Phase diagram in the vector meson extended PQM model

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Exploring the QCD Phase Diagram through Energy Scans
INT, Seattle

Collaborators: Zsolt Szép, Péter Kovács
Overview

1. Introduction
   - Motivation

2. The model
   - Axial(vector) meson extended linear $\sigma$-model
   - Parametrization at $T = 0$
   - Polyakov loop

3. eLSM at finite $T/\mu_B$
   - Extremum equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$

4. Results
   - $T$ dependence of the order parameters
   - Critical endpoint
   - Phase diagram

5. Summary
Phase diagram in the $T - \mu_B - \mu_I$ space

- At $\mu_B = 0$ $T_c = 151$ MeV

- Is there a CP?
  ($T_{CP}=162$ MeV, $\mu_{CP}=360$ MeV, Fodor-Katz)

- At $T = 0$ in $\mu_B$ where is the phase boundary?

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)
Motivation

- At $\mu = 0$ we know the properties of strong interactions from the lattice in theoretical side and from STAR/PHENIX and from ALICE in the experimental side. On the other hand, for $\mu \gg 0$ at the moment no theory and no experiment provide reasonable information.
- What is the order of phase transition on the $T=0$ line? Is there a CEP?
- Equation of state for neutron stars.
- How the masses change in medium?

Idea

- Build an effective model having the right global symmetry pattern.
- Compare the thermodynamics of the model with lattice at $\mu = 0$
- Extrapolate to high $\mu$. 
Since QCD is very hard to solve \(\rightarrow\) low energy effective models were set up \(\rightarrow\) reflecting the global symmetries of QCD

- Nambu-Jona-Lasinio model (+Kobayashi-Maskawa-t’Hooft)
- Chiral perturbation theory
- Linear and nonlinear (it does not contain degrees of freedom relevant at high T) sigma model
- To study the phase diagram, we introduced the constituent quarks
- For mimicking confinement, we add the Polyakov loop variables.

extended Polyakov-Quark-Meson model
Chiral symmetry

If the quark masses are zero (chiral limit) \(\implies\) QCD invariant
under the following global transformation (chiral symmetry):
\[
q_L = \frac{(1 - \gamma_5)}{2}q, \quad q_R = \frac{(1 + \gamma_5)}{2}q
\]
only the mass term mixes
\[
U(3)_V q = \exp(-i\alpha t)q \quad U(3)_A q = \exp(-i\beta\gamma_5 t)q
\]
\[
U(3)_L \times U(3)_R \cong U(3)_V \times U(3)_A =
SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A
\]

\(U(1)_V\) term \(\rightarrow\) baryon number conservation

\(U(1)_A\) term \(\rightarrow\) broken through axial anomaly

\(SU(3)_A\) term \(\rightarrow\) broken down by any quark mass

\(SU(3)_V\) term \(\rightarrow\) broken down to \(SU(2)_V\) if \(m_u = m_d \neq m_s\)
\(\rightarrow\) totally broken if \(m_u \neq m_d \neq m_s\) (in nature)
Meson fields - pseudoscalar and scalar meson nonets

\[ \Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \sqrt{2} & \eta_S \end{pmatrix} \left( \sim \bar{q}_i \gamma_5 q_j \right) \]

\[ \Phi_S = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_0^+ \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_0^0 \\ K_S^- & \sqrt{2} & \sigma_S \end{pmatrix} \left( \sim \bar{q}_i q_j \right) \]

Particle content:

Pseudoscalars: \( \pi(138), K(495), \eta(548), \eta'(958) \)

Scalars: \( a_0(980 \text{ or } 1450), K_0^*(800 \text{ or } 1430), (\sigma_N, \sigma_S) : 2 \text{ of } f_0(500, 980, 1370, 1500, 1710) \)
Structure of scalar mesons

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<th>Mass (MeV)</th>
<th>width (MeV)</th>
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<tbody>
<tr>
<td>$A_0(980)$</td>
<td>980 ± 20</td>
<td>50 – 100</td>
<td>$\pi\pi$ dominant</td>
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<tr>
<td>$A_0(1450)$</td>
<td>1474 ± 19</td>
<td>265 ± 13</td>
<td>$\pi\eta, \pi\eta', K\bar{K}$</td>
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<td>$K_s(800) = \kappa$</td>
<td>682 ± 29</td>
<td>547 ± 24</td>
<td>$K\pi$</td>
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<td>$K_s(1430)$</td>
<td>1425 ± 50</td>
<td>270 ± 80</td>
<td>$K\pi$ dominant</td>
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<td>$f_0(500) = \sigma$</td>
<td>400–550</td>
<td>400 – 700</td>
<td>$\pi\pi$ dominant</td>
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<tr>
<td>$f_0(980)$</td>
<td>980 ± 20</td>
<td>40 – 100</td>
<td>$\pi\pi$ dominant</td>
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<td>$f_0(1370)$</td>
<td>1200–1500</td>
<td>200 – 500</td>
<td>$\pi\pi \approx 250, K\bar{K} \approx 150$</td>
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<tr>
<td>$f_0(1500)$</td>
<td>1505 ± 6</td>
<td>109 ± 7</td>
<td>$\pi\pi \approx 38, K\bar{K} \approx 9.4$</td>
</tr>
<tr>
<td>$f_0(1710)$</td>
<td>1722 ± 6</td>
<td>135 ± 7</td>
<td>$\pi\pi \approx 30, K\bar{K} \approx 71$</td>
</tr>
</tbody>
</table>

Possible scalar states: $\bar{q}q, \bar{q}q\bar{q}q$, meson-meson molecules, glueballs
pseudoscalar nonet: $\pi, K, \eta, \eta'$, scalar nonet: $A_0, K_0, 2\ f_0$
multiquark states: $f_0(980), A_0(980) \ f_0(600), K_0(800) ??$
meson-meson bound state ($K\bar{K}$): $f_0(980) ??$
glueballs: $f_0(1500)$ (weak coupling to $\gamma\gamma$), $f_0(1710) ??$
Included fields - vector meson nonets

\[ V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \sqrt{2} K^{*0} & \omega_S \end{pmatrix}^\mu \]

\[ A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \sqrt{2} K_1^0 & f_{1S} \end{pmatrix}^\mu \]

Particle content:
Vector mesons: \( \rho(770), K^*(894), \omega_N = \omega(782), \omega_S = \phi(1020) \)
Axial vectors: \( a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426) \)
Lagrangian (2/1)

\[
\mathcal{L}_{\text{Tot}} = \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 - \frac{1}{4} \text{Tr}(L^2_{\mu \nu} + R^2_{\mu \nu}) + \text{Tr} \left[ \left( \frac{m_1^2}{2} + \Delta \right) (L^2_\mu + R^2_\mu) \right] + \text{Tr}[H(\Phi + \Phi^\dagger)] \\
+ c_1 (\text{det } \Phi + \text{det } \Phi^\dagger) + i \frac{g_2}{2} (\text{Tr}\{L_{\mu \nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu \nu}[R^\mu, R^\nu]\}) \\
+ \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L^2_\mu + R^2_\mu) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger). \\
+ g_3 [\text{Tr}(L_\mu L_\nu L^\mu L^\nu) + \text{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4 [\text{Tr}(L_\mu L^\mu L_\nu L^\nu) + \text{Tr}(R_\mu R^\mu R_\nu R^\nu)] \\
+ g_5 \text{Tr}(L_\mu L^\mu) \text{Tr}(R_\nu R^\nu) + g_6 [\text{Tr}(L_\mu L^\mu) \text{Tr}(L_\nu L^\nu) \\
+ \text{Tr}(R_\mu R^\mu) \text{Tr}(R_\nu R^\nu)] \\
+ \bar{\Psi} i \gamma^\mu \gamma^\nu \left( V_\mu + \frac{g_A}{g_\nu} \gamma_5 A_\mu \right) \Psi
\]

+ Polyakov loops

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke,
Lagrangian (2/2)

where

\[ D^\mu \Phi = \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu [T_3, \Phi] \]

\[ \Phi = \sum_{i=0}^{8} (\sigma_i + i\pi_i) T_i, \quad H = \sum_{i=0}^{8} h_i T_i \quad T_i : U(3) \text{ generators} \]

\[ R^\mu = \sum_{i=0}^{8} (\rho_i^\mu + b_i^\mu) T_i, \quad L^\mu = \sum_{i=0}^{8} (\rho_i^\mu - b_i^\mu) T_i \]

\[ L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu [T_3, L^\nu] - \{ \partial^\nu L^\mu - ieA_e^\nu [T_3, L^\mu] \} \]

\[ R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{ \partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu] \} \]

\[ \bar{\Psi} = (\bar{u}, \bar{d}, \bar{s}) \]

non strange – strange base:

\[ \varphi_N = \sqrt{2/3} \varphi_0 + \sqrt{1/3} \varphi_8, \]
\[ \varphi_S = \sqrt{1/3} \varphi_0 - \sqrt{2/3} \varphi_8, \quad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i) \]

broken symmetry: non-zero condensates \( \langle \sigma_N \rangle, \langle \sigma_S \rangle \rightarrow \phi_N, \phi_S \)
Symmetry properties of the model

Global $U(3)_L \times U(3)_R$ transformation:

$$\begin{align*}
\Phi & \rightarrow U_L \Phi U_R^\dagger \\
L^\mu & \rightarrow U_L L^\mu U_L^\dagger \\
R^\mu & \rightarrow U_R R^\mu U_R^\dagger
\end{align*}$$

Consequences (using the unitarity of $U$’s):

$$\begin{align*}
D^\mu \Phi & \rightarrow U_L D^\mu \Phi U_R^\dagger \\
L^{\mu\nu} & \rightarrow U_L L^{\mu\nu} U_L^\dagger \\
R^{\mu\nu} & \rightarrow U_R R^{\mu\nu} U_R^\dagger
\end{align*}$$

$$(\text{Tr}(\Phi^\dagger \Phi))' = \text{Tr}(U_R \Phi^\dagger U_L^\dagger U_L \Phi U_R^\dagger) = \text{Tr}(U_R \Phi^\dagger \Phi U_R^\dagger) = \text{Tr}(\Phi^\dagger \Phi U_R^\dagger U_R) = \text{Tr}(\Phi^\dagger \Phi)$$

All terms are invariant except the determinant and the explicit symmetry breaking term.
Determinant term

\[ U_L = e^{-i\omega_L^a T^a} \quad U_R = e^{-i\omega_R^a T^a} \]

\[ \omega_V^a = 0.5(\omega_L^a + \omega_R^a) \quad \omega_A^a = 0.5(\omega_L^a - \omega_R^a) \]

By $SU(3)_L \times SU(3)_R$ transformation (if $\omega_L^0 = \omega_R^0 = 0 = \omega_V^0 = \omega_A^0$)

\[(\det \Phi)' = \det(U_L \Phi U_R^\dagger) = \det U_L \det \Phi \det U_R^\dagger = \det \Phi \]

Similarly $\det \Phi^\dagger$ is also invariant.

If $\omega_V^0 \neq 0$ and all the other $\omega$'s are 0 ($[T^a, T^0] = 0$)

\[(\det \Phi)' = \det(e^{-i\omega_V^0 T^0} \Phi e^{i\omega_V^0 T^0}) = \det(e^{-i\omega_V^0 T^0} e^{i\omega_V^0 T^0} \Phi) = \det \Phi \]

On the other hand, if $\omega_A^0 \neq 0$ and all the other $\omega$'s are 0

\[(\det \Phi)' = \det(e^{-i\omega_A^0 T^0} \Phi e^{-i\omega_A^0 T^0}) = \det(e^{-i\omega_A^0 T^0} e^{-i\omega_A^0 T^0} \Phi) = e^{-i2\omega_A^0} \det \Phi \text{Tr} T^0 \]

So the determinant term is invariant under $U(3)_V \times SU(3)_A$ transformation and breaks explicitly the $U(1)_A$ symmetry.
Explicit breaking term: $\text{Tr}[\hat{\epsilon}(\Phi + \Phi^\dagger)]$

\[
\hat{\epsilon} = \sum_{i=0}^{8} \epsilon_i T_i = \begin{pmatrix}
\frac{\epsilon_N}{2} & 0 & 0 \\
0 & \frac{\epsilon_N}{2} & 0 \\
0 & 0 & \frac{\epsilon_S}{\sqrt{2}}
\end{pmatrix}
\]

only $\epsilon^0, \epsilon^8 \neq 0$

- axial transformation: if at least $\epsilon^0 \neq 0$ $U(3)_A$ is broken:

  \[
  (\text{Tr}[\hat{\epsilon}(\Phi)])' = \text{Tr}(e^{-i2\omega^a_A T^a} \hat{\epsilon} \Phi)
  \]

- vector transformation

  \[
  (\text{Tr}[\hat{\epsilon}(\Phi)])' = \text{Tr}(e^{-i\omega^a_V T^a} \hat{\epsilon} e^{i\omega^a_V T^a} \Phi)
  \]

Since $[\hat{\epsilon}, T^0] = 0$, $U(1)_V$ symmetry is preserved.

If all $\epsilon^a = 0$ except $\epsilon^0$, $U(3)_V$ is preserved.

If $\epsilon^8$ also non zero, then since $[T^K, T^8] = 0$ if $k = 1, 2, 3$, $U(1)_V \times SU(2)_V$ survives (isospin symmetry)

(If $\epsilon^3 \neq 0$ too, then the isospin symmetry is broken, only $U(1)_V$.)
Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

\[ \sigma_{N/S} \rightarrow \sigma_{N/S} + \phi_{N/S} \]

\[ \phi_{N/S} \equiv <\sigma_{N/S}> \]

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like \( \text{Tr}[(D_{\mu} \Phi)^\dagger(D_{\mu} \Phi)] \):

\[
\pi_N - a_{1N}^\mu : -g_1 \phi_N a_{1N}^\mu \partial_\mu \pi_N, \\
\pi - a_{1}^\mu : -g_1 \phi_N (a_{1}^\mu \partial_\mu \pi^- + a_{1}^\mu 0 \partial_\mu \pi^0) + \text{h.c.}, \\
\pi_S - a_{1S}^\mu : -\sqrt{2} g_1 \phi_S a_{1S}^\mu \partial_\mu \pi_S, \\
K_S - K^{\ast}_{\mu} : \frac{ig_1}{2} (\sqrt{2} \phi_S - \phi_N)(\bar{K}_{\mu}^{\ast 0} \partial_\mu K_S^0 + K_{\mu}^{\ast -} \partial_\mu K_S^{+}) + \text{h.c.}, \\
K - K_{1}^\mu : -\frac{g_1}{2} (\phi_N + \sqrt{2} \phi_S)(K_{1}^{\mu 0} \partial_\mu \bar{K}^0 + K_{1}^{\mu +} \partial_\mu K^-) + \text{h.c.}
\]

Diagonalization \(\rightarrow\) Wave function renormalization
Determination of the parameters of the Lagrangian

16 unknown parameters \((m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A)\) \(\rightarrow\) Determined by the min. of \(\chi^2:\)

\[
\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[ \frac{Q_i(x_1, \ldots, x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2,
\]

where \((x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots)\), \(Q_i(x_1, \ldots, x_N)\) calculated from the model, while \(Q_i^{\text{exp}}\) taken from the PDG

multiparametric minimalization \(\rightarrow\) MINUIT

- PCAC \(\rightarrow\) 2 physical quantities: \(f_\pi, f_K\)
- Tree-level masses \(\rightarrow\) 15 physical quantities:
  \(m_u/d, m_s, m_\pi, m_\eta, m_\eta', m_K, m_\rho, m_\Phi, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_S}, m_{f_0^L}, m_{f_0^H}\)
- Decay widths \(\rightarrow\) 12 physical quantities:
  \(\Gamma_{\rho \rightarrow \pi \pi}, \Gamma_{\Phi \rightarrow KK}, \Gamma_{K^* \rightarrow K \pi}, \Gamma_{a_1 \rightarrow \pi \gamma}, \Gamma_{a_1 \rightarrow \rho \pi}, \Gamma_{f_1 \rightarrow KK^*}, \Gamma_{a_0 \rightarrow K K_S}, \Gamma_{K_S \rightarrow K \pi}, \Gamma_{f_0^L \rightarrow \pi \pi}, \Gamma_{f_0^L \rightarrow KK}, \Gamma_{f_0^H \rightarrow \pi \pi}, \Gamma_{f_0^H \rightarrow KK}\)
- \(T_c = 155\) MeV from lattice
### Results

<table>
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<tr>
<th>Parameter</th>
<th>Cal (GeV)</th>
<th>Mass</th>
<th>Parameter</th>
<th>Cal (GeV)</th>
<th>Width</th>
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<td>$m_\pi$</td>
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<td>$f_\pi$</td>
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<td>$m_K$</td>
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<td>$f_K$</td>
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<td>$m_\eta$</td>
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<td>$m_\rho$</td>
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<td>$m_{f_0H}$</td>
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<td>$m_{ud}$</td>
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<td>0.308</td>
<td>$m_s$</td>
<td>0.4577</td>
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### Parameters

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<th>Parameter</th>
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<td>$\phi_N$  [GeV]</td>
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<td>$g_2$</td>
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<td>$M_0$  [GeV]</td>
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With this set $f_0^l = 0.2837\text{ GeV}$
Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\text{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\text{Tr}_c \bar{L}(\vec{x})}{N_c}$ with $L(x) = \mathcal{P} \exp \left[ i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right]$

→ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement transition

low $T$: confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$

high $T$: deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space

$G_{4,d}(\vec{x}) = \phi_3(\vec{x}) \lambda_3 + \phi_8(\vec{x}) \lambda_8$; $\lambda_3, \lambda_8$ : Gell-Mann matrices.

In this gauge the Polyakov loop operator is $L(\vec{x}) = \text{diag}(e^{i\beta \phi_+(\vec{x})}, e^{i\beta \phi_-(\vec{x})}, e^{-i\beta (\phi_+(\vec{x})+\phi_-(\vec{x}))})$

where $\phi_{\pm}(\vec{x}) = \pm \phi_3(\vec{x}) + \phi_8(\vec{x})/\sqrt{3}$
Form of the potential

I.) Simple **polynomial potential** invariant under $\mathbb{Z}_3$ and charge conjugation: R.D.Pisarski, PRD 62, 111501

$$\frac{U_{\text{poly}}(\Phi, \bar{\Phi})}{T^4} = -\frac{b_2(T)}{2} \Phi \bar{\Phi} - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\bar{\Phi} \Phi)^2$$

with

$$b_2(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \frac{T_0^2}{T^2} + a_3 \frac{T_0^3}{T^3}$$

II.) **Logarithmic potential** coming from the $SU(3)$ Haar measure of group integration


$$\frac{U_{\text{log}}(\Phi, \bar{\Phi})}{T^4} = -\frac{1}{2} a(T) \Phi \bar{\Phi} + b(T) \ln \left[ 1 - 6 \Phi \bar{\Phi} + 4 (\Phi^3 + \bar{\Phi}^3) - 3 (\Phi \bar{\Phi})^2 \right]$$

with

$$a(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \frac{T_0^2}{T^2}, \quad b(T) = b_3 \frac{T_0^3}{T^3}$$

$U(\Phi, \bar{\Phi})$ models the free energy of a pure gauge theory

$\longrightarrow$ the parameters are fitted to the pure gauge lattice data
Polyakov loop potential

“Color confinement”
\[ \langle \Phi \rangle = 0 \rightarrow \text{no breaking of } \mathbb{Z}_3 \]
one minimum

“Color deconfinement”
\[ \langle \Phi \rangle \neq 0 \rightarrow \text{spontaneous breaking of } \mathbb{Z}_3 \]
minima at \( 0, 2\pi/3, -2\pi/3 \)
one of them spontaneously selected

from H. Hansen et al., PRD75, 065004 (2007)
Effects of quarks: Improved Polyakov loops potential

- $N_f$-dependence of $T_0$ estimated: $T_0(N_f = 2 + 1) = 182$ MeV for $m_s=95$ MeV
  L.M. Haas et al., PRD 87, 076004 see also B.-J. Schaefer et al., PRD 76, 074023
- Within FRG, the glue potential $U_{glue}(\Phi, \bar{\Phi})$ coming from the gauge dof propagating in the presence of dynamical quarks can be matched to the potential $U^{YM}(\Phi, \bar{\Phi})$ of the SU(3) YM theory by relating the reduced temperatures:

$$\frac{U^{glue}}{T^4}(\Phi, \bar{\Phi}, t_{glue}) = \frac{U^{YM}}{(T^{YM})^4}(\Phi, \bar{\Phi}, t_{YM}(t_{glue})), \quad t_{YM}(t_{glue}) \approx 0.57 t_{glue}$$

$$t_{glue} \equiv \frac{T - T_{c}^{glue}}{T_{c}^{glue}}, \quad t_{YM} \equiv \frac{T^{YM} - T_{0}^{YM}}{T_{0}^{YM}}, \quad T_{c}^{glue} \in (180, 270) \text{MeV}$$

L.M.Haas et al., PRD 87, 076004 (2013)
Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

\[
f(E_p - \mu_q) \rightarrow f^+_\Phi(E_p) = \frac{\left(\Phi + 2\Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}{1 + 3 \left(\Phi + \Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}
\]

\[
f(E_p + \mu_q) \rightarrow f^-\Phi(E_p) = \frac{\left(\Phi + 2\Phi e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}{1 + 3 \left(\Phi + \Phi e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}
\]

\[
\Phi, \bar{\Phi} \rightarrow 0 \implies f^+\Phi(E_p) \rightarrow f(3(E_p \pm \mu_q)) \quad \Phi, \bar{\Phi} \rightarrow 1 \implies f^\pm\Phi(E_p) \rightarrow f(E_p \pm \mu_q)
\]

three-particle state appears: mimics confinement of quarks within baryons

at \( T = 0 \) there is no difference between models with and without Polyakov loop
Extremum equations for $\phi^N/S$ and $\Phi$, $\bar{\Phi}$

\[ T/\mu_B \] dependence of the Polyakov-loops

\[
\frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \bar{\Phi}} \bigg|_{\phi^N=\phi^N, \phi^S=\phi^S} = 0, \quad \Omega : \text{grand canonical potential}
\]

\[
-\frac{d}{d\Phi}\left(\frac{U(\Phi, \bar{\Phi})}{T^4}\right) + \frac{2N_c}{T^3} \sum_{q=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left( \frac{e^{-\beta E_q^-}(p)}{g_q^-(p)} + \frac{e^{-2\beta E_q^+}(p)}{g_q^+(p)} \right) = 0
\]

\[
-\frac{d}{d\bar{\Phi}}\left(\frac{U(\Phi, \bar{\Phi})}{T^4}\right) + \frac{2N_c}{T^3} \sum_{q=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left( \frac{e^{-\beta E_q^+}(p)}{g_q^+(p)} + \frac{e^{-2\beta E_q^-}(p)}{g_q^-(p)} \right) = 0
\]

\[
g_q^+(p) = 1 + 3 \left( \bar{\Phi} + \Phi e^{-\beta E_q^+}(p) \right) e^{-\beta E_q^+}(p) + e^{-3\beta E_q^+}(p)
\]

\[
g_q^-(p) = 1 + 3 \left( \Phi + \bar{\Phi} e^{-\beta E_q^-}(p) \right) e^{-\beta E_q^-}(p) + e^{-3\beta E_q^-}(p)
\]

\[
E_q^\pm(p) = E_q(p) \mp \mu_B/3, \quad E_{u/d}(p) = \sqrt{p^2 + m_{u/d}^2}, \quad E_s(p) = \sqrt{p^2 + m_s^2}
\]
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Extremum equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$

$T/\mu_B$ dependence of the condensates ($\phi_{N/S}$)

$$\frac{\partial \Omega}{\partial \phi_N} = \left. \frac{\partial \Omega}{\partial \phi_S} \right|_{\Phi, \bar{\Phi}} = 0, \quad \text{(after the SSB)}$$

Hybrid approach: fermions at one-loop, mesons at tree-level (their effects are much smaller)

$$m_0^2 \phi_N + \left( \lambda_1 + \frac{1}{2} \lambda_2 \right) \phi_N^3 + \lambda_1 \phi_N \phi_S^2 - h_N + \frac{g_F}{2} N_c \left( \langle u\bar{u} \rangle_T + \langle d\bar{d} \rangle_T \right) = 0$$

$$m_0^2 \phi_S + (\lambda_1 + \lambda_2) \phi_S^3 + \lambda_1 \phi_N^2 \phi_S - h_S + \frac{g_F}{\sqrt{2}} N_c \langle s\bar{s} \rangle_T = 0$$

$$\langle q\bar{q} \rangle_T = -4m_q \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_q(p)} \left( 1 - f^-_\phi(E_q(p)) - f^+_\phi(E_q(p)) \right)$$
Extremum equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$

Masses

\[
M_{i,ab}^2 = \left. \frac{\partial^2 \Omega(T, \mu_f)}{\partial \phi_i, a \partial \phi_i, b} \right|_{\min} = m_{i,ab}^2 + \Delta_0 m_{i,ab}^2 + \Delta_T m_{i,ab}^2,
\]

$m_{i,ab}^2 \rightarrow$ tree-level mass matrix,
\[
\Delta_0 / T m_{i,ab}^2 \rightarrow$ fermion vacuum/thermal fluctuation,
\]

\[
\Delta_0 m_{i,ab}^2 = \left. \frac{\partial^2 \Omega_{\text{vac}}^{\phi q \bar{q}}}{\partial \phi_i, a \partial \phi_i, b} \right|_{\min} = -\frac{3}{8\pi^2} \sum_{f=u,d,s} \left[ \left( \frac{3}{2} + \log \frac{m_f^2}{M^2} \right) m_{f,a}^2 m_{f,b}^2 + m_f^2 \left( \frac{1}{2} + \log \frac{m_f^2}{M^2} \right) m_{f,ab}^2 \right],
\]

\[
\Delta_T m_{i,ab}^2 = \left. \frac{\partial^2 \Omega_{\text{th}}^{\phi q \bar{q}}}{\partial \phi_i, a \partial \phi_i, b} \right|_{\min} = 6 \sum_{f=u,d,s} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_f(p)} \left[ (f_f^+(p) + f_f^-(p)) \left( m_{f,ab}^2 - \frac{m_{f,a}^2 m_{f,b}^2}{2E_f^2(p)} \right) \right.
\]
\[
+ \left. (B_f^+(p) + B_f^-(p)) \frac{m_{f,a}^2 m_{f,b}^2}{2TE_f(p)} \right],
\]

where $m_{f,a}^2 \equiv \partial m_f^2 / \partial \phi_i, a$, $m_{f,ab}^2 \equiv \partial^2 m_f^2 / \partial \phi_i, a \partial \phi_i, b$
Features of our approach

- D.O.F’s: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with $U^{YM}$ or $U^{glue}$
- $u,d,s$ constituent quarks, $(m_u = m_d)$
- no mesonic fluctuations included in the grand canonical potential:
  \[
  \Omega(T, \mu_q) = -\frac{1}{\beta V} \ln(Z)
  \]
- Fermion vacuum and thermal fluctuations
- quarks do not couple to (axial) vector meson yet
- Four order parameters ($\phi_N, \phi_S, \Phi, \bar{\Phi}$) $\rightarrow$ four $T/\mu$-dependent equations
- thermal contribution of $\pi, K, f_0^L$ included in the pressure
$\phi_{N/S}(T), \Phi/\bar{\Phi}(T)$ with Polyakov loop $m_{f_0^L} = 1326$ MeV

Condensates and Polyakov loop variables with vacuum fluctuations
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$T$ dependence of the order parameters

With low mass scalars, $m_{f_0^L} = 300$ MeV

Our pattern: $m_\eta \leq m_{\eta_N} < m_{\eta_S} \leq m_{\eta'}$ in contrast to others

Schaefer, PRD79 014018, Tiwari PRD88, 074017
With low mass scalars 1st order phase transition

Order params. in T, A0LKSf013, $\mu_B=0.849$ GeV, $m_{f0L}=0.25$ GeV

$\Phi = \Phi^*$
T-dependence of the order parameters

T-dependence of condensates compared to lattice results

$$\Delta = \frac{\left(\Phi_N - h_N/h_S\Phi_S\right)_T}{\left(\Phi_N - h_N/h_S\Phi_S\right)_{T=0}}$$

$U_{\text{glue}}$ with $T_c^{\text{glue}} \in (210 - 240)$ MeV gives good agreement with the lattice result of Borsanyi et al., JHEP 1009, 073 (2010)

- lattice shows smooth transition
- our result is completely off
- renormalization of the Polyakov loop may explain part of the discrepancy
  Andersen et al., PRD92, 114504
Thermodynamical Observables

We include mesonic thermal contribution to $p$ for $(\pi, K, f_0^l)$

$$\Delta p(T) = -nT \int \frac{d^3 q}{(2\pi)^3} \ln(1 - e^{-\beta E(q)}), \quad E(q) = \sqrt{q^2 + m^2}$$

- pressure: $p(T, \mu_q) = \Omega_H(T = 0, \mu_q) - \Omega_H(T, \mu_q)$
- entropy density: $s = \frac{\partial p}{\partial T}$
- quark number density: $\rho_q = \frac{\partial p}{\partial \mu_q}$
- energy density: $\epsilon = -p + Ts + \mu_q \rho_q$
- scaled interaction measure: $\frac{\Delta}{T^4} = \frac{\epsilon - 3p}{T^4}$
- speed of sound at $\mu_q = 0$: $c_s^2 = \frac{\partial p}{\partial \epsilon}$
we use $U_{\text{glue}}$ with $T_c^{\text{glue}} = 270$ MeV
pion dominates at low $T$
at high $T$ pressure overshoots the lattice data

overshooting increases with decreasing $T_c^{\text{glue}}$
lattice: Borsányi et al., JHEP 1011, 077 (2010)
Observables

**Interaction measure**

\[ \frac{(p-3\epsilon)}{T^4} \]

**Speed of sound**

\[ c_s^2(t) \]

- U_{YM}, T_0=182 MeV, only \( \pi \)
- U_{YM}, T_0=182 MeV
- U_{glue}, T_c^{glue}=150 MeV
- U_{glue}, T_c^{glue}=182 MeV
- U_{glue}, T_c^{glue}=210 MeV
- U_{glue}, T_c^{glue}=240 MeV
- U_{glue}, T_c^{glue}=270 MeV
- U_{glue}, T_c^{glue}=270 MeV, only \( \pi \)
- lattice
$T - \mu_B$ phase diagram

- we use $U^{\text{glue}}$ with $T_c^{\text{glue}} = 210$ MeV
- freeze-out curve from Cleymans et al., J.Phys.G32, S165
general feature of PQM (or similar models): at $T=0$ the phase transition at very low density, pushing it to higher value the phase transition become cross over.
**Introduction**

**The model eLSM at finite $T/\mu_B$**

**Results Summary**

**Phase diagram**

**Observables at $\mu_B \neq 0$**
Summary and Conclusions

- The thermodynamics of the ePQM was studied after parametrizing of the model with a modification of the method used in Parganlija et al., PRD87, 014011.
- 40 possible assignments of the scalars to the nonet states were investigated. Lowest $\chi^2$ for $a_0^q \rightarrow a_0(980)$, $K_0^q \rightarrow K_0^*(980)$, $f_0^q \rightarrow f_0(500)$, $f_0^h \rightarrow f_0(980)$.
- The phase transition temperature requires low mass ($\leq 400$ MeV) $f_0$.
- For the best set of parameters CEP was found in the $T - \mu_B$ plane.
- The $T$-dependence of various thermodynamical observables measured on the lattice is reasonable well reproduced with an improved Polyakov loop potential. L.M. Hass et al., PRD87, 076004.
- The model’s predictions are unrealistic at large $\mu_B$. 

→ To do . . .

→ Improve the vacuum phenomenology by tetraquarks (and glueballs)
→ coupling the quarks to the (axial)vectors
→ including mesonic fluctuations
→ find a way to improve the high density behaviour
Thank you for your attention!