NEUTRON STAR RADII AND CORE-CRUST TRANSITION

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Hyperons

- reduce the pressure in the inner core. i.e. softening of the EoS;
- reduce the maximum mass.

Claims that \( M_{\text{max}} \geq 2 \, M_\odot \) rules out hyperonic equations of state ...
Hyperonic equations of state and radii


$M - R$ plot

- 14 hyperonic EoSs (Y.EoSs), all consistent with $M_{\text{max}} \geq 2 \, M_\odot$, all but one (Yamamoto et al. PRC 2014) are RMF models;
- 3 nucleonic ones (N.EoSs) as reference.

Y.EoSs

for $M \in [1.0 - 1.6] \, M_\odot$, $R > 13$ km.

Nucleonic EoSs
Hyperonic EoSs
Hyperonic equations of state and radii


Pressure at $n_0 = 0.16 \text{ fm}^{-3}$ near the core-crust transition

$\nabla$ large radius for Y.EoSs correlated with a large pressure at $n_0$.

$\rightarrow M_{\text{max}} \geq 2 M_\odot$ possible if the decrease in the pressure at high density due to Y is compensated by a large pressure at low density.
Hyperonic equations of state and radii


Pressure at \( n_0 = 0.16 \text{ fm}^{-3} \) near the core-crust transition

- large radius for Y.EoS correlated with a large pressure at \( n_0 \).
- \( M_{\text{max}} \geq 2 M_\odot \) possible if the decrease in the pressure at high density due to Y is compensated by a large pressure at low density.
- over-pressure at \( n_0 \) for hyperonic EOS inconsistent with this constraint.

Recent work

Oertel et al. JPG (2015): hyperonic EoS consistent with Hebeler et al. constraint and with \( M_{\text{max}} \geq 2 M_\odot \).
How to glue core and crust: NL3 & DH?
Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, arXiv:1604.01944

- core glued to BPS+BBP EOS at 0.01 fm$^{-3}$;
- transition at the crossing density between the 2 EoSs;
- transition at the core-crust transition density $n_t$;
- transition at $n_0 = 0.16$ fm$^{-3}$;
- crust below $0.5n_0$ and core above $n_0$;
- crust below $0.1n_0$ and core above $n_t$;
- reference: unified EoS.

Uncertainty on $R$

- due to the treatment of the core-crust transition: up $\sim 4\%$ (up to $\sim 30\%$ on the crust thickness),
- decreases if crust and core EOS with similar saturation properties.
How to glue core and crust: NL3 & DH?
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### Uncertainty on $R$
- due to the treatment of the core-crust transition: up $\sim 4\%$
- with NICER, Athena or LOFT(?): expected precision $\sim 5\%$ . . . .
- how to, if not solve, at least handle this problem?
1. Thermodynamically consistent ‘gluing’

In principle one should match $P, \rho, n$.

$n = \left( \frac{dP}{d\mu} \right), \rho(\mu) = n(\mu)\mu - P(\mu)$.

- thermodynamic consistency: $n$ is an increasing function of $P \leftrightarrow P(\mu)$ is increasing and convex:

  $n_1 < \frac{P_2 - P_1}{\mu_2 - \mu_1} < n_2$

- causality:

  $(dP/d\rho)^{1/2} = v_{\text{sound}}/c \leq 1$
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2. Unified equations of state

Very few unified EoSs for NSs exist eg. DH (Douchin & Haensel 2001), BSk (Brussels Uni.)
Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, arXiv:1604.01944

9 RMF models

NL3, NL3$\omega\rho$, DDME2, GM1, TM1, DDH$\delta$, DD2, BSR2, and BSR6 with

- outer-crust non consistently calculated but hardly affect the $M - R$ relations
- inner-crust with pasta phase from Thomas-Fermi calculations
- noY: a purely nucleonic core
- Y: transition to hyperonic matter in the core: SU(6) with the $\phi$ meson;
  $U^N_\Lambda(n_0) = -28$ MeV, $U^N_\Sigma(n_0) = 30$ MeV, $U^N_\Xi(n_0) = -18$ MeV
- Yss: transition to hyperonic matter in the core: SU(6) with the $\phi$ and $\sigma^*$ mesons;
  $U^A_\Lambda(n_0) = -5$ MeV, $U^\Xi_\Xi \simeq 2U^A_\Lambda$, $g_{\sigma^*\Sigma} = g_{\sigma^*\Lambda}$

24 Skyrme models

Ska, Skb, Skl2, Skl3, Skl4, Skl5, Skl6, SLy2, SLy230a, SLy9, SkMP, SkOp, KDE0V, KDE0V1, Sk255, Sk272, Rs, BSk20, BSk21, BSk22, BSk23, BSk24, BSk25, and BSk26 with

- purely nucleonic core, causal up to $2M_\odot$
- compressible liquid drop model
- no shell effect and curvature terms
2. Unified equations of state

33 nucleonic EoSs and 15 hyperonic EoSs

- tables with $n$, $\rho$, $P$ as supplemental material to the paper
- available on the open-source CompOSE database: http://compose.obspm.fr/
Comparison with nuclear constraints

1. Low-density

Gandolfi et al. PRC (2012): Quantum Monte Carlo technique

- RMF: DDME2; $\pm 10\%$: NL3$\omega\rho$, DD2
- Skyrme: SLy2, KDE0v, KDE0v1; $\pm 10\%$: SLy9

2. Incompressibility

$K = 230 \pm 40$ MeV: all but GM1 and TM1
Comparison with nuclear constraints

3. $L - J$ plane

- neutron skin thickness of $^{208}$Pb
- heavy ion collisions (HIC)
- electric dipole polarizalibility $\alpha_D$

References: see eg. Lattimer & Steiner, EPJA (2015)
Comparison with nuclear constraints

EOS fulfilling:

- all constraints 1.+2.+3.: DDME2
- constraint 1.±10%+ constraints 2.+3.: DD2, NL3ωρ and SLy9.

\[ R_{1.4} = 13.10 \pm 0.65 \text{ km}. \]
Correlations

- $R$ vs $L$: dispersion larger for higher masses due to higher order terms
- $R$ vs $K$: for isoscalar properties higher order terms can not be neglected
Nucleonic DUrca process

- $n \rightarrow p + e^- + \bar{\nu}_e$ and $p + e^- \rightarrow n + \nu_e$
- momentum conservation $\rightarrow$ density $n_{\text{DU}}$ and mass $M_{\text{DU}}$ threshold

Beznogov & Yakovlev MNRAS (2015): DUrca process needed to explain the thermal emission of isolated and accreting NS.


For $L \gtrsim 70$ MeV, DUrca process always on for $M > 1.5 \, M_\odot$.

For $L \lesssim 70$ MeV, EOS with DUrca and others without.

$L - J$ plane: the intersection of all constraints gives $L \lesssim 70$ MeV.

Additional DUrca processes for hyperonic EOS.
3. Approximate formula for the radius and crust thickness

Zdunik, Fortin, and Haensel, in prep.

Thickness of a shell in a catalyzed crust

Assuming that in the crust $m \approx M$ and $4\pi r^3 P/mc^2 \ll 1$ in the TOV equation one obtains:

$$\frac{dP}{\rho + P/c^2} = -\frac{GM}{r^2 (1 - 2GM/rc^2)} \frac{dr}{r}.$$ 

With

$$\frac{dP}{\rho c^2 + P} = \frac{d\mu}{\mu}$$

one gets

$$\frac{\sqrt{1 - 2GM/r_2 c^2}}{\sqrt{1 - 2GM/r_1 c^2}} = \frac{\mu_2}{\mu_1}$$

valid for no jump in the chemical potential.
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$$\frac{dP}{\rho c^2 + P} = \frac{d\mu}{\mu} \quad \text{one gets} \quad \frac{\sqrt{1 - 2GM/r_2c^2}}{\sqrt{1 - 2GM/r_1c^2}} = \frac{\mu_2}{\mu_1}$$

valid for no jump in the chemical potential.

Taking $r_1 = R$ and $r_2 = R_{\text{core}}$

$$\frac{\sqrt{1 - 2GM/Rc^2}}{\sqrt{1 - 2GM/R_{\text{core}}c^2}} = \frac{\mu_b}{\mu_0}$$

with $\mu_0 = \mu(P = 0) = 930.4$ MeV - minimum energy per nucleon of a bcc lattice of $^{56}\text{Fe}$ and $\mu_b$ at the core-crust transition.
3. Approximate formula for the radius and crust thickness

Zdunik, Fortin, and Haensel, in prep.

- All you need is . . . : the core EOS down to a chosen density \( n_b \) with \( \mu(n_b) = \mu_b \).
- Obtain the \( M(R_{\text{core}}) \) relation solving the TOV equations.
- Obtain \( M(R) \) with
  \[
  R = R_{\text{core}} \sqrt{1 - \left( \frac{\mu_b}{\mu_0} - 1 \right) \left( \frac{R_{\text{core}} c^2}{2GM} - 1 \right)}.
  \]

Results

- uncertainty in the radius: \( \lesssim 0.2\% \) for \( M > 1 M_\odot \)
- uncertainty in the crust thickness: \( \lesssim 1\% \) for \( M > 1 M_\odot \)

Solution of the TOV equation with a unified EoS

TOV solution for the core \( M(R_{\text{core}}) \)

Approximate \( M(R) \) for \( n_b = 0.077 \text{ fm}^{-3} \)
3. Approximate formula for the radius and crust thickness

Zdunik, Fortin, and Haensel, in prep.

- All you need is . . . : the core EOS down to a chosen density $n_b$ with $\mu(n_b) = \mu_b$.
- Obtain the $M(R_{\text{core}})$ relation solving the TOV equations.
- Obtain $M(R)$ with
  \[ R = \frac{R_{\text{core}}}{\left(1 - \left(\frac{\mu_b^2}{\mu_0^2} - 1\right)\left(\frac{R_{\text{core}} c^2}{2GM} - 1\right)\right)}. \]

How to choose the core-crust transition density $n_b$?

- Inversely proportional to $L$ (Horowitz & Piekarewicz 2001)
- Ducoin et al. (2011): for EOS with $30 \leq L \leq 120$ MeV, obtain:
  \[ 0.06 \lesssim n_b \lesssim 0.10 \text{ fm}^{-3} \]
  \[ \Rightarrow n_b \approx n_0/2 = 0.08 \text{ fm}^{-3} \]

Solution of the TOV equation with a unified EoS

TOV solution for the core $M(R_{\text{core}})$

Approximate $M(R)$ for $n_b = 0.16, 0.13, 0.11, 0.09, 0.077$ fm$^{-3}$ from left to right.
3. Approximate formula for the radius and crust thickness
Zdunik, Fortin, and Haensel, in prep.

Thickness of a shell in an accreted crust

For a catalyzed crust

\[ \frac{\sqrt{1 - \frac{2GM}{r_2 c^2}}}{\sqrt{1 - \frac{2GM}{r_1 c^2}}} = \frac{\mu_2}{\mu_1} \]

For an accreted crust

\[ \frac{\sqrt{1 - \frac{2GM}{Rc^2}}}{\sqrt{1 - \frac{2GM}{R_1 c^2}}} = \frac{\mu_1^+}{\mu_0} \]

\[ \ldots \]

\[ \frac{\sqrt{1 - \frac{2GM}{R_{i+1} c^2}}}{\sqrt{1 - \frac{2GM}{R_i c^2}}} = \frac{\mu_{i+1}^+}{\mu_i} \]

\[ \ldots \]

\[ \frac{\sqrt{1 - \frac{2GM}{R_{n} c^2}}}{\sqrt{1 - \frac{2GM}{R_{\text{core}} c^2}}} = \frac{\mu_b}{\mu_n} \]
3. Approximate formula for the radius and crust thickness
Zdunik, Fortin, and Haensel, in prep.

Thickness of a shell in an accreted crust

For a catalyzed crust

\[
\frac{\sqrt{1 - 2GM/r^2_c}}{\sqrt{1 - 2GM/r_1^2}} = \frac{\mu_2}{\mu_1}
\]

For an accreted crust

\[
\frac{\sqrt{1 - \frac{2GM}{R_c^2}}}{\sqrt{1 - \frac{2GM}{R_{\text{core}}^2}}} = \frac{\mu_1^+}{\mu_1} \cdot \frac{\mu_2^+}{\mu_2} \cdots \frac{\mu_i^+}{\mu_i} \cdots \frac{\mu_n^+}{\mu_n} \cdot \frac{\mu_b}{\mu_0}
\]

\[
= \frac{\mu_b}{\mu_0} \cdot \prod_{i=1}^{n} \frac{\mu_i^+}{\mu_i}
\]

Energy release at \( P_i \): \( Q_i = \mu_i^+ - \mu_i \).

\[
\frac{\sqrt{1 - \frac{2GM}{R_c^2}}}{\sqrt{1 - \frac{2GM}{R_{\text{core}}^2}}} \simeq \frac{\mu_b}{\mu_0} \cdot (1 + \frac{Q_{\text{tot}}}{\mu_{\text{IC}}})
\]

with \( Q_{\text{tot}} = \sum_{i=1}^{n} Q_i \) the total energy release in the crust and the mean chemical potential in the inner-crust
\( \mu_{\text{IC}} \simeq 941 \) MeV.
3. Approximate formula for the radius and crust thickness
Zdunik, Fortin, and Haensel, in prep.

Catalyzed vs. accreted crusts

\[
\sqrt{1 - \frac{2GM}{Rc^2}} \approx \frac{\mu_b}{\mu_0} \cdot \left(1 + \frac{Q_{\text{tot}}}{\mu_{\text{IC}}}\right)
\]

Radius of a star with a catalyzed crust: \( R_{\text{cat}} \)
with an accreted crust \( R_{\text{acc}} \).

\[
R_{\text{acc}} = \frac{R_{\text{cat}}}{1 - (\alpha - 1)\left(\frac{R_{\text{cat}}c^2}{2GM} - 1\right)}
\]

with \( \sqrt{\alpha} \equiv \prod_{i=1}^{n} \frac{\mu_i^+}{\mu_i} = (1 + \frac{Q_{\text{tot}}}{\mu_{\text{IC}}}) \).

Difference in the radius between a NS with an accreted crust and a catalyzed crust

\[
\Delta R \simeq 144 \text{ m} \cdot \left(\frac{Q_{\text{tot}}}{2 \text{ MeV}}\right) \left(\frac{R_{\text{cat}}}{10 \text{ km}}\right)^2 \left(\frac{M}{M_{\odot}}\right) \left(1 - \frac{2GM}{R_{\text{cat}}c^2}\right)
\]
Conclusions

- Most hyperonic EoSs consistent with $2 M_\odot$ have a large $R_{1.4}$ and overpressure close to saturation density (Fortin+, A&A 2015)
- Treatment of the gluing of non-unified core and crust EoSs introduces an uncertainty on the radius that can be as large as the expected precision from NICER, Athena or LOFT(?) (Fortin+, arXiv:1604.01944)
- Development of unified nucleonic and hyperonic EoSs based on 9 RMF and 24 Skyrme models (Fortin+, arXiv:1604.01944);
- available on the CompOSE database: http://compose.obspm.fr and as supplemental material to the paper;
- Approximate formula for $M(R)$ as a function of $M(R_{\text{core}})$ for catalyzed and accreted crusts. (Zdunik+, in prep.)

Perspectives

- Fits by piecewise polytropes of the unified EOS for various applications.
- Study of rotating NS (Keplerian frequency, minimum mass, ...) with LORENE and of the surface gravitational redshift (spectral lines).
- Development of more EOS consistent with Hebeler et al. constraint and $2 M_\odot$. 
1. Thermodynamically consistent ‘gluing’

Example: as a function of $n$

Matched EOS

For $n_1 < n < n_2$:

- crust: SLy4 for $n \leq n_1 = 0.076 \text{ fm}^{-3}$,
- core: NL3 for $n \geq n_2 = 0.1 \text{ fm}^{-3}$
1. Thermodynamically consistent ‘gluing’

Example: as a function of $n$

Matched EOS

For $n_1 < n < n_2$:

- assume a form for $P(n)$.
- $\mu(n) = \mu(n_1) + \int_{n_1}^{n} \frac{dP(n)}{n}$
- $\rho(n) = n\mu(n) - P(n)$

- crust: SLy4 for $n \leq n_1 = 0.076 \text{ fm}^{-3}$,
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- $\rho(n) = n\mu(n) - P(n)$

- but in general jump in $\mu$ at $n_2$
- for $n > n_2$, $\mu(n) \rightarrow \mu(n) + (\mu(n_2) - \mu_{co}(n_2))$
- for $n > n_2$, $\rho(n) \rightarrow \rho(n) + (n - n_2)(\mu(n_2) - \mu_{co}(n_2))$.

But $P(\rho)$ enters the TOV equation
$\rightarrow M - R$ relation affected.

$\triangleright$ crust: SLy4 for $n \leq n_1 = 0.076$ fm$^{-3}$,
$\triangleright$ core: NL3 for $n \geq n_2 = 0.1$ fm$^{-3}$
1. Thermodynamically consistent ‘gluing’

Example: as a function of $\rho$

Matched EOS
For $\rho_1 < \rho < \rho_2$:

- crust: SLy4 for $\rho \leq \rho_1 = 1.27 \times 10^{14}$ g cm$^{-3}$,
- core: NL3 for $\rho \geq \rho_2 = 1.69 \times 10^{14}$ g cm$^{-3}$
1. Thermodynamically consistent ‘gluing’

Example: as a function of $\rho$

\[ P(\rho) \text{ (dyn cm}^{-2}) \]
\[ \mu \text{ (MeV)} \]
\[ n \text{ (fm}^{-3}) \]

\[ \rho \text{ (}10^{14}\text{ g cm}^{-3}) \]

Matched EOS

For $\rho_1 < \rho < \rho_2$:

- assume a form for $P(\rho)$.

\[ n(\rho) = n_1 \exp \left( \int_{\rho_1}^{\rho} \frac{d\rho}{P(\rho)+\rho} \right). \]

- crust: SLy4 for $\rho \leq \rho_1 = 1.27 \times 10^{14} \text{ g cm}^{-3}$,
- core: NL3 for $\rho \geq \rho_2 = 1.69 \times 10^{14} \text{ g cm}^{-3}$
1. Thermodynamically consistent ‘gluing’

Example: as a function of $\rho$

![Graph showing $P(\rho)$, $\mu$, and $n(\rho)$ as functions of $\rho$.]

**Matched EOS**

For $\rho_1 < \rho < \rho_2$:
- assume a form for $P(\rho)$.
- $n(\rho) = n_1 \exp \left( \int_{\rho_1}^{\rho} \frac{d\rho}{P(\rho)+\rho} \right)$.
- but in general jump in $\mu$ at $\rho_2$
- for $\rho > \rho_2$, $n(\rho) = n_{\text{co}}(\rho)n(\rho_2)/n_{\text{co}}(\rho_2)$.

$P(\rho)$ and $M - R$ relation unaffected but the microscopic approach given by $n(\rho)$ modified.

- crust: SLy4 for $\rho \leq \rho_1 = 1.27 \times 10^{14}$ g cm$^{-3}$,
- core: NL3 for $\rho \geq \rho_2 = 1.69 \times 10^{14}$ g cm$^{-3}$

Morgane Fortin (CAMK)