Nuclear Pasta Observables

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Neutron stars

- The crust is a crystalline lattice, while the core is uniform nuclear matter, like a nucleus. What’s in between these two phases?
Non-Spherical Nuclei

- First theoretical models of the shapes of nuclei near $n_0$
  1983: Ravenhall, Pethick, & Wilson
  1984: Hashimoto, H. Seki, and M. Yamada

- **Frustration**: Competition between proton-proton Coulomb repulsion and strong nuclear attraction

- Nucleons adopt non-spherical geometries near the saturation density to minimize surface energy
Nuclear Pasta

Fig. 1. Candidates for nuclear shapes. Protons are confined in the hatched regions, which we call nuclei. Then the shapes are: (a) sphere, (b) cylinder, (c) board or plank, (d) cylindrical hole and (e) spherical hole. Note that many cells of the same shape and orientation are piled up to form the whole space, and thereby the nuclei are joined to each other except for the spherical nuclei (a).
Classical Pasta Formalism

- Classical Molecular Dynamics with IUMD on Big Red II

\[ V_{np}(r_{ij}) = ae^{-r_{ij}^2/\Lambda} + [b - c]e^{-r_{ij}^2/2\Lambda} \]
\[ V_{nn}(r_{ij}) = ae^{-r_{ij}^2/\Lambda} + [b + c]e^{-r_{ij}^2/2\Lambda} \]
\[ V_{pp}(r_{ij}) = ae^{-r_{ij}^2/\Lambda} + [b + c]e^{-r_{ij}^2/2\Lambda} + \frac{\alpha}{r_{ij}}e^{-r_{ij}/\lambda} \]

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>\Lambda</th>
<th>\lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>110 MeV</td>
<td>-26 MeV</td>
<td>24 MeV</td>
<td>1.25 fm^2</td>
<td>10 fm</td>
</tr>
</tbody>
</table>

- Short range **nuclear force**
- Long range **Coulomb force**

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Monte-Carlo \langle V_{tot} \rangle (MeV)</th>
<th>Experiment (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>^{16}O</td>
<td>-7.56±0.01</td>
<td>-7.98</td>
</tr>
<tr>
<td>^{40}Ca</td>
<td>-8.75±0.03</td>
<td>-8.45</td>
</tr>
<tr>
<td>^{90}Zr</td>
<td>-9.13±0.03</td>
<td>-8.66</td>
</tr>
<tr>
<td>^{208}Pb</td>
<td>-8.2 ±0.1</td>
<td>-7.86</td>
</tr>
</tbody>
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Classical Pasta Formalism

- Classical Molecular Dynamics with IUMD on Big Red II
- Short range nuclear force
- Long range Coulomb force

\[
V_{np}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b - c] e^{-r_{ij}^2/2\Lambda}
\]
\[
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</tr>
<tr>
<td>(^{90}\text{Zr})</td>
<td>-9.13±0.05</td>
</tr>
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<td>(^{208}\text{Pb})</td>
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\[ n = 0.1200 \text{fm}^{-3} \]
Pass around the 3D printed cubes

- I always forget so I made a slide to remind myself that I have them.
Classical and Quantum MD

- We can use the classical pasta to initiate the quantum codes
- Classical structures remain stable when evolved via HF

Classical Points $\rightarrow$ Folded with Gaussian $\rightarrow$ Equilibrated Wavefunctions
Classical and Quantum MD

800 nucleons
24 fm
Classical and Quantum MD

51,200 nucleons

800 nucleons
24 fm

100 fm

\( n = 0.312n_0 \)
Molecular Dynamics

- We have evolved simulations of 409,600 nucleons, 819,200 nucleons, 1,638,400 nucleons, and 3,276,800 nucleons.
Phase Diagrams
Phase Diagrams

- Simulate pasta with constant temperature and proton fraction
- Observe phase transitions as a function of density

Y_p = 0.2

Y_p = 0.4

Decreasing Density
“Thermodynamic” Curvature

• Use curvature as a thermodynamic quantity
• Discontinuities in curvature indicate phase changes

\[
\int_M K \, dA + \int_{\partial M} \kappa_g \, ds = 2\pi \chi(M)
\]

\[
\chi(M) = 2 - 2g
\]

• Pieces + Cavities - Holes

<table>
<thead>
<tr>
<th>Volume</th>
<th>Surface Area</th>
<th>Mean Breadth</th>
<th>Euler Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
<td>( A = \int_{\partial K} dA )</td>
<td>( B = \int_{\partial K} (\kappa_1 + \kappa_2) / 4\pi , dA )</td>
<td>( \chi = \int_{\partial K} (\kappa_1 \cdot \kappa_2) / 4\pi , dA )</td>
</tr>
</tbody>
</table>

- Sphere
  - \( 2 \)
- Torus (Product of two circles)
  - \( 0 \)
- Double torus
  - \(-2\)
- Triple torus
  - \(-4\)
“Thermodynamic” Curvature

- Use curvature as a thermodynamic quantity
- Discontinuities in curvature indicate phase changes

\[
V = \int_{\partial K} \, dA \\
A = \int_{\partial K} (\kappa_1 + \kappa_2) / 4\pi \, dA \\
B = \int_{\partial K} (\kappa_1 \cdot \kappa_2) / 4\pi \, dA
\]

\[
B = \int_{\partial K} \frac{k_1 + k_2}{4\pi} \, dA
\]

\[
X = \int_{\partial K} \frac{k_1 \cdot k_2}{4\pi} \, dA
\]
Phases

i-Antignocchi  i-Antispaghetti  i-Lasagna  i-Spaghetti  i-Gnocchi

Uniform  Defects  Waffles

r-Antignocchi  r-Antispaghetti  r-Lasagna  r-Spaghetti  r-Gnocchi
Self Assembly

• Well studied in phospholipids: hydrophilic heads and hydrophobic tails self assemble in an aqueous solution
Self Assembly

Seddon, BBA 1031, 1–69 (1990)
Self Assembly

• Left: Electron microscopy of helicoids in mice endoplasmic reticulum

Terasaki et al, Cell 154.2 (2013)

• Right: Defects in nuclear pasta MD simulations

Horowitz et al, PRL.114.031102 (2015)

Parking Garage Structures in astrophysics and biophysics (arXiv:1509.00410)
• Glass transition – impure substance forms an amorphous solid when quenched

• Solid:
  • Long Range Order
  • Nondiffusive
  • First order phase transition

• Glass:
  • Short range order
  • Low diffusion
  • Second order phase transition
Quench Rate

- Cooling quickly and cooling slowly
- Proton fractions: $Y_P = 0.0 - 0.2$
- Density: $n = 0.12 \text{ fm}^{-3}$

Caplan et al (In Prep)
Glassy Systems

Caplan et al (In Prep)
Observables
Nuclear Pasta

• Important to many processes:
  • Thermodynamics: Late time crust cooling
  • Magnetic field decay: Electron scattering in pasta
  • Gravitational wave amplitude: Pasta elasticity and breaking strain
  • Neutrino scattering: Neutrino wavelength comparable to pasta spacing
  • R-process: Pasta is ejected in mergers
Lepton Scattering

• Supernova: Hot, proton rich
  • Neutrino scattering?
• Neutron Stars: Cold, proton poor
  • Electron scattering?
Defects

• In the same way that crystals have defects, pasta does too!
• Electrons don’t scatter from order, they scatter from disorder

• Horowitz et al, PRL.114.031102 (2015)
Pasta Defects

- Defects act as a site for *scattering*
Pasta Defects

- The magnetic field decays within about 1 million years, consistent with observations (Pons et al. 2013)

Lepton Scattering

- Lepton scattering from pasta influences a variety of transport coefficients:

  - Shear viscosity:

    \[ \eta = \frac{\pi v_F^2 n_e}{20 \alpha^2 \Lambda_{ep}^\eta}, \]

  - Electrical conductivity:

    \[ \sigma = \frac{v_F^2 k_F}{4\pi \alpha \Lambda_{ep}^\sigma} \]

    \[ \Lambda_{ep}^\eta = \int_0^{2k_F} \frac{dq}{q e^2(q)} \left(1 - \frac{q^2}{4k_F^2}\right) \left(1 - \frac{v_F^2 q^2}{4k_F^2}\right) S_p(q) \]

  - Thermal conductivity:

    \[ \kappa = \frac{\pi v_F^2 k_F k_B^2 T}{12 \alpha^2 \Lambda_{ep}^\kappa}. \]
Lepton Scattering

\[ S_i(q) = \langle \rho_i^*(q, t) \rho_i(q, t) \rangle_t - \langle \rho_i^*(q, t) \rangle_t \langle \rho_i(q, t) \rangle_t \]

\[ \rho_i(q, t) = N_i^{-1/2} \sum_{j=1}^{N_i} e^{i q \cdot r_j(t)} \]

\[ \Lambda_{ep} \approx \frac{\Delta q^* Z^*}{q^*} \]

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$\bar{n}$ (fm$^{-3}$)</th>
<th>$\bar{n}$ (10$^3 k_B$ MeV/fm)</th>
<th>$\bar{Z}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>perfect</td>
<td>87.7</td>
<td>6.66</td>
<td>5.5</td>
</tr>
<tr>
<td>defects</td>
<td>55.5</td>
<td>4.15</td>
<td>50.2</td>
</tr>
</tbody>
</table>
Crust Cooling
Thermal Resistivity

- So we’ve seen that pasta is electrically resistive...
- Electrons are a thermal carrier too! How does poor thermal conductivity effect the neutron star?
Low Mass X-Ray Binaries

COMPANION (HOT)

MASS FLOW

NEUTRON STAR (COLD)

HOT CRUST

ACCRETION
Low Mass X-Ray Binaries

COMPANION (HOT)

COOLING CRUST

Crust Temperature

Core Temp

Time
Observables – Thermal Properties

• Guess an effective impurity parameter for defects and try to fit neutron star cooling curves

• Cooling curves: low mass X-ray binary MXB 1659-29
  
  • Blue: normal isotropic matter
    \[ Q_{\text{imp}} = 3.5 \]
    \[ T_c = 3.05 \times 10^7 \text{ K} \]

  • Red: impure pasta layer
    \[ Q_{\text{imp}} = 1.5 \text{ (outer crust)} \]
    \[ Q_{\text{imp}} = 30 \text{ (inner crust)} \]
    \[ T_c = 2 \times 10^7 \text{ K} \]

\[ Q_{\text{imp}} \equiv n_{\text{ion}}^{-1} \sum_i n_i (Z_i - \langle Z \rangle)^2 \]

r-process
Simulate pasta with constant temperature and proton fraction

Observe ‘nuclei’ that form as pasta decompresses

Y_p = 0.2

Y_p = 0.4
Cluster Masses

Use a cluster algorithm to count $A$, $Z$ of gnocchi
Simulations at 1 MeV reproduce NSE for sufficiently slow evolution!

<table>
<thead>
<tr>
<th>$Y_P$</th>
<th>$\log_{10}\xi$</th>
<th>$M_{\text{free neutrons}}$</th>
<th>$\overline{A} \pm \sigma_A$</th>
<th>$\overline{Z} \pm \sigma_Z$</th>
<th>$M_{\text{free neutrons}}$</th>
<th>$\overline{A}$</th>
<th>$\overline{Z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-5</td>
<td>0.6538</td>
<td>90.12 ± 24.20</td>
<td>26.25 ± 7.69</td>
<td>0.6538</td>
<td>179.3</td>
<td>53.80</td>
</tr>
<tr>
<td></td>
<td>-6</td>
<td>0.6561</td>
<td>94.08 ± 18.01</td>
<td>27.57 ± 5.69</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>-7</td>
<td>0.6578</td>
<td>99.18 ± 17.42</td>
<td>29.20 ± 5.57</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>-5</td>
<td>0.3748</td>
<td>147.75 ± 33.88</td>
<td>47.31 ± 10.95</td>
<td>0.3335</td>
<td>179.3</td>
<td>53.80</td>
</tr>
<tr>
<td></td>
<td>-6</td>
<td>0.3731</td>
<td>146.22 ± 24.96</td>
<td>46.69 ± 8.30</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>-7</td>
<td>0.3704</td>
<td>136.12 ± 20.32</td>
<td>43.29 ± 6.72</td>
<td></td>
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<tr>
<td>0.3</td>
<td>-5</td>
<td>0.1359</td>
<td>186.47 ± 73.19</td>
<td>64.75 ± 24.59</td>
<td>0.0475</td>
<td>184.4</td>
<td>58.07</td>
</tr>
<tr>
<td></td>
<td>-6</td>
<td>0.1377</td>
<td>175.13 ± 34.48</td>
<td>60.94 ± 12.16</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>-7</td>
<td>0.1367</td>
<td>166.74 ± 34.71</td>
<td>57.95 ± 12.44</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>-5</td>
<td>0.0109</td>
<td>369.37 ± 426.03</td>
<td>149.32 ± 167.45</td>
<td>0.0001</td>
<td>194.4</td>
<td>77.75</td>
</tr>
<tr>
<td></td>
<td>-6</td>
<td>0.0121</td>
<td>179.40 ± 29.83</td>
<td>72.64 ± 11.93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-7</td>
<td>0.0115</td>
<td>190.25 ± 29.34</td>
<td>76.94 ± 11.83</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-8</td>
<td>0.0111</td>
<td>191.74 ± 24.55</td>
<td>77.55 ± 9.79</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table of Nuclides

• Nuclear pasta simulations at constant proton fraction reproduce nuclear statistical equilibrium:

• Right distribution of isotopes and number of free neutrons for a given T

Summary

• Classical models give reasonable results, provided you stay in the range they are valid.
• They can inform transport properties of the neutrino star crust, and be used to interpret neutron star observables.