Integrating ab initio structure models into accurate reaction calculations using EFT

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Halo nuclei

Exotic nuclear structures are found far from stability. In particular, halo nuclei with peculiar quantal structure:

- Light, n-rich nuclei
- Low $S_n$ or $S_{2n}$

Exhibit large matter radius due to strongly clusterised structure: neutrons tunnel far from the core and form a halo.

One-neutron halo

$^{11}\text{Be} \equiv ^{10}\text{Be} + n$

$^{15}\text{C} \equiv ^{14}\text{C} + n$

Two-neutron halo

$^{6}\text{He} \equiv ^{4}\text{He} + n + n$

$^{11}\text{Li} \equiv ^{9}\text{Li} + n + n$

Proton haloes are possible but less probable: $^{8}\text{B}$, $^{17}\text{F}$
Reactions with halo nuclei

Halo nuclei are fascinating objects but difficult to study \[ \tau_{1/2}(^{11}\text{Be})= 13 \text{ s} \]
⇒ require indirect techniques, like reactions

**Elastic scattering**

**Breakup** ≡ dissociation of halo from core by interaction with target

Need good understanding of the reaction mechanism
i.e. an accurate theoretical description of reaction coupled to a realistic model of projectile
Framework

**Projectile** \( P \) modelled as a two-body system:
- **core** \( c \)+loosely bound **nucleon** \( f \) described by

\[
H_0 = T_r + V_{cf}(r)
\]

\( V_{cf} \) adjusted to reproduce
- bound state \( \Phi_0 \)
- and resonances

**Target** \( T \) seen as
structureless particle

\( P-T \) interaction simulated by optical potentials

\[ \Rightarrow \] breakup reduces to **three-body** scattering problem:

\[
\left[ T_R + H_0 + V_{cT} + V_{fT} \right] \Psi(r, R) = E_T \Psi(r, R)
\]

with initial condition \( \Psi(r, R) \xrightarrow{Z \to -\infty} e^{iKZ+\cdots} \Phi_0(r) \)
Dynamical eikonal approximation (DEA)

Three-body scattering problem:

\[ [T_R + H_0 + V_{cT} + V_{fT}] \Psi(r, R) = E_T \Psi(r, R) \]

with condition \( \Psi \xrightarrow{Z \to -\infty} e^{iKZ} \Phi_0 \)

Eikonal approximation: factorise \( \Psi = e^{iKZ} \hat{\Psi} \)

\[ T_R \Psi = e^{iKZ} [T_R + \nu P_Z + \frac{\mu P_T}{2} v^2] \hat{\Psi} \]

Neglecting \( T_R \) vs \( P_Z \) and using \( E_T = \frac{1}{2} \mu P_T v^2 + \epsilon_0 \)

\[ i\hbar v \frac{\partial}{\partial Z} \hat{\Psi}(r, b, Z) = [H_0 - \epsilon_0 + V_{cT} + V_{fT}] \hat{\Psi}(r, b, Z) \]

solved for each \( b \) with condition \( \hat{\Psi} \xrightarrow{Z \to -\infty} \Phi_0(r) \)

This is the dynamical eikonal approximation (DEA)
[Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

(Usual) eikonal includes the adiabatic approximation: \( (H_0 - \epsilon_0) \approx 0 \)
$^{11}\text{Be} \equiv ^{10}\text{Be} \otimes n$

- $\frac{1}{2}^+$ ground state:
  $\epsilon_{\frac{1}{2}^+} = -0.504 \text{ MeV}$
  In our model, seen as $1s_{\frac{1}{2}}$ neutron bound to $^{10}\text{Be}(0^+)$

- $\frac{1}{2}^-$ bound excited state:
  $\epsilon_{\frac{1}{2}^-} = -0.184 \text{ MeV}$
  In our model, seen as $0p_{\frac{1}{2}}$ neutron bound to $^{10}\text{Be}(0^+)$

- $\frac{5}{2}^+$ bound excited state:
  $\epsilon_{\frac{5}{2}^+} = 1.274 \text{ MeV}$
  In our model, seen as a $d_{\frac{5}{2}}$ resonance
Usual phenomenological description

In reaction models, projectile $\equiv$ two-body system:

$$H_0 = T_r + V_{cn}(r),$$

where $V_{cn}$ is a phenomenological Woods-Saxon that reproduces the basic nuclear properties of the projectile (binding energy, $J^\pi$, ...)

Nowadays **ab initio** calculations of such exotic nuclei are available. Can we use them within a reaction code?

But do we need to go that far?

Breakup reactions are mostly peripheral, i.e., probe:

- ANC of the ground state [P.C. & Nunes, PRC 75, 054609 (2007)]
- Phaseshifts in the continuum [P.C. & Nunes, PRC 73, 014615 (2006)]

$\Rightarrow$ **constrain** two-body description by **ab initio** prediction
Ab initio description of $^{11}$Be

A recent ab initio calculation of $^{11}$Be has been performed [A. Calci et al. PRL 117, 242501 (2016)]

Difficult to reproduce the shell inversion

⇒ include phenomenology to obtain the correct ordering
Ab initio description of $^{11}$Be bound states

- $\frac{1}{2}^+$ ground state:
  \[ \epsilon_{1^+} = -0.500 \text{ MeV} \]
  \[ C_{\frac{1}{2}^+} = 0.786 \text{ fm}^{-1/2} \]
  \[ S_{1^+S_{1/2}} = 0.90 \]

- $\frac{1}{2}^-$ bound excited state:
  \[ \epsilon_{1^-} = -0.184 \text{ MeV} \]
  \[ C_{\frac{1}{2}^-} = 0.129 \text{ fm}^{-1/2} \]
  \[ S_{0p_{1/2}} = 0.85 \]
Ab initio description of $^{10}$Be-n continuum

Provides the most accurate calculation for the $^{10}$Be-n continuum

![Graph of n + $^{10}$Be phase shifts as a function of kinetic energy](image)

**FIG. 3.** The $n + ^{10}$Be phase shifts as a function of the kinetic energy in the center-of-mass frame. NCSMC phase shifts for the $N^2LO_{\text{SAT}}$ interaction are compared for two model spaces indicated by $N_{\text{max}}$.

**Idea:** constrain the $^{10}$Be-n potential in the reaction code to reproduce **ab initio** bound states ANC and $\delta_{l,j}$.
**$^{10}$Be-n potential**

Replace the $^{10}$Be-n interaction by **effective** potentials in each partial wave

Use the spirit of **halo EFT**: separation of scales (in energy or in distance)

Use narrow Gaussian potentials

\[
V_{l,j}(r) = V_0 \, e^{-\frac{r^2}{2\sigma^2}} + V_2 \, r^2 \, e^{-\frac{r^2}{2\sigma^2}}
\]

Fit $V_0$ and $V_2$ to reproduce $\epsilon_{l,j}$, and $C_{l,j}$ (bound states) or $\Gamma_{l,j}$ for resonances

$\sigma = 1.2, 1.5 \text{ or } 2$ fm is a parameter used to evaluate the sensitivity of the calculations to this effective model
$s_{1/2}^\uparrow$: potentials fitted to $\epsilon_{1s_{1/2}}$ and $C_{1s_{1/2}}$

Potentials fitted to $\epsilon_{1s_{1/2}} = -0.504$ MeV and $C_{1s_{1/2}} = 0.786$ fm$^{-1/2}$

Ground-state wave function

- Wave functions: *same* asymptotics but *different* interior
- $\delta_{1s_{1/2}}$: all effective potentials are in *good agreement* with *ab initio* up to 1.5 MeV (same effective-range expansion)
- Similar results obtained for $p_{1/2}$ (excited bound state)
\( d_{5/2}^5 \): potentials fitted to \( \epsilon_{5/2}^{\text{res}} \) and \( \Gamma_{5/2}^{5+} \)

- **Identical** \( \delta_{d_{5/2}} \) up to 1.5 MeV
- up to 5 MeV for the narrow potentials (\( \sigma = 1.2 \) or 1.5 fm)
- **Excellent agreement** with **ab initio** results up to 2 MeV
$p^{\frac{3}{2}}$ and $d^{\frac{3}{2}}$: potentials fitted to $\epsilon^\text{res}$ and $\Gamma$

- **$p^{\frac{3}{2}}$**
  - Large variation in $\delta$ obtained by effective potentials
  - Broad potential ($\sigma = 2 \text{ fm}$) cannot reproduce correct behaviour
  - Fair agreement with *ab initio* results up to 2.5 MeV
  - $^{10}$Be core excitation @ 3.4 MeV not described in effective model
$^{11}\text{Be} + \text{Pb} \rightarrow ^{10}\text{Be} + \text{n} + \text{Pb} @ 69\text{A MeV}$

Total breakup cross section and $p$ contributions

- Major differences in $p_{3/2}$ partial wave; due to differences in $\delta_{p_{3/2}}$
- Broad potential ($\sigma = 2$ fm) produces unrealistic $p_{3/2}$ contribution
- Excellent agreement with data [Fukuda et al. PRC 70, 054606 (2004)]
Role of $\delta_{p3/2}$

Calculations repeated with different potentials ($\sigma = 1.2, 1.5$ or $2 \text{ fm}$) but in $p_{3/2}$, where $\sigma = 1 \text{ fm}$ (perfect agreement with ab initio).

All potentials provide the same $p_{3/2}$ contribution

- confirms the peripherality of reaction (no influence of the internal part)
- shows the significant role of phaseshifts
$^{11}$Be+Pb → $^{10}$Be+n+Pb @ 69 AMeV

- Good agreement with experiment [Fukuda et al. PRC 70, 054606 (2004)]
- All potentials provide similar cross sections ($\sigma = 2$ fm slightly lower)
**$	ext{^{11}Be+C} \rightarrow \text{^{10}Be+n+C @ 67AMeV}$**

Total breakup cross section and dominant contributions

Folded with experimental resolution [Fukuda et al. PRC 70, 054606 (2004)]

- All potentials produce similar breakup cross sections (but $\sigma = 2$ fm)
- In nuclear breakup, resonances play significant role
- Order of magnitude of experiment well reproduced
- But resonant breakup not correctly described due to short-range details missing in the effective model (?)
Effect of core-excitation in resonant breakup
\[ ^{11}\text{Be}+\text{C} \rightarrow ^{10}\text{Be}+\text{n}+\text{C} @ 67\text{AMeV} \]
computed in an extended DWBA model including core excitation
[A. Moro & J.A. Lay, PRL 109, 232502 (2012)]

- Breakup due to the excitation of the \textit{valence} neutron and of the \textit{core} are considered
- \textbf{Both} are needed to reproduce the oscillatory pattern of experiment
- \textbf{Core excitation} dominates the \( \frac{3}{2}^+ \) resonant breakup
- Confirms the missing short-range details in our effective model
SF vs ANC

Calci et al. predict $S_{1s\frac{1}{2}} = 0.90$, but we use $S_{1s\frac{1}{2}} = 1.\ldots$

⇒ repeat calculations with $S_{1s\frac{1}{2}} = 0.90$ (keeping $C_{\frac{1}{2}^+} = 0.786 \text{ fm}^{-1/2}$)

$^{11}\text{Be} + \text{Pb} \rightarrow ^{10}\text{Be} + n + \text{Pb}$

$^{11}\text{Be} + \text{C} \rightarrow ^{10}\text{Be} + n + \text{C}$

No difference ⇒ SF cannot be extracted from these measurements
One exception: resonant breakup, where SF plays a role

⇒ influence of the short-range details (?)
Summary and prospect

- Exotic nuclei studied mostly through reactions
- Mechanism of reactions with halo nuclei understood
  How to improve the projectile description in reaction models?
- Ab initio models too expensive to be used in reaction codes
  ⇒ include the predictions that matter in effective model
- Using Gaussian potentials, we reproduce the ANC and phase shifts predicted by ab initio calculations
- Our study confirms
  - peripherality of breakup reactions
  - influence of the continuum through phase shifts
- Using ab initio predictions gives excellent agreement with data
  - efficient way to include the significant degrees of freedom
  - provides an estimate the influence of omitted mechanisms
  e.g., resonances include short-range details
Thanks to my collaborators

Daniel Baye
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Achim Schwenk
Hans-Werner Hammer

Daniel Phillips
Filomena Nunes
$p_{\frac{1}{2}}$ : potentials fitted to $\epsilon_{\frac{1}{2}}$ and $C_{\frac{1}{2}}$

Potentials fitted to $\epsilon_0 p_{\frac{1}{2}} = -0.184$ MeV and $C_0 p_{\frac{1}{2}} = 0.129$ fm$^{-1/2}$

Excited-state wave function

- Wave functions: same asymptotics but different interior
- Larger variation in $\delta p_{\frac{1}{2}}$ obtained by effective potentials
  Fair agreement with *ab initio* results up to 1 MeV
$^{11}\text{Be} + \text{Pb} \rightarrow ^{10}\text{Be} + \text{n} + \text{Pb} @ 69\text{AMeV}$ (forward angles)

Total breakup cross section and $p$ contributions

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- Major differences in $p_{3/2}$ partial wave; due to differences in $\delta_{p_{3/2}}$
- Broad potential ($\sigma = 2\text{ fm}$) produces unrealistic $p_{3/2}$ contribution
- Excellent agreement with experiment

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