Does LQCD $g_A$ agree with QCD experiments?

First workshop with global-fit community to address LQCD

http://www.physics.ox.ac.uk/confs/PDFlattice2017
Lattice Aspects
The Trouble with Nucleons

Nucleons are more complicated than mesons because...

§ Noise issue
- Signal diminishes at large $t_E$ relative to noise
- Gets worse when quark mass decreases

§ Excited-state contamination
- Nearby excited state: Roper(1440)

§ Hard to extrapolate in pion mass
- $\Delta$ resonance nearby; multiple expansions, poor convergence...
- Less an issue in the physical pion-mass era

§ Requires larger volume and higher statistics
- Ensembles are not always generated with nucleons in mind
- **High-statistics**: large measurement and long trajectory
The Trouble with Nucleons

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The Trouble with Nucleons

“Welcome to the lattice and its dangerous animals.”

Karl Jansen
“Welcome to the lattice and its dangerous animals.”

Re-examine all systematics!

PROCEED WITH CAUTION

The Trouble with Nucleons

Huey-Wen Lin — INT, Seattle
Nucleon Matrix Elements

Lattice-QCD calculation of $\langle p|\bar{u}\Gamma d|n\rangle$

§ Control all systematic errors:

☞ Finite-volume effects

☞ Chiral extrapolations to physical $u$ and $d$ quark masses

☞ Extrapolation to the continuum limit (lattice spacing $a \to 0$)

☞ Nonperturbative renormalization using the RI/SMOM scheme

☞ Contamination from excited states

☞ Statistical effects
Precision Neutron-Decay Matrix Elements

https://sites.google.com/site/pndmelqcd/

Tanmoy Bhattacharya  Rajan Gupta  HWL  Vincenzo Cirigliano

Saul Cohen  Anosh Joseph  Yong-Chull Jang  Boram Yoon
§ Much effort has been devoted to controlling systematics
§ A state-of-the art calculation (PNDME)

<table>
<thead>
<tr>
<th>$a$ (fm)</th>
<th>$V$</th>
<th>$M_\pi L$</th>
<th>$M_\pi$ (MeV)</th>
<th>$t_{\text{sep}}$</th>
<th># Meas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>$24^3 \times 64$</td>
<td>4.55</td>
<td>310</td>
<td>8,10,12</td>
<td>64.8k</td>
</tr>
<tr>
<td>0.12</td>
<td>$24^3 \times 64$</td>
<td>3.29</td>
<td>220</td>
<td>8,10,12</td>
<td>24k</td>
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<td>0.12</td>
<td>$32^3 \times 64$</td>
<td>4.38</td>
<td>220</td>
<td>8,10,12</td>
<td>7.6k</td>
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<tr>
<td>0.12</td>
<td>$40^3 \times 64$</td>
<td>5.49</td>
<td>220</td>
<td>8,10,12,14</td>
<td>64.6k</td>
</tr>
<tr>
<td>0.09</td>
<td>$32^3 \times 96$</td>
<td>4.51</td>
<td>310</td>
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<td>7.0k</td>
</tr>
<tr>
<td>0.09</td>
<td>$48^3 \times 96$</td>
<td>4.79</td>
<td>220</td>
<td>10,12,14</td>
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<td>$64^3 \times 96$</td>
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<td>130</td>
<td>10,12,14</td>
<td>56.5k</td>
</tr>
<tr>
<td>0.06</td>
<td>$48^3 \times 144$</td>
<td>4.52</td>
<td>310</td>
<td>16,20,22,24</td>
<td>64.0k</td>
</tr>
<tr>
<td>0.06</td>
<td>$64^3 \times 144$</td>
<td>4.41</td>
<td>220</td>
<td>16,20,22,24</td>
<td>41.6k</td>
</tr>
<tr>
<td>0.06</td>
<td>$96^3 \times 192$</td>
<td>3.80</td>
<td>130</td>
<td></td>
<td>On-going</td>
</tr>
</tbody>
</table>
Excited-State Contamination

§ Trade off: signal-to-noise versus contamination

☞ Noise issue (P. Lepage; D. Kaplan)
☞ Consider a baryon correlator \( C = \langle 0 \rangle = \langle q\bar{q}(t)\bar{q}q(0) \rangle \)
☞ Variance (noise squared) of \( C \propto \langle O\dagger O \rangle - \langle O^2 \rangle \)

What you want:

\[
\begin{align*}
N & \quad \Rightarrow \quad N
\end{align*}
\]

Signal falls exponentially as \( e^{-m_N t} \)
Excited-State Contamination

§ Trade off: signal-to-noise versus contamination

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<table>
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<tr>
<th>What you want:</th>
<th>What you get:</th>
</tr>
</thead>
<tbody>
<tr>
<td>N → π → N$^\dagger$</td>
<td>π ← N ← π</td>
</tr>
</tbody>
</table>

Signal falls exponentially as $e^{-m_N t}$
Noise falls as $e^{-\frac{3}{2}m_\pi t}$

§ Difficulties in Euclidean space

☞ True ground state (nucleon in this case) at large Euclidean time
Much effort has been devoted to controlling systematics

A state-of-the-art calculation (PNDME) $a = 0.12 \text{ fm}$, 310-MeV pion

Move the excited-state systematic into the statistical error

$$C^{3\text{pt}}(t_f, t, t_i) = |\mathcal{A}_0|^2 \langle 0 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_0(t_f-t_i)} + \mathcal{A}_0 \mathcal{A}_1^* \langle 0 | \mathcal{O}_\Gamma | 1 \rangle e^{-M_0(t-t_i)} e^{-M_1(t_f-t)} + \mathcal{A}_0^* \mathcal{A}_1 \langle 1 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_0(t-t_i)} e^{-M_1(t_f-t)} + |\mathcal{A}_1|^2 \langle 1 | \mathcal{O}_\Gamma | 1 \rangle e^{-M_1(t_f-t_i)}$$

No obvious contamination between 0.96 and 1.44 fm separation

$O_\Gamma = \gamma_\mu \gamma_5$

$O_\Gamma = 1$

$O_\Gamma = \sigma_{\mu\nu}$
Systematic Control

Much effort has been devoted to controlling systematics

A state-of-the-art calculation (PNDME) $a = 0.09 \text{ fm}, 310$-MeV pion

Move the \textbf{excited-state systematic} into the statistical error

$$C^{3\text{pt}}(t_f, t, t_i) = |A_0|^2 \langle 0 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_0(t_f-t_i)}$$

$$+ A_0 A_1^*(0) \langle 0 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_1(t_f-t)} e^{-M_0(t_f-t_i)}$$

$$+ A_0^* A_1 (1 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_1(t_f-t_i)} e^{-M_0(t_f-t)}$$

$$+ |A_1|^2 \langle 1 | \mathcal{O}_\Gamma | 1 \rangle e^{-M_1(t_f-t_i)}$$

Much stronger effect at finer lattice spacing!

Needs to be studied case by case

Huey-Wen Lin — INT, Seattle
§ Much effort has been devoted to controlling systematics
§ A state-of-the-art calculation (PNDME)

\[
2.6k \quad g^\text{bare}_T \quad 41.6k
\]

\[
a = 0.06 \text{ fm}, 220-\text{MeV pion}
\]

Plots by Boram Yoon

Huey-Wen Lin — INT, Seattle
§ Much effort has been devoted to controlling systematics

§ A state-of-the-art calculation (PNDME)

Statistical effect

\[ g_{\text{bare}}^{S} = 0.06 \text{ fm}, \text{220-MeV pion} \]

Plots by Boram Yoon

Huey-Wen Lin — INT, Seattle
Much effort has been devoted to controlling systematics.

A state-of-the-art calculation (PNDME) showed a statistical effect (worst case).

\[ a = 0.06 \text{ fm}, 220-\text{MeV pion} \]

Plots by Boram Yoon

Huey-Wen Lin — INT, Seattle
Much effort has been devoted to controlling systematics.

A state-of-the-art calculation (PNDME):

Robustness of the 2-state fit

\[ g^\text{bare}_A = 0.06 \text{ fm}, \text{220-MeV pion} \]

Plots by Boram Yoon

MICHIGAN STATE UNIVERSITY

Huey-Wen Lin — INT, Seattle
Extrapolations

§ Finite-volume/statistical effects

\[ g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L} \]
§ Finite-volume/statistical effects

$M_{\pi}^2$ (GeV$^2$)

2016 Results
Here we are

Summary

- $2\sigma$ might go away with greater statistics

Implications?

- Lattice 2016 Prelim.
  - $\text{RBC}^* \ 2+1f \ 1.15(4)$
  - $\text{PACS}^* \ 2+1f \ 1.8(4)$

New physics?

- $\lambda = \frac{g_A}{g_V} f_{NP}$
  - $A_0 = \frac{-2(\lambda^2 - |\lambda|)}{1 + 3\lambda^2}$
Conclusions (?)

§ $g_A$ is not a gold-plated quantity

 arma early idea that $g_A$ would be easy underestimated systematics

§ High-statistics and large-volume studies are needed!

§ Can you trust other lattice calculations?

... from groups who do due diligence for every ensemble and carefully study systematics

§ Disappointment?

 arma certainly not.

 We are just entering into the precision era to explore these issues...

§ Difficulties = opportunities

 arma getting $g_A$ to subpercent precision will be very hard

§ New physics?

 arma $\lambda = g_A / g_V f_{NP}$

$$A_0 = \frac{-2(\lambda^2 - |\lambda|)}{1 + 3\lambda^2}$$

Stay tuned...

Huey-Wen Lin — INT, Seattle
Can We Trust LQCD?

The disappearance of X(750)

A signal with cross-section as the largest excess in 2015+8TeV would look like this

Can we trust LHC?

http://resonaances.blogspot.co.uk

Huey-Wen Lin — INT, Seattle
Backup Slides
Other Results

§ Isovector form factors

\[
\frac{G_A}{g_A}
\]

Plots by Yong-Chull Jang

\[
\frac{G_E}{g_V}
\]

§ Flavor-dependent couplings, 1\textsuperscript{st} moments of PDFs, ...

公认的 qEDM by Cirigliano (this afternoon)
Available Time Separations

\[ t_{\text{sep}} = t_{\text{max}} \text{ (fm)} \]

\[ m_\pi^2 \text{ (GeV}^2) \]

- LHPC 2+1f Clover
- RBC/UKQCD 2+1f DSDR
- PNDME 2+1+1f Mixed
- ETMC 2+1+1f TM
- ETMC 2f TM
- CLS/Mainz 2f Clover

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Excited-State Contamination

§ Tradeoff: signal-to-noise versus contamination

Noise issue (P. Lepage; D. Kaplan 2011)

For example, CLS/Mainz

2f NP clover,

\[ M_\pi \approx 320 \text{ MeV} \]

\[ a \approx 0.063 \text{ fm} \]

Fix \( N_{\text{meas}} = 200 \)

1205.0180 & private communication
Tradeoff: signal-to-noise versus contamination

- Noise issue (P. Lepage; D. Kaplan 2011)

Options

- Stay at large $t_{\text{sink}}$: RBC/UKQCD (must check smaller pion mass)
- Include excited-state degrees of freedom
  - Multistate fitting or variational method from 3pt correlator matrix
  - HWL (Lat 2008); ETMC/LHPC/Mainz-CLS (2011); CSSM 2012 (mesons)
- Extend to small $t_{\text{sink}}$ to pick up better signal and apply “summation” method

$$S(t_s) := \sum_{t=0}^{t_s} R(t, t_s) \underset{t_s \gg 0}{\to} c + t_s \left(g_A^{\text{bare}} + O(e^{-\Delta t_s})\right)$$

- $g_A$ obtained from slope
Summation Method

§ CLS/Mainz

2f, NP Clover ("summation" vs "plateau" $t_{\text{sep}} = 1.1 \text{ fm}$)

§ LHPC

consistent results using largest $t_{\text{sep}}$ and summation

My two cents: Not clearly superior
Renormalization
Renormalization

§ QCDSF hypothesis: $Z_A$ might be a problem?

![Graph showing $g_A/f_\pi$ vs. $m^2_\pi$ (GeV^2)]
§ QCDSF hypothesis: $Z_A$ might be a problem?

§ Residual $O(a)$ artifacts in

\[ A^R_\mu = Z_A \left( 1 + b_A a m_q \right) \left( A_\mu + a c_A \partial_\mu P \right) \]

Chiral fermions: $m_{\text{res}}$

§ Other systematic cancellations (such as volume, ...)

Need higher statistics to be conclusive
Renormalization

§ QCDSF hypothesis: $Z_A$ might be a problem?

§ Residual $O(a)$ artifacts in

☞ Clover: $A^R_\mu = Z_A (1 + b_A a m_q) (A_\mu + a c_A \hat{\partial}_\mu P)$

☞ Chiral fermions: $m_{\text{res}}$

§ Other systematic
cancellations
(such as volume, ...)

§ Need higher statistics to be conclusive
Chiral Extrapolation
Chiral Extrapolation

- Chiral extrapolation
- Small shift matters?

CLS/Mainz, 1205.0180

\[ m_{\pi,\text{phys}}^2 \] vs. \[ m_{\pi}^2 \text{ [GeV}^2\text{]} \]

- \( a = 0.079\text{fm} \)
- \( a = 0.063\text{fm} \)
- \( a = 0.050\text{fm} \)
Chiral Extrapolation

§ Chiral extrapolation

§ Small shift matters?

§ Blind analysis?

§ More precise studies are needed

CLS/Mainz, 1205.0180

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Chiral Extrapolation

- Chiral extrapolation
- Same formula, similar LECs fixed, different ChPT behavior

Graphs showing data points and curves with different symbols and labels indicating various parameters.
Chiral Extrapolation

§ Chiral extrapolation

§ Same formula, similar LECs fixed, different ChPT behavior

Huey-Wen Lin — INT, Seattle
Finite-Volume Effects
How big $M_\pi L$ is required?

ChPT volume correction/used to estimate systematics

ETMC, QCDSF, CLS/Mainz: possibly underestimated?

Example study (RBC/UKQCD)

$A + B m_\pi^2 + C f_V(m_\pi L)$


Available Volumes

§ How big $M_\pi L$ is required?
§ How big $M_\pi L$ is required?
§ ChPT volume correction/used to estimate systematics
☞ ETMC, QCDSF, CLS/Mainz: possibly underestimated?
§ How big $M_\pi L$ is required?
§ ChPT volume correction/used to estimate systematics

ETF, QCDSF, CLS/Mainz: possibly underestimated?

Highly sensitive to what parameters used in ChPT

\[
\Delta g_A (L) = - \frac{g_A^0 m_\pi^2}{4\pi^2 F_\pi^2} \sum_n \frac{K_1 (L|\bar{n}|m_\pi)}{L|\bar{n}|m_\pi}
+ \frac{(g_A^0)^3 m_\pi^2}{6\pi^2 F_\pi^2} \sum_n \left[ K_0 (L|\bar{n}|m_\pi) - \frac{K_1 (L|\bar{n}|m_\pi)}{L|\bar{n}|m_\pi} \right]
+ \frac{25c_A^2 g_1}{81\pi^2 F_\pi^2} \int_0^\infty dy \sum_n \left[ K_0 (L|\bar{n}|f(m_\pi, y)) - \frac{L|\bar{n}|f(m_\pi, y)}{3} K_1 (L|\bar{n}|f(m_\pi, y)) \right]
- \frac{c_A^2 g_A^0}{\pi^2 F_\pi^2} \int_0^\infty dy \sum_n \left[ K_0 (L|\bar{n}|f(m_\pi, y)) - \frac{L|\bar{n}|f(m_\pi, y)}{3} K_1 (L|\bar{n}|f(m_\pi, y)) \right]
+ \frac{8c_A^2 g_A^0}{27\pi^2 F_\pi^2} \int_0^\infty dy \sum_n \frac{f(m_\pi, y)^2}{\Delta_0} \left[ K_0 (L|\bar{n}|f(m_\pi, y)) - \frac{K_1 (L|\bar{n}|f(m_\pi, y))}{L|\bar{n}|f(m_\pi, y)} \right]
- \frac{4c_A^2 g_A^0}{27\pi^3 F_\pi^2} \frac{m_\pi^3}{\Delta_0} \sum_n \frac{1}{L|\bar{n}|m_\pi} e^{-L|\bar{n}|m_\pi} + \mathcal{O}(\epsilon^4)
\]

fix $\Delta_0 = 0.271$ GeV, $c_A = 1.5$, $F_\pi = 86.2$ MeV
§ How big $M_\pi L$ is required?
§ ChPT volume correction/used to estimate systematics

étrMC, QCDSF, CLS/Mainz: possibly underestimated?
How big $M_\pi L$ is required?

ChPT volume correction/used to estimate systematics

ETMC, QCDSF, CLS/Mainz: possibly underestimated?

ETMC $N_f=2$ example

<table>
<thead>
<tr>
<th>$m_\pi$</th>
<th>$Lm_\pi$</th>
<th>$g_A$</th>
<th>$g_A(L \to \infty)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\beta = 3.9$</td>
</tr>
<tr>
<td>0.4675</td>
<td>5.04</td>
<td>1.163(18)</td>
<td>1.167</td>
</tr>
<tr>
<td>0.4319</td>
<td>4.66</td>
<td>1.134(25)</td>
<td>1.140</td>
</tr>
<tr>
<td>0.3770</td>
<td>4.06</td>
<td>1.140(27)</td>
<td>1.150</td>
</tr>
<tr>
<td>0.3032</td>
<td>3.27</td>
<td>1.111(34)</td>
<td>1.133</td>
</tr>
<tr>
<td>0.2978</td>
<td>4.28</td>
<td>1.103(32)</td>
<td>1.106</td>
</tr>
<tr>
<td>0.2600</td>
<td>3.74</td>
<td>1.156(47)</td>
<td>1.162</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\beta = 4.05$</td>
</tr>
<tr>
<td>0.4653</td>
<td>5.28</td>
<td>1.173(24)</td>
<td>1.177</td>
</tr>
<tr>
<td>0.4035</td>
<td>4.58</td>
<td>1.175(31)</td>
<td>1.182</td>
</tr>
<tr>
<td>0.2925</td>
<td>3.32</td>
<td>1.194(66)</td>
<td>1.218</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\beta = 4.2$</td>
</tr>
<tr>
<td>0.4698</td>
<td>4.24</td>
<td>1.130(26)</td>
<td>1.144</td>
</tr>
<tr>
<td>0.2622</td>
<td>3.55</td>
<td>1.138(43)</td>
<td>1.146</td>
</tr>
</tbody>
</table>

$\Delta g_A = 0.02$

$\Delta g_A = 0.008$
§ Sensitivity to the parameters chosen in ChPT

Ref. [20] using a variety of constraints to $F_\pi = 86.2$ MeV, $c_A = 1.5$, $g_1 = 2.6$, $g_A^0 = 1.15$, Ref. [33] use SU(6) relations to derive $g_A = 1 + (2/3)\cos^2\psi$, $g_{\Delta N} = -2\cos\psi$, $g_{\Delta A} = -3$. 

Huey-Wen Lin — INT, Seattle
§ Sensitivity to the parameters chosen in ChPT

Ref. [20] using a variety of constraints to $F_\pi = 86.2$ MeV, $c_A = 1.5$, $g_1 = 2.6$, $g_A^0 = 1.15$.

Ref. [33] use SU(6) relations to derive $g_A = 1 + (2/3)\cos^2\psi$, $g_{\Delta N} = -2\cos\psi$, $g_{\Delta\Delta} = -3$. 
§ How big $M_\pi L$ is required?

§ How global data changes with a cut

$$\begin{align*}
A + B m_\pi^2 + C f_V(m_\pi L) \\
\sim e^{-m_\pi L} \\
\sim (m_\pi L)^{-3} \\
\sim m_\pi^2 e^{-m_\pi L}(m_\pi L)^{-0.5}
\end{align*}$$

No cut

$\theta$
§ How big $M_\pi L$ is required?
§ How global data changes with a cut

$$A + B m_\pi^2 + C f_V(m_\pi L)$$

$$f_V \sim e^{-m_\pi L}$$

$$f_V \sim (m_\pi L)^{-3}$$

$$f_V \sim m_\pi^2 e^{-m_\pi L}(m_\pi L)^{-0.5}$$

Cut by $m_\pi L > 4$

\[ \begin{align*}
A & = \text{Experiment} \\
B & = \text{QCDSF 2f Clover} \\
C & = \text{RBC/UKQCD 2+1f DSDR} \\
D & = \text{PNDME 2+1+1f Mixed} \\
E & = \text{ETMC 2+1+1f TM} \\
F & = \text{HSC 2+1f Anisoclover} \\
G & = \text{LHPC 2+1f Clover} \\
H & = \text{ETMC 2f TM} \\
I & = \text{CLS/Mainz 2f Clover}
\end{align*} \]
Finite-Volume Effects

§ How big $M_\pi L$ is required?
§ How global data changes with a cut $A + B m_\pi^2 + C f_V(m_\pi L)$

Cut by $m_\pi L > 4$

$f_V \sim e^{-m_\pi L}$

6-fm box

4-fm box

Experiment
QCDSF 2f Clover
RBC/UKQCD 2+1f DSDR
PNDME 2+1+1f Mixed
ETMC 2+1+1f TM
HSC 2+1f Anisoclover
LHPC 2+1f Clover
ETMC 2f TM
CLS/Mainz 2f Clover