Search for the QCD Critical Point -
Fluctuations of Conserved Quantities in High-Energy Nuclear Collisions at RHIC

Xiaofeng Luo

Central China Normal University
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Outline

- **Introduction**

- **Data Analysis**: *Net-P, Net-Q, Net-K*
  1) Centrality determination and Volume Fluctuations.
  2) Efficiency correction, particle identification.
  3) Error Estimation.

- **Results and Discussion**: *CP Search*
  1) Cumulants and Their Ratios: up to 4th order
  2) Centrality, rapidity and energy dependence.
  3) Deuteron Formation, UrQMD, JAM, NJL etc.
  4) Correlation functions from data and model.

- **Summary and Outlook**
QCD Phase Diagram (Conjectured)

QCD Phase Structure: Emergent properties of the strong interaction.

Search for the QCD Critical Point in HIC:
Challenges:
1. finite size/time.
2. Contribution of non-CP physics background.
3. signal survive after dynamical expansion.

Critical Point and Critical Phenomena

First CP is discovered in 1869 for CO$_2$ by Andrews.

\[ T_c = 31^\circ C \]

Can we discovery the Critical Point of Quark Matter? (Put a permanent mark in the QCD phase diagram in text book.)

\[ T_c \sim \text{Trillion } (10^{12}) \, ^\circ C \]
**Location of CEP: Theoretical Prediction**

**Lattice QCD:**
   $(\mu_E^b, T_E) = (360, 162)$ MeV (Reweighting)
2) Gavai & Gupta, NPA 904, 883c (2013) 
   $(\mu_E^b, T_E) = (279, 155)$ MeV (Taylor Expansion)
3) F. Karsch ($\mu_E^b / T_E > 2$, CPOD2016)

**DSE:**
   $(\mu_E^b, T_E) = (372, 129)$ MeV 
2) Hong-shi Zong et al., JHEP 07, 014 (2014). 
   $(\mu_E^b, T_E) = (405, 127)$ MeV
3) C. S. Fischer et al., PRD90, 034022 (2014). 
   $(\mu_E^b, T_E) = (504, 115)$ MeV

$\mu_E^b = 266 \sim 504$ MeV, $T_E = 115 \sim 162$, $\mu_E^b / T_E = 1.8 \sim 4.38$
Finite Size Scaling: HBT and Net-P Fluctuations

1) Scaling function validates the location of the CEP and the (static) critical exponents.

2) 2\textsuperscript{nd} order PT (3D Ising Model): $\nu = 0.66$, $\gamma = 1.2$, $T \sim 165$ MeV, $\mu_B \sim 95$ MeV

Do we understand the non-critical contributions??

Roy Lacey et al., arXiv:1606.08071
Fluctuations as Signature of Phase Transition

Fluctuations are sensitive to the phase transition and critical point.

1. Derivations of thermodynamic quantities (EoS) are related to E-by-E fluctuations in HIC.

\[ \chi_n = \frac{\partial^n(P/T^4)}{\partial(\mu/T)^n} \bigg|_T \]

\[ \chi_1 = \frac{1}{VT^3} \langle N \rangle, \quad \chi_2 = \frac{1}{VT^3} \langle (\Delta N)^2 \rangle, \quad \chi_3 = \frac{1}{VT^3} \langle (\Delta N)^3 \rangle, \]

\[ \chi_4 = \frac{1}{VT^3} \langle (\Delta N)^4 \rangle_c = \frac{1}{VT^3} \left( \langle (\Delta N)^4 \rangle - 3 \langle (\Delta N)^2 \rangle^2 \right). \]

2. It reveals more details: Sign change and diverge.

Higher Order Fluctuations of Conserved Quantities

1. Higher sensitivity to correlation length ($\xi$) and probe non-gaussian fluctuations.

$$\langle (\delta N)^3 \rangle \approx \xi^{4.5}, \quad \langle (\delta N)^4 \rangle \approx \xi^7$$

$$C_{1,x} = \langle x \rangle, C_{2,x} = \langle (\delta x)^2 \rangle,$$

$$C_{3,x} = \langle (\delta x)^3 \rangle, C_{4,x} = \langle (\delta x)^4 \rangle - 3 \langle (\delta x)^2 \rangle^2$$


2. Connection to the susceptibility of the system.

$$\frac{\chi^4_q}{\chi^2_q} = \kappa \sigma^2 = \frac{C_{4,q}}{C_{2,q}} \quad \frac{\chi^3_q}{\chi^2_q} = S \sigma = \frac{C_{3,q}}{C_{2,q}}$$

$$\chi^{(n)}_q = \frac{1}{VT^3} \times C_{n,q} = \frac{\partial^n (p/T^4)}{\partial (\mu_q)^n}, q = B, Q, S$$

STAR Detector System

- Large, Uniform Acceptance at Mid-y
- Excellent Particle Identification
<table>
<thead>
<tr>
<th>√s_{NN} (GeV)</th>
<th>Events (10^6)</th>
<th>Year</th>
<th>*μ_B (MeV)</th>
<th>*T_{CH} (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>350</td>
<td>2010</td>
<td>25</td>
<td>166</td>
</tr>
<tr>
<td>62.4</td>
<td>67</td>
<td>2010</td>
<td>73</td>
<td>165</td>
</tr>
<tr>
<td>39</td>
<td>39</td>
<td>2010</td>
<td>112</td>
<td>164</td>
</tr>
<tr>
<td>27</td>
<td>70</td>
<td>2011</td>
<td>156</td>
<td>162</td>
</tr>
<tr>
<td>19.6</td>
<td>36</td>
<td>2011</td>
<td>206</td>
<td>160</td>
</tr>
<tr>
<td>14.5</td>
<td>20</td>
<td>2014</td>
<td>264</td>
<td>156</td>
</tr>
<tr>
<td>11.5</td>
<td>12</td>
<td>2010</td>
<td>316</td>
<td>152</td>
</tr>
<tr>
<td>7.7</td>
<td>4</td>
<td>2010</td>
<td>422</td>
<td>140</td>
</tr>
</tbody>
</table>

*(μ_B, T_{CH}) : J. Cleymans et al., PRC73, 034905 (2006)*

1) Access broad region of the QCD phase diagram.
2) STAR: Large and homogeneous acceptance, excellent PID capabilities.

**STAR is a unique detector with huge discovery potential in exploring the QCD phase structure at high baryon density.**
## Analysis Details

<table>
<thead>
<tr>
<th>Net-Charge</th>
<th>Net-Proton</th>
<th>Net-Kaon</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kinematic cuts</strong></td>
<td><strong>Kinematic cuts</strong></td>
<td><strong>Kinematic cuts</strong></td>
</tr>
<tr>
<td>$0.2 &lt; p_T (\text{GeV/c}) &lt; 2.0$</td>
<td>$0.4 &lt; p_T (\text{GeV/c}) &lt; 2.0$</td>
<td>$0.2 &lt; p_T (\text{GeV/c}) &lt; 1.6$</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 0.5$</td>
</tr>
<tr>
<td><strong>Particle Identification</strong></td>
<td><strong>Particle Identification</strong></td>
<td><strong>Particle Identification</strong></td>
</tr>
<tr>
<td>Reject protons form spallation for $p_T &lt; 0.4 \text{ GeV/c}$</td>
<td>$0.4 &lt; p_T (\text{GeV/c}) &lt; 0.8 \rightarrow \text{TPC}$</td>
<td>$0.2 &lt; p_T (\text{GeV/c}) &lt; 0.4 \rightarrow \text{TPC}$</td>
</tr>
<tr>
<td></td>
<td>$0.8 &lt; p_T (\text{GeV/c}) &lt; 2.0 \rightarrow \text{TPC+TOF}$</td>
<td>$0.4 &lt; p_T (\text{GeV/c}) &lt; 1.6 \rightarrow \text{TPC+TOF}$</td>
</tr>
</tbody>
</table>

**Centrality definition, \( \rightarrow \) to avoid auto-correlations**

- Uncorrected charged primary particles multiplicity distribution
- Uncorrected charged primary particles multiplicity distribution, without (anti-)protons
- Uncorrected charged primary particles multiplicity distribution, without (anti-)kaons

| $0.5 < |\eta| < 1.0$ | $|\eta| < 1.0$ | $|\eta| < 1.0$ |
Effects needed to be addressed to get final moments/cumulants:
1. Auto-correlation effects: Centrality definition.
2. Effects of volume fluctuation: Centrality bin width correction
3. Finite detector efficiency: Factorial moments

Efficiency Correlation and Error Estimation

- We can express the cumulants in terms of the factorial moments, which can be easily efficiency corrected by assuming binomial response function for efficiency.

\[ F_{u,v,j,k}(N_{p_1}, N_{p_2}, N_{\bar{p}_1}, N_{\bar{p}_2}) = \frac{f_{u,v,j,k}(n_{p_1}, n_{p_2}, n_{\bar{p}_1}, n_{\bar{p}_2})}{(\varepsilon_{p_1})^u (\varepsilon_{p_2})^v (\varepsilon_{\bar{p}_1})^j (\varepsilon_{\bar{p}_2})^k} \]

X. Luo, PRC91, 034907 (2015);

- Statistical Errors based on Delta Theorem.
  With same N events: error(net-charge) > error(net-kaon) > error(net-proton)

<table>
<thead>
<tr>
<th>Au+Au 14.5GeV</th>
<th>Net-Charge</th>
<th>Net-Proton</th>
<th>Net-Kaon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical Width((\sigma))</td>
<td>12.2</td>
<td>4.2</td>
<td>3.4</td>
</tr>
<tr>
<td>Average efficiency((\varepsilon))</td>
<td>65%</td>
<td>75%</td>
<td>38%</td>
</tr>
<tr>
<td>(\sigma^2/\varepsilon^2)</td>
<td>355</td>
<td>32</td>
<td>82</td>
</tr>
</tbody>
</table>

Those numbers are for illustration purpose and not used in actual analysis.
Efficiencies for Protons and Anti-protons

Au + Au Collisions at RHIC

Efficiencies (|y| < 0.5)

Proton
- low p_T (TPC)
- high p_T (TPC+ToF)

Anti-proton
- low p_T (TPC)
- high p_T (TPC+ToF)

Efficiency decrease with increasing energies and centralities.

Proton Efficiency > Anti-proton Efficiency

Due to TOF matching eff., high p_T efficiency (~50%) are smaller than low p_T (~80%).
Efficiencies for $K^+$ and $K^-$

- $0.2 < p_T < 0.4 \text{ (GeV/c)}$, TPC only
- $0.4 < p_T < 1.6 \text{ (GeV/c)}$, TPC+TOF

Efficiency = Efficiency(Tracking) * Efficiency(TOF match)

Ji Xu, SQM 2016.
Net-Proton Cumulants ($C_1$-$C_4$) Vs. Centrality

1. In general, cumulants are linearly increasing with $<N_{part}>$.
2. Efficiency corrections are important.
3. At low energies, the proton cumulants are close to net-proton.
C₃ and C₄ generally consistent with Poisson expectation.
The higher the order of cumulants, the larger deviations from Poisson expectations for net-proton and proton.

In general, the cumulants for net-kaon, kaon and antikaon are consistent with Poisson baseline within uncertainties.
1) Within errors, the results of net-Q and net-Kaon show flat energy dependence.
2) More statistics are needed at low energies.
Theoretical and Model Calculations

Motivation:

1): Signals from Criticality.
- NJL, VDW liquid-gas EoS, PQM, σ model etc.
- DSE, Lattice QCD.

Theoretical predictions are important for us!

2) Study non-CP background in HIC.
Transport model: UrQMD, AMPT, JAM etc.
Non-monotonic energy dependence is observed for 4\textsuperscript{th} order net-proton fluctuations in most central Au+Au collisions.
NJL Model Calculations

Baryon (B)  Charge (Q)  Strangeness (S)

\[ C_3/C_2 \quad m_1(B) \]
\[ C_4/C_2 \quad m_2(B) \]

1) CP Signals from baryon fluctuations are much stronger than Q and S.
2): Forth and third order fluctuations have very different behavior.

Comparison Between NJL Model and STAR Data

\[ m_1 = \frac{C_3}{C_2} \]
\[ m_2 = \frac{C_4}{C_2} \]

Along the assumed freeze-out curve, the NJL Model can qualitatively describe the non-monotonic behavior observed in data.

Model Calculations

Effects of Deuteron Formation

At $\sqrt{s_{NN}} \leq 10$ GeV: Data: $\kappa \sigma^2 > 1$ Model: $\kappa \sigma^2 < 1$

- Model simulation indicates: *Baryon conservations*, *Mean-field potential*, *Deuteron formation*, *Softening of EOS*. *All suppress the net-proton fluctuations.*


Acceptance Dependence : Test Power Law Behavior

Acceptance dependence of the critical contribution

\[ C_1 = \langle N \rangle \]
\[ C_2 = \langle N \rangle + \hat{\kappa}_2 \]
\[ C_3 = \langle N \rangle + 3\hat{\kappa}_2 + \hat{\kappa}_3 \]
\[ C_4 = \langle N \rangle + 7\hat{\kappa}_2 + 6\hat{\kappa}_3 + \hat{\kappa}_4 \]

\[ \hat{\kappa}_2, \hat{\kappa}_3, \hat{\kappa}_4 : 2,3,4\text{-particle correlation function} \]

Generating function for the factorial cumulants: (corr. fun.):
\[ g(x) \equiv \sum_{k=1}^{\infty} \hat{\kappa}_k \frac{x^k}{k!} = \ln \langle (1 + x)^N \rangle. \]

\[ \Delta y \text{corr} \text{ : The correlation range in rapidity} \]

Adam\&Volker, arXiv:1607.07375

If \( \Delta y \ll \Delta y \text{corr} \):
\[ C_n \propto \hat{\kappa}_n \propto \langle N \rangle^n \sim (\Delta y)^n \]

If \( \Delta y \gg \Delta y \text{corr} \):
\[ C_n \propto \hat{\kappa}_n \propto \langle N \rangle \sim (\Delta y) \]
Net-Proton Fluctuations vs. Rapidity

STAR Data: 0-5% Au+Au Collisions at 7.7 GeV
- $0.4 < p_T < 2 \text{ GeV/c}$
- $0.4 < p_T < 0.8 \text{ GeV/c}$
- BES-II Error

$f = 1 - p_0 x + p_1 x^3$

1) BES-I results: Poisson + Baryon conservation + $v^3$, criticality?

2) BES-II: iTPC extend the rapidity coverage to $\Delta y = 1.6$, allowing to studying kinematic dependence and precision measurement of higher moments

Without the four particle correlation, the non-monotonic behavior observed in forth order net-proton fluctuations disappears.
- The cumulants can be strongly suppressed due to baryon conservations. A. Bzdak, V. Koch, V. Skokov, Phys. Rev. C 87 (2013) 014901.
- The two and three-proton correlation function are negative.
1) Event statistics driven by QCD CP search and di-electron measurements.
2) The STAR Fix-target mode is also planned in BESII. ($\sqrt{s_{NN}}$: 4.5, 3.9, 3.6, 3.0 GeV)
Future Experiments for High Baryon Density

Longer future: search for the “peak structure” at lower energies $350 < \mu_B < 750$ MeV ($2 < \sqrt{s_{NN}} < 8$ GeV). FXT experiment is more effective.
We show cumulants of net-P, net-K and net-Q for Au+Au collisions at 7.7, 11.5, 14.5, 19.6, 27, 39, 62.4 and 200 GeV.

Non-monotonic energy dependence is observed at central Au+Au collisions for net-proton kurtosis, which is consistent with the presence of critical point. Observation of the criticality?

Acceptance Studies: Looking for the power law behavior. Understand the background contribution to corr. func.

Study the QCD phase structure at high baryon density with high precision:

(1) BES-II at RHIC (2019-2020, both collider and fix target mode).
(2) Fix-target at low energies: FAIR/CBM(starting at 2022), JPARC.
Thank you!