Single particle states in bound nuclei can be populated by transfer to bound states.

S.P states in unbound nuclei can be populated by transfer to a resonance state or fragmentation of a bound projectile. Also elastic resonant method on p-target:

*Are the structure information extracted equivalent*? Do we understand the interplay with the reaction mechanism?

Do we understand the distortion involved in using “surrogate” reactions since the n-N scattering cannot be measured when N is a radioactive nucleus?

Are we interested only in the “absorption” part or also “elastic”? 
~300 stable nuclei, ~7000 “exotic”

Enter the world of exotic nuclei: probing the unbound by walking at the drip line.

- What life is there beyond the dripline?
- How can we discover it without getting lost?
- Extend our understanding of the residual nuclear force.
- Check the limits of validity of structure models such as the SHELL MODEL or "ab initio" models.
- Challenges in peripheral reaction theory.
Flagships are still flagship!!!!
(Uesaka san DREB2016 summary talk)
The case of $^{10}$Li

Transfer to continuum (Resonances)

Fragmentation ($^{10}$Li best example)


Table 1

<table>
<thead>
<tr>
<th>$E_x$ (MeV)</th>
<th>$E_x$ (MeV)</th>
<th>$E_x$ (MeV)</th>
<th>$E_x$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$ (MeV)</td>
<td>$E_x$ (MeV)</td>
<td>$E_x$ (MeV)</td>
<td>$E_x$ (MeV)</td>
</tr>
<tr>
<td>(0.65)</td>
<td>(0.35)</td>
<td>(0.45)</td>
<td>(0.75)</td>
</tr>
</tbody>
</table>

Fig. 2. Reaction: Location energy spectra for transfer to the $s$ and continuum states in $^{10}$Li given in Table 1.

Unbound Nuclei “levels”

Investigation of the role of $^{10}$Li resonances in the halo structure of $^{11}$Li through the $^{11}$Li(p, d)$^{10}$Li transfer reaction

A. Saniellino$^{1,2}$, R. Kajumbe$^{3,4}$, J. Tanaka$^{5}$, M. Akosta$^{6}$, C. Andreo$^{7}$, P. Brandt$^{8}$, A.A. Chen$^{9}$, G. Chukanov$^{10}$, R. Davis$^{11}$, J. Fallis$^{12}$, J.P. Fertin$^{13}$, N. Galli$^{14}$, A.T. Gallant$^{15}$, P.E. Garrett$^{16}$, G. Hackman$^{17}$, B. Hadjimagi$^{18}$, S. Ichimoto$^{19}$, M. Klee$^{20}$, R. Krickem$^{21}$, J. Lighthall$^{22}$, E. McNeil$^{23}$, D. Miller$^{24}$, J. Pocchi$^{25}$, L. Randle$^{26}$, T. Roger$^{27}$, A. Rojas$^{28}$, H. Sawaj$, A. Shuter$^{29}$, I. Tanahashi$^{30}$, J. Thompson$^{31}$, C. Uehara$^{32}$, P. Voss$^{33}$, Z. Wang$^{34}$.

The resonance energy spectrum (Fig. 2b) of $^{10}$Li is constructed in the missing mass technique using the measured energy and scattering angle of the deuterons. A very prominent resonance peak is seen at $E_r = 0.62 \pm 0.04$ MeV and full width $\Gamma = 0.33 \pm 0.07$ MeV is obtained from fitting the spectrum with a Voigt function with an energy-dependent Breit–Wigner function width [22].

### Table 3

<table>
<thead>
<tr>
<th>$2s_{1/2}$</th>
<th>0.63</th>
<th>0.35</th>
<th>-17.2</th>
<th>-10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1p_{1/2}$</td>
<td>1.55</td>
<td>0.18</td>
<td>-9.8</td>
<td></td>
</tr>
</tbody>
</table>

H. Simon et al., NPA791, 267 (2007)
Another intriguing nucleus: $^{10}$Be populated by $1n$-transfer

n-$^9$Be potential from PRC89, 024619
see slide 19

FIG. 2. (Color online) Inclusive excitation energy spectrum (the $^{9}$Be$(^{18}$O,$^{17}$O)$^{10}$Be reaction at 84-MeV incident energy and $3^{\circ}<\theta_{\text{lab}}<10^{\circ}$. The background that comes from $^{12}$C and $^{16}$O impurities has been subtracted. Peaks marked with an asterisk refer to the $^{17}$O ejectile emitted in its first excited state at 0.87 MeV. Total $1-n$ breakup calculations that result from the use of the DOM and the $AB$ potentials (see text) [12] are shown as the green-continuous and the violet-dashed lines, respectively. The experimental data [22] of the $^{9}$Be$(n,nn)^8$Be [23] and $^{9}$Be$(n,\alpha)^6$He [24] reactions are reported as the red-dotted and blue-dotted-dashed lines, respectively. The $1n-(S_n)$, $2n-(S_{2n})$, and $\alpha-(S_\alpha)$ separation energies are also indicated.
Can Ab Initio Theory Explain the Phenomenon of Parity Inversion in $^{11}\text{Be}$?

Angelo Calci, Petr Navrátil, Robert Roth, Jérémy Dohet-Eraly, Sofia Quaglioni, and Guillaume Hupin


$S_n = 0.5\text{MeV}$  \hspace{1cm}  $S_{2n} = 7.3\text{MeV}$  \hspace{1cm}  $\epsilon_f - \epsilon_i = 1.25\text{MeV}$

---

The continuum of $^{11}\text{Be}$ populated by the $(^{16}\text{O},^{10}\text{O})$ two-neutron transfer reaction*

D. Carboni, A. Bonaccorso, C. Agodi, M. Bondi, F. Cappuzzello, M. Cavallaro, A. Consolo, M. De Napoli

---

*PRELIMINARY
$^{18}\text{O} + ^{9}\text{Be} \rightarrow ^{16}\text{O} + ^{11}\text{Be}$

- 2n transfer to bound states
- 1n transfer to a bound state
- 1n transfer to a resonance
- 2n transfer to the continuum
F. Cappuzzello et al.
Signatures of the Giant Pairing Vibration in the 14C and 15C atomic nuclei.
*Nature Communications, 2015, 6, 6743*

Cristina Rea thesis, Pisa 2010
A consistent formalism for all breakup reaction mechanisms

The core-target movement is treated in a semiclassical way, but neutron-target and/or neutron-core with a full QM method.


\[
\frac{d\sigma}{d\xi} = C^2 S \int_0^\infty db_c \frac{dP_{-n}(b_c)}{d\xi} P_{ct}(b_c),
\]

\[\xi \to \epsilon_f, k_z, P_{//}, \text{ also } ANC = \sqrt{C^2 S C_i^2}\]

Use of the simple parametrization

\[P_{ct}(b_c) = |S_{ct}|^2 = e^{-\ln 2 \exp[(R_s - b_c)/a]}\]

\[R_s \approx r_s (A_p^{1/3} + A_t^{1/3}) \quad r_s \approx 1.4 \text{fm}\]

'strong absorption radius'

AB&F.Carstoiu, NPA706 (2002) 322
AB&A.Ibraheem, NPA748 (2005) 414
Transfer to the continuum: from resonances to knockout reactions

First order time dependent perturbation theory amplitude: **

\[ A_{fi} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} dt < \phi_f(r) | V(r) | \phi_i(r - R(t)) > e^{-i(\omega t - mvz/\hbar)} \]  

\[ \omega = \epsilon_i - \epsilon_f + \frac{1}{2}mv^2 \quad R(t) = b_c + vt \]

\[ \frac{dP_{-n}(b_c)}{d\epsilon_f} = \frac{1}{8\pi^3 \hbar^2 k_f 2l_i + 1} \sum m_i |A_{fi}|^2 \]

\[ \approx \frac{4\pi}{2k_f^2} \sum j_f (2j_f + 1)(1 - |\bar{S}_{j_f}|^2 + 1 - |\tilde{S}_{j_f}|^2) \mathcal{F}, \]

\( \phi_f \) see (*)

\[ \mathcal{F} = (1 + F_{l_f,l_i,j_f,j_i}) B_{l_f,l_i} \]

\[ B_{l_f,l_i} = \frac{1}{4\pi} \left[ \frac{k_f}{mv^2} \right] |C_i|^2 \frac{e^{-2\eta b_c}}{2\eta b_c} M_{l_f,l_i} \]
In this case the final continuum state contains interaction n-target

**Wave functions**

Final continuum state:

\[
\phi_f(r) = C_f k^{\frac{i}{2}} (h_{l_f}^{(+)}(kr) - \bar{S}_{l_f} h_{l_f}^{(-)}(kr)) Y_{l_f,m_f}(\Omega_f),
\]

\(\bar{S}_{l_f}(\varepsilon_f)\) is an optical model n-t (n-core in fragmentation reactions) S-matrix.

or using the potential \(V = V_{nt} + V_{eff}\) sum of the neutron-target optical and Coulomb potentials, a distorted wave of the eikonal-type

\[
\phi_f(r, k) = \exp \left\{ ik \cdot r + i\chi_{eik}(r, t) \right\}
\]

the eikonal phase shift is simply

\[
\chi_{eik}(r, t) = \frac{1}{\hbar} \int_t^\infty V(r, R(t'))dt'.
\]

Initial state:

\[
\phi_i(r) = -C_i i/\gamma h_{l_i}^{(1)}(i\gamma r) Y_{l_i,m_i}(\Omega_i).
\]

A historical application: 40 continuum states involved (partial waves) to study strength of resonances in $^{209}$Pb and damping of s.p. modes

FIG. 5. Inclusive spectrum of the reaction $^{208}$Pb($^{10}$Ar,$^{30}$Ar)$^{208}$Pb at $E_{\text{inc}} = 41$ MeV/nucleon [10]. The solid curve superimposed onto the experimental spectrum is the result of our calculation for the cross section due to transfer from the $1d_{3/2}$ initial state in Ar. In the lower part of the figure the dotted curve shows the contribution of the $1k_{17/2}$ final state. The solid curve is the total contribution due to $l_f = 8$. The second peak is due to the $1k_{15/2}$ state. The dashed line is the contribution of $l_f = 9$ and the dot-dashed line is for $l_f = 10$. The tightly dotted curve is the elastic breakup.
Fragmentation of a 2n-halo nucleus

Inelastic-like excitations can be described by the first order time dependent perturbation theory amplitude \( \otimes \):

\[
A_{fi} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} dt \langle \psi_f(t) | V_2(r - R(t)) | \psi_i(t) \rangle
\]

This method has the advantage that different potentials can be used for the determination of \( \psi_i \) and \( \psi_f \).

Which components of the initial wave function show up in the continuum??

In this case the final continuum state contains interaction n-core of origin
(cf. Eq. (2.5) of Ref. [29]). The quantity $S$ is the $S$-matrix representing the final state interaction of the neutron with the projectile core. Then

$$A_{fi} = -\frac{v_2 C_i C_f}{h\nu} \int_{-\infty}^{\infty} dz \frac{e^{-(\gamma - ik)r} - S^* e^{-(\gamma + ik)r}}{r^2} \cos qz. \quad (13)$$

Let us define

$$I_R = \text{Re} \int_{-\infty}^{\infty} dz \frac{e^{-(\gamma - ik)r}}{r^2} \cos qz, \quad (14)$$

and

$$I_I = \text{Im} \int_{-\infty}^{\infty} dz \frac{e^{-(\gamma - ik)r}}{r^2} \cos qz, \quad (15)$$

such that:

$$I(k, q) = I_R + i I_I = |I|e^{i\nu} \quad (16)$$

while

$$\bar{S} = S e^{2i\nu} = e^{2i(\delta + \nu)} \quad (17)$$

then

$$A_{fi} = C(I - S^* I^*) \quad (18)$$

and

$$|A_{fi}|^2 = C^2 |I|^2 |1 - \bar{S}|^2. \quad (19)$$

Where now $C = -\frac{v_2 C_i C_f}{h\nu} \frac{1}{8\pi}$. Only elastic breakup term appears→coincidence experiment
Determination of the bound and unbound (via optical model n-core S-matrix) states in $^{11}\text{Li} \rightarrow ^{10}\text{Li}$

\[ U(r) = V_{WS} + \delta V \]

\[ \delta V(r) = 16\alpha \frac{e^{2(r-R)/a}}{(1 + e^{(r-R)/a})^4} \]

\[ V_{WS} = \text{Woods-Saxon + Spin orbit} \]

\[ \delta V = \text{Correction to the potential originated from p.v. coupling (N. Vinh Mau and J. C. Pacheco, NPA607 (1996) 163)} \]
$^{10}$Li spectrum from $^{11}$Li fragmentation

G. Blanchon, A. Bonaccurso, D.M. Brink, N. Vinh Mau

$\sigma/\varepsilon_f$ (mb/MeV)

$\varepsilon_f$ (MeV)
Hypothesis on the reaction mechanism for 2n transfer on $^{13}$C: dependence on matching conditions
(D M Brink, PLB 1985)

$E_{\text{inc}} = 84 \text{MeV} \rightarrow \sim 4 \text{AMeV}$

$\varepsilon_f - \varepsilon_i = 0.132 \text{MeV}$

Excellent matching for 1n transfer to bound state in the first step, no time delay assumed. Simultaneous 2n transfer assumed.
Fig. 3. Inclusive energy spectrum of the reaction $\text{n}+^{14}\text{C}$ and theoretical calculations of various break-up components (see text). The red dotted (el_n1), red double-dashed-dotted (el_n2), green dotted (abs_n2), and red double-dashed-dotted (el+n2) curves represent the one- and two-neutron elastic break-up, respectively. The green dotted curve (abs_n2) represents the two-neutron absorption term. The orange dashed-dotted curve ((el-abs)_n2) is the sum of two-neutron elastic break-up and absorption. Finally, the violet full curve ((el+n1+el+abs)_n2) is the sum of all contributions folded with the experimental resolution. In the experimental spectrum, plotted with a bin size of 70 keV, the background from the $^{12}\text{C}$ impurity has been subtracted (see Fig. 1). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

Fig. 4. Dominant contributions to the partial wave decomposition of the theoretical energy spectrum shown in Fig. 3. The legend indicates the single particle angular momentum of each individual strength distribution. The lines starting at $E_n = 1.218$ MeV refer to the $^{14}\text{C}+\text{n}$ system while those starting at 9.394 MeV to the $^{12}\text{C}+\text{n}+\text{n}$ (see text).
Resonances described by $WS+\left(\delta V(r) = 16\alpha \frac{e^{2(r-R^R)/a^R}}{(1 + e^{(r-R^R)/a^R})^4}\right)$ consistent with dispersive contribution

$n^{-9}\text{Be scattering data + calculations}$
Can Ab Initio Theory Explain the Phenomenon of Parity Inversion in $^{11}\text{Be}$?

Angelo Calci, Petr Navrátil, Robert Roth, Jérémie Dohet-Eraly, Sofia Quaglioni, and Guillaume Hupin


In this case successive transfer → two steps seem to dominate

FIG. 2. (color online) NCSMC: energies of the $^{10}\text{Be}$ states. Light boxes indicate resonance.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>(\text{NN + 3N(400)})</th>
<th>(\text{N}^2\text{LO_{SAT}})</th>
<th>(\text{NCSMC-pheno})</th>
<th>(\text{exp.})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(E) (\Gamma)</td>
<td>(E) (\Gamma)</td>
<td>(E) (\Gamma)</td>
<td>(E) (\Gamma)</td>
</tr>
<tr>
<td>1/2*$-$</td>
<td>-0.001 (-)</td>
<td>-0.40 (-)</td>
<td>-0.50 (-)</td>
<td>-0.50 (-)</td>
</tr>
<tr>
<td>1/2*</td>
<td>-0.27 (-)</td>
<td>-0.35 (-)</td>
<td>-0.18 (-)</td>
<td>-0.18 (-)</td>
</tr>
<tr>
<td>5/2*</td>
<td>3.03 (0.44)</td>
<td>1.47 (0.12)</td>
<td>1.31 (0.10)</td>
<td>1.28 (0.1)</td>
</tr>
<tr>
<td>3/2$^-$</td>
<td>2.34 (0.35)</td>
<td>2.14 (0.21)</td>
<td>2.15 (0.19)</td>
<td>2.15 (0.21)</td>
</tr>
<tr>
<td>3/2*</td>
<td>3.48 (-)</td>
<td>2.90 (0.014)</td>
<td>2.92 (0.06)</td>
<td>2.898 (0.122)</td>
</tr>
<tr>
<td>5/2*</td>
<td>3.43 (0.001)</td>
<td>2.25 (0.0001)</td>
<td>3.30 (0.0002)</td>
<td>3.3874 (&lt;0.008)</td>
</tr>
<tr>
<td>3/2*</td>
<td>5.52 (0.20)</td>
<td>6.62 (0.29)</td>
<td>5.72 (0.19)</td>
<td>3.45 (0.01)</td>
</tr>
<tr>
<td>9/2*</td>
<td>7.44 (2.30)</td>
<td>5.42 (0.80)</td>
<td>5.59 (0.62)</td>
<td>- (-)</td>
</tr>
</tbody>
</table>

TABLE I. Excitation spectrum of $^{11}\text{Be}$ with respect to the n+$^{10}\text{Be}$ threshold. Energies and widths in MeV. NCSMC(-pheno) calculations are carried out at \(N_{\text{max}} = 9\).
Calculated n-\textsuperscript{10}Be free particle cross section. Thanks to Angelo Calci and Jérémy Dohet-Eraly for providing the n-\textsuperscript{10}Be S-matrix.
Traditional methods

- Renewed interest in highly numerical DWBA and CDCC calculations of various aspects of breakup: A. Moro & Co., B.V. Carlson et al., K.Ogata et al. So far mainly d-induced reactions.
- Resonant scattering and R-matrix: P. Descouvemont, G. Rogachev
- Semiclassical and few body (Hussein, Canto & Co), P. Capel
- Reaction cross sections OK with eikonal approach (Ogawa): improved folding models for the optical potential (Furumoto opt pot)

New methods:

- Chiral interactions used for: ab initio no-core shell model with continuum, and its applications to nucleon and deuterium scattering on light nuclei: P. Navratil, S. Quaglioni, G. Hupin, J. Dohet-Eraly
- Optical potential microscopic calculations from chiral interactions

ECT* workshop

Open questions:

- Unbound nuclei via projectile fragmentation. SEMICLASSICAL OK, but numerical (DWBA-like still in progress)
- Tetraneutron
Thanks to:

DOM Bob Charity

Some more historical collaborators:

David Brink, Nicole Vinh Mau, Guillaume Blanchon, Cristina Rea, the MAGNEX group at LNS-INFN, Catania.
“If you have heart you will certainly have brain…”
(paraphrased from Julian Fellowes)