Isoscalar and Isovector Densities and Symmetry Energy

Pawel Danielewicz,¹ Pardeep Singh¹,² and Jenny Lee³

¹Natl Superconducting Cyclotron Lab, Michigan State U, ²Deenbandhu Chhotu Ram U Science & Techn, Murthal, India and ³U of Hong Kong

INT-16-2b Program Workshop
Laboratory and Astronomical Observations of Dense Matter

July 18-22, 2016, University of Washington, Seattle
Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under \( n \leftrightarrow p \) interchange

An isoscalar quantity \( F \) does not change under \( n \leftrightarrow p \) interchange. E.g. nuclear energy. Expansion in asymmetry \( \eta = (N - Z)/A \), for smooth \( F \), yields even terms only:

\[
F(\eta) = F_0 + F_2 \eta^2 + F_4 \eta^4 + \ldots
\]

An isovector quantity \( G \) changes sign. Example:
\[
\rho_{np}(r) = \rho_n(r) - \rho_p(r).
\]
Expansion with odd terms only:

\[
G(\eta) = G_1 \eta + G_3 \eta^3 + \ldots
\]

Note: \( G/\eta = G_1 + G_3 \eta^2 + \ldots \)

In nuclear practice, analyticity requires shell-effect averaging!

Charge invariance: invariance of nuclear interactions under rotations in \( n-p \) space
Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under $n \leftrightarrow p$ interchange

An isoscalar quantity $F$ does not change under $n \leftrightarrow p$ interchange. E.g. nuclear energy. Expansion in asymmetry $\eta = (N - Z)/A$, for smooth $F$, yields even terms only:

$$F(\eta) = F_0 + F_2 \eta^2 + F_4 \eta^4 + \ldots$$

An isovector quantity $G$ changes sign. Example: $\rho_{np}(r) = \rho_n(r) - \rho_p(r)$. Expansion with odd terms only:

$$G(\eta) = G_1 \eta + G_3 \eta^3 + \ldots$$

Note: $G/\eta = G_1 + G_3 \eta^2 + \ldots$

In nuclear practice, analyticity requires shell-effect averaging!

Charge invariance: invariance of nuclear interactions under rotations in $n$-$p$ space
Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under $n \leftrightarrow p$ interchange

An isoscalar quantity $F$ does not change under $n \leftrightarrow p$ interchange. E.g. nuclear energy. Expansion in asymmetry $\eta = (N - Z)/A$, for smooth $F$, yields even terms only:

$$F(\eta) = F_0 + F_2 \eta^2 + F_4 \eta^4 + \ldots$$

An isovector quantity $G$ changes sign. Example:

$$\rho_{np}(r) = \rho_n(r) - \rho_p(r).$$

Expansion with odd terms only:

$$G(\eta) = G_1 \eta + G_3 \eta^3 + \ldots$$

Note: $G/\eta = G_1 + G_3 \eta^2 + \ldots$

In nuclear practice, analyticity requires shell-effect averaging!

Charge invariance: invariance of nuclear interactions under rotations in $n-p$ space
Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under \( n \leftrightarrow p \) interchange

An isoscalar quantity \( F \) does not change under \( n \leftrightarrow p \) interchange. E.g. nuclear energy. Expansion in asymmetry \( \eta = (N - Z)/A \), for smooth \( F \), yields even terms only:

\[
F(\eta) = F_0 + F_2 \eta^2 + F_4 \eta^4 + \ldots
\]

An isovector quantity \( G \) changes sign. Example:
\[
\rho_{np}(r) = \rho_n(r) - \rho_p(r).
\]
Expansion with odd terms only:

\[
G(\eta) = G_1 \eta + G_3 \eta^3 + \ldots
\]

Note: \( G/\eta = G_1 + G_3 \eta^2 + \ldots \)

In nuclear practice, analyticity requires shell-effect averaging!

Charge invariance: invariance of nuclear interactions under rotations in \( n-p \) space
Charge Symmetry & Charge Invariance

**Charge symmetry:**

\( n \leftrightarrow p \) invariance

**Charge invariance:**

symmetry under rotations in \( n-p \) space

Isospin doublets

\[ p : (\tau, \tau_z) = \left( \frac{1}{2}, \frac{1}{2} \right) \]

\[ n : (\tau, \tau_z) = \left( \frac{1}{2}, -\frac{1}{2} \right) \]

Net isospin

\[ \vec{T} = \sum_{i=1}^{A} \vec{\tau}_i \]

\[ T = \frac{3}{2}, \ldots \quad T = \frac{1}{2}, \frac{3}{2}, \ldots \quad T = \frac{3}{2}, \ldots \]

Nuclear states:

\[ (T, T_z), \quad T \geq |T_z| = \frac{1}{2}|N - Z| \]

Isobars: Nuclei with the same \( A \)

\( A=7 \)

\( Z=2 \)
\( Z=3 \)
\( Z=4 \)
\( Z=5 \)
Charge Symmetry & Charge Invariance

Charge symmetry: $n \leftrightarrow p$ invariance

Charge invariance: symmetry under rotations in $n$-$p$ space

Isospin doublets

$p : (\tau, \tau_z) = (\frac{1}{2}, \frac{1}{2})$

$n : (\tau, \tau_z) = (\frac{1}{2}, -\frac{1}{2})$

Net isospin

$\vec{T} = \sum_{i=1}^{A} \vec{\tau}_i$

Nuclear states: $(T, T_z), \quad T \geq |T_z| = \frac{1}{2} |N - Z|$
Charge Symmetry & Charge Invariance

Charge symmetry: 
\( n \leftrightarrow p \) invariance

Charge invariance: 
symmetry under rotations in 
\( n-p \) space

Isospin doublets

\( p : (\tau, \tau_z) = (\frac{1}{2}, \frac{1}{2}) \)

\( n : (\tau, \tau_z) = (\frac{1}{2}, -\frac{1}{2}) \)

Net isospin

\[ \vec{T} = \sum_{i=1}^{A} \vec{\tau}_i \]

Nuclear states: 
\( (T, T_z), \quad T \geq |T_z| = \frac{1}{2}|N - Z| \)

Isobars: Nuclei with the same \( A \)

A=7

\[ Z=2 \quad Z=3 \quad Z=4 \quad Z=5 \]
Energy in Uniform Matter

\[
\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + S(\rho) \left( \frac{\rho_n - \rho_p}{\rho} \right)^2 + O(\ldots^4)
\]

symmetric matter (a)symmetry energy \( \rho = \rho_n + \rho_p \)

Known: \( a_a \approx 16 \text{ MeV} \quad K \sim 235 \text{ MeV} \)

Unknown: \( a_a^V \? \quad L \? \)
Importance of Slope

\[ \frac{E}{A} = \frac{E_0}{A} (\rho) + S(\rho) \left( \frac{\rho_n - \rho_p}{\rho} \right)^2 \]

\[ S \approx a_a^V + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0} \]

In neutron matter:
\[ \rho_p \approx 0 \& \rho_n \approx \rho. \]

Then,
\[ \frac{E}{A} (\rho) \approx \frac{E_0}{A} (\rho) + S(\rho) \]

Pressure:
\[ P = \rho^2 \frac{d}{d\rho} \frac{E}{A} \approx \rho^2 \frac{dS}{d\rho} \approx \frac{L}{3\rho_0} \rho^2 \]

43 \( \leq \) L \( \leq \) 60 MeV ??
Importance of Slope

\[
\frac{E}{A} = \frac{E_0}{A} (\rho) + S(\rho) \left( \frac{\rho_n - \rho_p}{\rho} \right)^2 \\
S \simeq a_a^V + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0}
\]

In neutron matter: 
\( \rho_p \approx 0 \) \& \( \rho_n \approx \rho \).

Then, \( \frac{E}{A}(\rho) \approx \frac{E_0}{A}(\rho) + S(\rho) \)

Pressure:
\[
P = \rho^2 \frac{d}{d\rho} \frac{E}{A} \simeq \rho^2 \frac{dS}{d\rho} \simeq \frac{L}{3\rho_0} \rho^2
\]

43 \( \lesssim L \lesssim 60 \) MeV ??
Isoscalar and Isovector Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N - Z)$ for given $A$. [Coulomb suppressed . . . ]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A \rho_{np}(r)/(N - Z)$ isoscalar!

$A/(N - Z)$ normalizing factor global . . . Similar local normalizing factor, in terms of intense quantities, $2a^V_a/\mu_a$, where $a^V_a \equiv S(\rho_0)$

Isoscalar formfactor for isovector density:

$$\rho_a(r) = \frac{2a^V_a}{\mu_a} \left[ \rho_n(r) - \rho_p(r) \right]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on $\eta$!

In any nucleus:

$$\rho_{n,p}(r) = \frac{1}{2} \left[ \rho(r) \pm \frac{\mu_a}{2a^V_a} \rho_a(r) \right]$$

where $\rho(r)$ & $\rho_a(r)$ have universal features! (subject to shell effects)

No shell-effects, $\rho$’s as dynamic vbles: Hohenberg-Kohn functional

Isoscalar Skin

Danielewicz, Singh, Lee
Isoscalar and Isovector Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N - Z)$ for given $A$. [Coulomb suppressed . . .]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A\rho_{np}(r)/(N - Z)$ isoscalar!

$A/(N - Z)$ normalizing factor global . . . Similar local normalizing factor, in terms of intense quantities, $2a^V_a/\mu_a$, where $a^V_a \equiv S(\rho_0)$

Isoscalar formfactor for isovector density:

$\rho_a(r) = \frac{2a^V_a}{\mu_a} [\rho_n(r) - \rho_p(r)]$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on $\eta$!

In any nucleus:

$\rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a^V_a} \rho_a(r)]$

where $\rho(r)$ & $\rho_a(r)$ have universal features! (subject to shell effects)

No shell-effects, $\rho$’s as dynamic vbles: Hohenberg-Kohn functional
Isoscalar and Isovector Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N - Z)$ for given $A$. [Coulomb suppressed...]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A \rho_{np}(r)/(N - Z)$ isoscalar!

$A/(N - Z)$ normalizing factor global. . . Similar local normalizing factor, in terms of intense quantities, $2a^V_a/\mu_a$, where $a^V_a \equiv S(\rho_0)$

Isoscalar formfactor for isovector density:

$$\rho_a(r) = \frac{2a^V_a}{\mu_a} [\rho_n(r) - \rho_p(r)]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on $\eta$!

In any nucleus:

$$\rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a^V_a} \rho_a(r)]$$

where $\rho(r)$ & $\rho_a(r)$ have universal features! (subject to shell effects)

No shell-effects, $\rho$’s as dynamic vbles: Hohenberg-Kohn functional
Isoscalar and Isovector Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N - Z)$ for given $A$. [Coulomb suppressed . . .]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A\rho_{np}(r)/(N - Z)$ isoscalar!

$A/(N - Z)$ normalizing factor global . . . Similar local normalizing factor, in terms of intense quantities, $2a_a^V/\mu_a$, where $a_a^V \equiv S(\rho_0)$

Isoscalar formfactor for isovector density:

$$\rho_a(r) = \frac{2a_a^V}{\mu_a} [\rho_n(r) - \rho_p(r)]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on $\eta$!

In any nucleus:

$$\rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a_a^V} \rho_a(r)]$$

where $\rho(r)$ & $\rho_a(r)$ have universal features! (subject to shell effects)

No shell-effects, $\rho$’s as dynamic vbles: Hohenberg-Kohn functional.
Isoscalar and Isovector Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N - Z)$ for given $A$. [Coulomb suppressed…]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A\rho_{np}(r)/(N - Z)$ isoscalar! $A/(N - Z)$ normalizing factor global… Similar local normalizing factor, in terms of intense quantities, $2a^V_a/\mu_a$, where $a^V_a \equiv S(\rho_0)$

Isoscalar formfactor for isovector density:

$$\rho_a(r) = \frac{2a^V_a}{\mu_a} [\rho_n(r) - \rho_p(r)]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on $\eta$!

In any nucleus:

$$\rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a^V_a} \rho_a(r)]$$

where $\rho(r)$ & $\rho_a(r)$ have universal features! (subject to shell effects)

No shell-effects, $\rho$’s as dynamic vbles: Hohenberg-Kohn functional
Isovector Density

\[ \rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a_a^v} \rho_a(r)] \]

Net density \( \rho \) usually parameterized w/Fermi function

\[ \rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r - R}{d}\right)} \quad \text{with} \quad R = r_0 A^{1/3} \]

Isovector density \( \rho_a \)?? Related to \( S(\rho) \)!

In uniform matter

\[ \mu_a = \frac{\partial E}{\partial (N - Z)} = \frac{\partial [S(\rho) \rho_{np}^2/\rho]}{\partial \rho_{np}} = \frac{2 S(\rho)}{\rho} \rho_{np} \]

\[ \Rightarrow \rho_a = \frac{2a_a^v}{\mu_a} \rho_{np} = \frac{a_a^v}{S(\rho)} \rho \]

\[ \Rightarrow \text{Hartree-Fock study of surface} \]
Isovector Density

\[ \rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a^V_a} \rho_a(r)] \]

Net density \( \rho \) usually parameterized w/Fermi function

\[ \rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{d}\right)} \quad \text{with} \quad R = r_0 A^{1/3} \]

Isovector density \( \rho_a \)?? Related to \( S(\rho) \)!

In uniform matter

\[ \mu_a = \frac{\partial E}{\partial (N - Z)} = \frac{\partial [S(\rho) \rho_{np}^2 / \rho]}{\partial \rho_{np}} = \frac{2 S(\rho)}{\rho} \rho_{np} \]

\[ \Rightarrow \rho_a = \frac{2a^V_a}{\mu_a} \rho_{np} = \frac{a^V_a \rho}{S(\rho)} \]

\[ \rightarrow \] Hartree-Fock study of surface
Isovector Density

\[
\rho_{n,p}(r) = \frac{1}{2} \left[ \rho(r) \pm \frac{\mu_a}{2a_a^V} \rho_a(r) \right]
\]

Net density \( \rho \) usually parameterized w/Fermi function

\[
\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{d}\right)}
\]

with \( R = r_0 A^{1/3} \)

Isovector density \( \rho_a \)

Related to \( S(\rho) \! \)

In uniform matter

\[
\mu_a = \frac{\partial E}{\partial (N - Z)} = \frac{\partial [S(\rho) \rho_{np}^2 / \rho]}{\partial \rho_{np}} = \frac{2 S(\rho)}{\rho} \rho_{np}
\]

\[
\Rightarrow \rho_a = \frac{2a_a^V}{\mu_a} \rho_{np} = \frac{a_a^V \rho}{S(\rho)}
\]

\( \rightarrow \) Hartree-Fock study of surface
Isovector Density

\[ \rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a_a} \rho_a(r)] \]

Net density \( \rho \) usually parameterized w/ Fermi function

\[ \rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{d}\right)} \quad \text{with} \quad R = r_0 A^{1/3} \]

Isovector density \( \rho_a \) Related to \( S(\rho) \)!

In uniform matter

\[ \mu_a = \frac{\partial E}{\partial(N-Z)} = \frac{\partial[S(\rho) \rho_{np}^2/\rho]}{\partial \rho_{np}} = \frac{2 S(\rho)}{\rho} \rho_{np} \]

\[ \Rightarrow \rho_a = \frac{2a_a^V}{\mu_a} \rho_{np} = \frac{a_a^V \rho}{S(\rho)} \]

\[ \Rightarrow \text{Hartree-Fock study of surface} \]
Half-Infinite Matter in Skyrme-Hartree-Fock

To one side infinite uniform matter & vacuum to the other

Wavefunctions: \( \Phi(\mathbf{r}) = \phi(z) e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \)

matter interior/exterior:
\[
\phi(z) \propto \sin(k_z z + \delta(\mathbf{k})) \\
\phi(z) \propto e^{-\kappa(\mathbf{k}) z}
\]

Discretization in \( \mathbf{k} \)-space. Set of 1D HF eqs solved using Numerov’s method until self-consistency:

\[
-\frac{d}{dz} \frac{\hbar^2}{2m^*(z)} \frac{d}{dz} \phi(z) + \left( \frac{\hbar^2 k_\perp^2}{2m^*(z)} + U(z) \right) \phi(z) = \epsilon(\mathbf{k}) \phi(z)
\]

PD&Lee, NPA818(09)36. Before: Farine et al, NPA338(80)86
Asymmetry Dependence of Net Density

Half-$\infty$ matter results for different Skyrme interactions and asymmetries

$$\eta = \frac{N-Z}{A}$$
Asymmetry Dependence of Isovector Density

$$\rho_a = \frac{2a_a^V}{\mu_a} (\rho_n - \rho_p)$$

Half-$$\infty$$ matter results for different Skyrme interactions and asymmetries

PD&Lee
NP818(09)36
Sensitivity to $S(\rho)$

Results for different Skyrme interactions in half-$\infty$ matter.

Isoscalar ($\rho = \rho_n + \rho_p$; blue) and isovector ($\rho_n - \rho_p$; green) densities displaced relative to each other.

As $S(\rho)$ changes, so does displacement.
Sensitivity to $S(\rho)$

Results from different Skyrme interactions in half-$\infty$ matter.

Isoscalar ($\rho=\rho_n+\rho_p$; blue) & isovector ($\rho_n-\rho_p$; green) densities displaced relative to each other.

As $S(\rho)$ changes, so does displacement.
Strategies for Independent Densities

Jefferson Lab
Direct: $\sim p$
Interference: $\sim n$

PD
elastic: $\sim p + n$
charge exchange: $\sim n - p$
**Why Isovector Rather than Neutron Skins**

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!
Not suppressed by low \((N-Z)/A!\)

Nucleon (Lane) optical potential in isospin space:

\[
U = U_0 + \frac{4\tau T}{A} U_1
\]

isoscalar potential \(U_0 \propto \rho\), isovector potential \(U_1 \propto (\rho_n - \rho_p)\)
In elastic scattering \(U = U_0 \pm \frac{N-Z}{A} U_1\)
In quasielastic charge-exchange (p,n) to IAS: \(U = \frac{4\tau - T}{A} U_1\)
Elastic scattering dominated by \(U_0\)
Quasielastic governed by \(U_1\)
Geometry usually assumed the same for \(U_0\) and \(U_1\)
E.g. Koning & Delaroche NPA713(03)231

?Isovector skin \(\Delta R\) from comparison of elastic and quasielastic (p,n)-to-IAS scattering?
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!
Not suppressed by low \((N - Z)/A\)!

Nucleon (Lane) optical potential in isospin space:

\[ U = U_0 + \frac{4\tau T}{A} U_1 \]

isoscalar potential \(U_0 \propto \rho\), isovector potential \(U_1 \propto (\rho_n - \rho_p)\)

In elastic scattering \(U = U_0 \pm \frac{N-Z}{A} U_1\)

In quasielastic charge-exchange (p,n) to IAS: \(U = \frac{4\tau - T_+}{A} U_1\)

Elastic scattering dominated by \(U_0\)
Quasielastic governed by \(U_1\)
Geometry usually assumed the same for \(U_0\) and \(U_1\)
e.g. Koning & Delaroche NPA713(03)231

?Isovector skin \(\Delta R\) from comparison of elastic and quasielastic (p,n)-to-IAS scattering?
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!

Not suppressed by low \((N - Z)/A\)!

Nucleon (Lane) optical potential in isospin space:

\[
U = U_0 + \frac{4\tau T}{A} U_1
\]

isoscalar potential \(U_0 \propto \rho\), isovector potential \(U_1 \propto (\rho_n - \rho_p)\)

In elastic scattering \(U = U_0 \pm \frac{N-Z}{A} U_1\)

In quasielastic charge-exchange \((p,n)\) to IAS: \(U = \frac{4\tau - T_+}{A} U_1\)

Elastic scattering dominated by \(U_0\)
Quasielastic governed by \(U_1\)

Geometry usually assumed the same for \(U_0\) and \(U_1\)
e.g. Koning & Delaroche NPA713(03)231

? Isovector skin \(\Delta R\) from comparison of elastic and quasielastic \((p,n)\)-to-IAS scattering?
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!
Not suppressed by low \((N - Z)/A\)!

Nucleon (Lane) optical potential in isospin space:

\[
U = U_0 + \frac{4\tau T}{A} U_1
\]

isoscalar potential \(U_0 \propto \rho\), isovector potential \(U_1 \propto (\rho_n - \rho_p)\)

In elastic scattering \(U = U_0 \pm \frac{N-Z}{A} U_1\)

In quasielastic charge-exchange (p,n) to IAS: \(U = \frac{4\tau - T_\pm}{A} U_1\)

Elastic scattering dominated by \(U_0\)
Quasielastic governed by \(U_1\)

Geometry usually assumed the same for \(U_0\) and \(U_1\)
e.g. Koning & Delaroche NPA713(03)231

?Isovector skin \(\Delta R\) from comparison of elastic and quasielastic (p,n)-to-IAS scattering?
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!
Not suppressed by low \((N - Z)/A\)!

Nucleon (Lane) optical potential in isospin space:

\[ U = U_0 + \frac{4\tau T}{A} U_1 \]

isoscalar potential \(U_0 \propto \rho\), isovector potential \(U_1 \propto (\rho_n - \rho_p)\)

In elastic scattering \(U = U_0 \pm \frac{N-Z}{A} U_1\)
In quasielastic charge-exchange \((p,n)\) to IAS: \(U = \frac{4\tau - T_+}{A} U_1\)
Elastic scattering dominated by \(U_0\)
Quasielastic governed by \(U_1\)

Geometry usually assumed the same for \(U_0\) and \(U_1\)
e.g. Koning & Delaroche NPA713(03)231
?Isovector skin \(\Delta R\) from comparison of elastic and quasielastic \((p,n)\)-to-IAS scattering?
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!
Not suppressed by low \((N - Z)/A\)!

Nucleon (Lane) optical potential in isospin space:

\[
U = U_0 + \frac{4\tau T}{A} U_1
\]

isoscalar potential \(U_0 \propto \rho\), isovector potential \(U_1 \propto (\rho_n - \rho_p)\)

In elastic scattering \(U = U_0 \pm \frac{N-Z}{A} U_1\)

In quasielastic charge-exchange \((p,n)\) to IAS: \(U = \frac{4\tau - T_\pm}{A} U_1\)

Elastic scattering dominated by \(U_0\)

Quasielastic governed by \(U_1\)

Geometry usually assumed the same for \(U_0\) and \(U_1\)

e.g. Koning & Delaroche NPA713(03)231

Isovector skin \(\Delta R\) from comparison of elastic and quasielastic \((p,n)\)-to-IAS scattering?
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!
Not suppressed by low \((N-Z)/A\)!

Nucleon (Lane) optical potential in isospin space:

\[ U = U_0 + \frac{4\tau T}{A} U_1 \]

isoscalar potential \(U_0 \propto \rho\), isovector potential \(U_1 \propto (\rho_n - \rho_p)\)

In elastic scattering \(U = U_0 \pm \frac{N-Z}{A} U_1\)

In quasielastic charge-exchange (p,n) to IAS: \(U = \frac{4\tau T \mp T_+}{A} U_1\)

Elastic scattering dominated by \(U_0\)
Quasielastic governed by \(U_1\)

Geometry usually assumed the same for \(U_0\) and \(U_1\)
e.g. Koning & Delaroche NPA713(03)231

?Isovector skin \(\Delta R\) from comparison of elastic and quasielastic (p,n)-to-IAS scattering?
Expectations on Isovector Skin?

Much Larger Than Neutron!
Surface radius \( R \approx \sqrt{\frac{5}{3}} \langle r^2 \rangle^{1/2} \)
rms neutron skin
\[
\langle r^2 \rangle_{\rho_n}^{1/2} - \langle r^2 \rangle_{\rho_p}^{1/2} \\
\approx 2 \frac{N - Z}{A} \left[ \langle r^2 \rangle_{\rho_n - \rho_p}^{1/2} - \langle r^2 \rangle_{\rho_n + \rho_p}^{1/2} \right]
\]
	ext{rms isovector skin}

Estimated \( \Delta R \approx 3 \left( \langle r^2 \rangle_{\rho_n}^{1/2} - \langle r^2 \rangle_{\rho_p}^{1/2} \right) \) for \(^{48}\text{Ca}/^{208}\text{Pb}\)!
Even before consideration of Coulomb effects that further enhances difference!
Direct Reaction Primer

**DWBA:**

$$\frac{d\sigma}{d\Omega} \propto \left| \int dr \psi_f^* U_1 \psi_i \right|^2$$

- Oscillations: 2-side interference/source size
- Fall-off: softness of source
- Filling of minimae: imaginary/real contributions, spin-orbit

---

**Doering et al**

*Phys Rev C 12, 378 (1975)*

---

**$^{48}$Ca**

QE (p,n)

$E_p = 35$ MeV

---

---

Isovector Skin

Danielewicz, Singh, Lee
Potentials Fit to Elastic in Quasielastic

E.g. Koning-Delaroche NPA713(03)231 same radii for neutrons/protons, isoscalar/isovector, focus on p elastic

\[ U_0 + \frac{N-Z}{A} U_1 \]

\[ \begin{align*}
\text{Elastic scattering} \\
\text{E}_p = 35 \text{ MeV} \\
\theta_{cm} \text{ (deg)}
\end{align*} \]

\[ \begin{align*}
\text{QuasiElastic} (p,n) \\
\theta_{cm} \text{ (deg)}
\end{align*} \]
Effect of Changing Isovector Radius

Koning-Delaroche
NPA713(03)231
same radii $R$ for $U_0$ & $U_1$!

$$U_1(r) \propto \frac{U_{01}}{1 + \exp \frac{r-R}{a}}$$

$R \to R + \Delta R_1$

charge-exchange cs oscillations grow
Effect of Changing Isoscalar Radius

Koning-Delaroche
NPA713(03)231
same radii $R$ for $U_0$ & $U_1$!

$$U_0(r) \propto \frac{U_{00}}{1 + \exp \frac{r-R}{a}}$$

$R \rightarrow R + \Delta R_0$

charge-exchange cs oscillations shrink
Impact of \( U \)-Radii on (p,n) Cross Section

\[
\frac{d\sigma}{d\Omega} \propto \left| \int dr \, \psi_p^*(r) \, U_1(r) \, \psi_n(i) \right|^2
\]

Isoscalar radius responsible for holes in wavefunctions \( \Psi \)

Isovector radius responsible for region where (p,n) conversion can occur
Modified Koning-Delaroche Fits: $^{48}$Ca

In Koning-Delaroche: $R_{0,1} = R + \Delta R_{0,1}$, $a_{0,1} = a + \Delta a_{0,1}$
Modified Koning-Delaroche Fits: $^{90}$Zr

In Koning-Delaroche:

\[ R_{0,1} = R + \Delta R_{0,1} \quad a_{0,1} = a + \Delta a_{0,1} \]
Modified Koning-Delaroche Fits: $^{120}\text{Sn}$

In Koning-Delaroche:

$$R_{0,1} = R + \Delta R_{0,1}, \quad a_{0,1} = a + \Delta a_{0,1}$$
Modified Koning-Delaroche Fits: $^{208}\text{Pb}$

In Koning-Delaroche: $R_{0,1} = R + \Delta R_{0,1}$, $a_{0,1} = a + \Delta a_{0,1}$
Size of Isovector Skin

Large $\sim 0.9\text{ fm skins}$! $\sim$ Independent of $A$...
Sharper isovector surface!
Constraints on Symmetry-Energy Parameters

![Graph showing constraints on symmetry-energy parameters.](image)
Constraints on $S(\rho)$

- $S$ (MeV)
- $\rho$ (fm$^{-3}$)
Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density.

- For large $A$, displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy.

- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions.

- Such an analysis produces large isovector skins $\Delta R \sim 0.9$ fm!

- Symmetry energy is stiff! $L = (65 - 90)$ MeV, $a^V_a = (33 - 36)$ MeV

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh et al
US PHY-1403906 + Indo-US Grant
Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density.

- For large $A$, displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy.

- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions.

- Such an analysis produces large isovector skins $\Delta R \sim 0.9\text{ fm}$.

- Symmetry energy is stiff! $L = (65 - 90)\text{ MeV}$, $a^V_a = (33 - 36)\text{ MeV}$.

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh et al
US PHY-1403906 + Indo-US Grant
Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density.
- For large $A$, displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy.
- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions.
  - Such an analysis produces large isovector skins: $\Delta R \sim 0.9$ fm.
  - Symmetry energy is stiff! $L = (65 - 90)$ MeV, $a^V_A = (33 - 36)$ MeV.

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh et al.
US PHY-1403906 + Indo-US Grant.
Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density.
- For large $A$, displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy.
- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions.
- Such an analysis produces large isovector skins, $\Delta R \sim 0.9$ fm.

Symmetry energy is stiff! $L = (65 - 90)$ MeV, $a_V^A = (33 - 36)$ MeV.

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh et al.
US PHY-1403906 + Indo-US Grant.
Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density.

- For large \( A \), displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy.

- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions.

- Such an analysis produces large isovector skins
  \[ \Delta R \sim 0.9 \text{ fm!} \]

- Symmetry energy is stiff! \( L = (65 - 90) \text{ MeV}, \ a^V_a = (33 - 36) \text{ MeV} \)

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh et al
US PHY-1403906 + Indo-US Grant
Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density.
- For large $A$, displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy.
- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions.
- Such an analysis produces large isovector skins $\Delta R \sim 0.9$ fm!
- Symmetry energy is stiff! $L = (65 - 90)$ MeV, $a_a^V = (33 - 36)$ MeV

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh et al
US PHY-1403906 + Indo-US Grant