Baryon Spectroscopy: Data Consistency and Model Discrimination

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INT Program INT-16-2a,
Bayesian Methods in Nuclear Physics,
June 13 - July 8, 2016
Introduction
Why Spectroscopy?

A spectrum reveals the underlying nature of the physical system.
### Baryon Summary Table

**Figure 1:** Particle Data Group listing 2014 [1]

<table>
<thead>
<tr>
<th>Baryon</th>
<th>Charge</th>
<th>Mass (MeV)</th>
<th>Spin</th>
<th>Parity</th>
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### References

Figure 2: Lattice QCD calculation of baryon spectrum. From [2]

- Both lattice- and quark model calculations predict more states than observed
Figure 3: Most resonance information is from partial wave analysis (PWA) of $\pi N$ scattering
Resonance decays to other channels

Figure 4: Some resonances predicted to decay into strange channels [3].
Figure 5: Comparison of photoproduction channels
Figure 6: Energy dependence of cross section
Figure 7: Possible production scenario
\( \gamma p \rightarrow K\Lambda \) Kinematics

**Figure 8:** Taken from [4]. Kinematic variables are \( W \) (hadronic mass) and \( \theta_{c.m.} \) (scattering angle).
The transversity basis

Transversity amplitudes \( b_j \) \((j = 1, 2, 3, 4)\): quantization axis perpendicular to reaction plane and the linear photon polarizations \( J_x \) and \( J_y \)

\[
\begin{align*}
    b_1 & = y\langle +|J_y|+\rangle_y, \\
    b_2 & = y\langle -|J_y|−\rangle_y, \\
    b_3 & = y\langle +|J_x|−\rangle_y, \\
    b_4 & = y\langle −|J_x|+\rangle_y.
\end{align*}
\]

Normalized transversity amplitudes (NTA) \( a_j \) \((j = 1, 2, 3, 4)\)

\[
a_j \equiv \frac{b_j}{\sqrt{|b_1|^2 + |b_2|^2 + |b_3|^2 + |b_4|^2}},
\]

The \( a_j \) are functions of \( W \) (hadronic mass) and \( \theta_{\text{c.m.}} \) (scattering angle)
### $\gamma p \rightarrow K\Lambda$ Reaction Amplitudes

<table>
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<th>Type</th>
<th>Observable</th>
<th>Transversity representation</th>
<th>Helicity representation</th>
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<td>$</td>
<td>a_1</td>
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<tr>
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<td>$T$</td>
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<td>$2\Re(h_1 h_3^* - h_2 h_4^*)$</td>
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<td>$L_z$</td>
<td>$2\Re(a_1 a_2^* + a_3 a_4^*)$</td>
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\[ \sigma_{Total} = \sigma_0 \left\{ 1 - P_L^\gamma P_T^T P_y^R \sin(\phi) \cos(2\phi) + \Sigma(-P_L^\gamma \cos(2\phi) + P_T^T P_y^R \sin(\phi)) \right\} \\
+ T(P_T^T \sin(\phi) - P_L^\gamma P_y^R \cos(2\phi)) + P(P_y^R - P_L^\gamma P_T^T \sin(\phi) \cos(2\phi)) \\
+ E(-P_C^\gamma P_L^T + P_L^\gamma P_T^T P_y^R \cos(\phi) \sin(2\phi)) + F(P_C^\gamma P_T^T \cos(\phi) + P_L^\gamma P_T^T P_y^R \sin(2\phi)) \\
- G(P_L^\gamma P_T^T \sin(2\phi) + P_L^\gamma P_T^T P_y^R \cos(\phi)) - H(P_L^\gamma P_T^T \cos(\phi) \sin(2\phi) - P_C^\gamma P_L^T P_y^R) \\
- C_x(P_C^\gamma P_x^R - P_L^\gamma P_T^T P_z^R \sin(\phi) \sin(2\phi)) - C_z(P_C^\gamma P_z^R + P_L^\gamma P_T^T P_x^R \sin(\phi) \sin(2\phi)) \\
- O_x(P_L^\gamma P_T^T \sin(2\phi) + P_C^\gamma P_T^T P_z^R \sin(\phi)) - O_z(P_L^\gamma P_T^T \sin(2\phi) - P_C^\gamma P_T^T P_x^R \sin(\phi)) \\
+ L_x(P_L^\gamma P_R^T + P_L^\gamma P_T^T P_z^R \cos(\phi) \cos(2\phi)) + L_z(P_L^\gamma P_T^T P_x^R \cos(\phi) \cos(2\phi)) \\
+ T_x(P_T^T P_x^R \cos(\phi) - P_L^\gamma P_L^T P_z^R \cos(2\phi)) + T_z(P_T^T P_z^R \cos(\phi) + P_L^\gamma P_T^T P_x^R \cos(2\phi)) \right\} \]

**Figure 10:** Cross section as a function of beam \((P_{C,L}^\gamma)\), target \((P_{L,T}^T)\) and recoil \((P_{x,y,z}^R)\) polarization
### Usual process:

- Progress by fitting observables (cross sections, asymmetries) for several channels
- o(10000) data points in total
- \(\chi^2\) minimization, occasionally event-by-event maximum likelihood
- Different model frameworks (i.e. different theory groups) and different model content (choice of resonances, etc.)

### Issues:

- How accurate do measurements require to be?
- How do we deal with measurements from different experiments?
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### Issues:

- How accurate do measurements require to be?
- How do we deal with measurements from different experiments?
Model Discrimination
Distinguishing Objects

- Resolve two objects
- Actual angular “distance”
- Instrumental resolution (aperture limit)
- **Rayleigh Criterion**: 1st diffraction minimum of object 1 ≤ distance to centre of object 2

Figure 11: Airy disk near Rayleigh Criterion.
Distinguishing Objects

Figure 12: Mapping between Amplitudes (X) and Observables (Y).
Model Discrimination from Cross Sections

\[ \mathcal{A}[A, B] = \left| \frac{d\sigma}{d\Omega}(A) - \frac{d\sigma}{d\Omega}(B) \right| \left| \frac{d\sigma}{d\Omega}(A) + \frac{d\sigma}{d\Omega}(B) \right| \]

- Measure for difference between the c.s. predictions
- Example: BnGa2014-02 vs. RPR-2011 predictions for \( \gamma p \rightarrow K^+\Lambda \)
- Experimental resolution: \( \Delta\sigma = \left( \frac{\Delta d\sigma}{d\Omega} \right) / \frac{d\sigma}{d\Omega} \)
- \( \overline{\mathcal{A}}(\text{th}) \approx \Delta\sigma(\text{expt}) \)
- ArXiv: [5]
Model discrimination: distance in amplitude space

Measure to discriminate between two models for $p(\gamma, K^+)\Lambda$ in amplitude space?

- 4D-vector representation for NTA
  
  $$\mathbf{M}_1(s, t) = (a_1, a_2, a_3, a_4)^T$$

  vectors on a 3-sphere in $\mathbb{C}^4$ (unit 7-sphere in $\mathbb{R}^8$)

- Distance between two models
  
  $$D[\mathbf{M}_1, \mathbf{M}_2] = \arccos \text{Re} \left( \mathbf{M}_1^\dagger \cdot \mathbf{M}_2 \right)$$

- Dependence on arbitrary phase:
  
  $\mathbf{M}_2(\alpha'_4 = 0)$ and vary $\alpha_4$ in $\mathbf{M}_1(\alpha_4 = 0)$ such that $D[\mathbf{M}_1, \mathbf{M}_2]$ is minimized
Figure 13: Distance measure in amplitude space for BnGa versus RPR-2011
Model discrimination: distance in amplitude space

- Blue line: random samples in NTA amplitude space
- $\mathcal{D}[\text{RPR-2011, RPR2011*} ]$: Resolution required to hunt a resonance ($D_{13}(1900)$)
- $\mathcal{D}[\text{RPR-2011, Regge}]$: Resolution required to determine “the” background
- $\mathcal{D}[\text{RPR-2011, KM}]$: Resolution required to discriminate between RPR-2011 and Kaon-MAID
Extract $r_3 e^{i\delta_3}$ at $(W = 1.8 \text{ GeV}, \theta_{c.m.} = -0.1)$ from data

1. **Bootstrap**: $M$ sets of data $\{A_i^j \pm \delta A_i^j, i = 1, ..., N\}, j = 1, ..., M$
2. $\chi^2$ fit to extract amplitudes for each set of synthetic data
3. Histogram solutions in amplitude space
Extract $r_3 e^{i\delta_3^4}$ at $(W = 1.8 \text{ GeV}, \theta_{c.m.} = -0.1)$ from data

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Red: accuracy = 0.1; Blue: accuracy = 0.01

(i) “mathematically complete set”: $\{A^\text{exp}_i\}_1 = \left\{ \frac{d\sigma}{d\Omega}, \Sigma, T, P, C_x, O_x, E, F \right\}$
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(ii) $\{A_{i}^{\exp}\}_2 = \{A_{i}^{\exp}\}_1 + \{C_z, O_z, G\}$
Extract $r_3 e^{i\delta_3^4}$ at $(W = 1.8 \text{ GeV}, \theta_{c.m.} = -0.1)$ from data

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(iii) \( \{A_i^{\exp}\}_3 = \{A_i^{\exp}\}_2 + \{H\} \)
(iv) \( \{A_i^{\exp}\}_4 = \{A_i^{\exp}\}_3 + \{T_x, T_z, L_x, L_z\} \)
Extract $r_3 e^{i \delta_3}$ at ($W = 1.8$ GeV, $\theta_{c.m.} = -0.1$) from data

Compare bootstrap method:
Extract \( r_3 e^{i \delta_3} \) at \((W = 1.8 \text{ GeV}, \theta_{c.m.} = -0.1)\) from data

Compare bootstrap method:

To MCMC (nested sampling):
Resolving power of $p(\gamma, K^+) \Lambda$ polarization data?

- All data in grids:
  1. $\Delta W = 20$ MeV
  2. $\Delta \cos \theta_{c.m.} = 0.1$

- 2241 single polarization observables ($\Sigma, P, T$)
- 452 double polarization observables (beam-recoil, target-recoil, beam-target)

The darker the color, the better the reaction amplitudes are determined by the data
**Model Discrimination**

**Key Points:**

- Introduced distance measure between models in amplitude space.
- Experimental data must lead to PDFs in amplitude space that have smaller dispersions than characteristic distances between models.
- The power of new measurements can be analysed using synthetic data from models, plus realistic experimental uncertainties.
- Bootstrap and MCMC (Nested Sampling) give similar distributions.

**Questions:**

- How to extend this for distributions over kinematic variables?
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Data Consistency
Fierz Identity Comparison: $\gamma + p \rightarrow K + \Lambda$

For $\gamma + N \rightarrow$ p.s. meson + baryon

$O_x^2 + O_z^2 + C_x^2 + C_z^2 + \Sigma^2 - T^2 + P^2 = 1$

**Figure 14:** Open circles - $C_x^2 + C_z^2$ [4]; Filled - $1 - \Sigma^2 + T^2 - P^2 - O_x^2 - O_z^2$ [6]
The constraints among observables, e.g.:

$$O_x^2 + O_z^2 + C_x^2 + C_z^2 + \Sigma^2 - T^2 + P^2 = 1$$

stem from the constraint among amplitudes:

$$|a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 = 1$$

i.e. surface of a unit 7-sphere in $\mathbb{R}^8$
Data Consistency Idea

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i.e. surface of a unit 7-sphere in \( \mathbb{R}^8 \)

- Can we map PDFs in observable space to PDF in amplitude space?
- If so, can we project amplitude PDF back into a joint observable PDF?
Test Case: $\pi$-N Scattering

Two amplitudes, four observables:

\[
\frac{d\sigma}{d\Omega} = |f|^2 + |g|^2
\]
\[
A = |f|^2 - |g|^2
\]
\[
R = -2 \text{Re} \ (fg^*)
\]
\[
P = 2 \text{Im} \ (fg^*)
\]

Normalize:

\[
|f|^2 + |g|^2 = 1
\]

Constraint:

\[
A^2 + R^2 + P^2 = 1
\]

Figure 15: $\pi^-$ $p$ (left) and $\pi^+$ $p$ (right) polarization observables
Test Case: $\pi$-N Scattering

- Generate “true” synthetic data
- Generate statistical uncertainty
- Sample from $\mathcal{N}(\mu, \sigma)$
- Add systematic error

<table>
<thead>
<tr>
<th>Observables</th>
<th>$A$</th>
<th>$R$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>“True” values</td>
<td>0.35</td>
<td>0.09</td>
<td>0.93</td>
</tr>
<tr>
<td>“Smeared”</td>
<td>$0.10 \pm 0.45$</td>
<td>$0.14 \pm 0.14$</td>
<td>$0.93 \pm 0.06$</td>
</tr>
<tr>
<td>Systematic Error</td>
<td>0.04</td>
<td>0.06</td>
<td>-0.09</td>
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Test Case: $\pi$-N Scattering

Unconstrained PDF

- Use emcee
- Sample from 3D Gaussian
- Mean and standard deviation from smeared data
- Assume uncorrelated measurements
- Corner plot with true values indicated

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<tr>
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Test Case: $\pi$-N Scattering

Constrained PDF

- Use emcee
- Sample from amplitude space
- Calculate likelihood from 3D Gaussian
- Corner plot with true values indicated

Observables

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Next steps

**π-N Scattering Roadmap**

- Generate large sample of synthetic data
- For each data set:
  - select different experimental uncertainty
  - select different systematic uncertainty
- Analyse all sets statistically
- Apply to measured data

**Further work**

- Apply procedure to pseudoscalar meson photoproduction
- Other reactions?
Question: How to cope with different bins?
Data Consistency

Key Points:

• Independent polarization measurements lead to observables that are projections of the same amplitudes
• Map observable PDFs into amplitude space PDFs and combine.
• Inverse map of amplitude PDF to observable space
• Needs to be extended to pseudoscalar meson photoproduction (4 amplitudes), and other reactions?

Questions:

• Can this be used to detect inconsistent data?
• How to deal with kinematic bins that partially overlap?
Key Points:

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Questions:
Data Consistency

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### Questions:

- Can this be used to detect inconsistent data?
- How to deal with kinematic bins that *partially* overlap?
Conclusion
### Summary

<table>
<thead>
<tr>
<th>Topic</th>
<th>Details</th>
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<tr>
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### Baryon Spectroscopy

- We are still not sure of the spectrum of baryons

### Model Discrimination

### Data Consistency
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## Summary

### Baryon Spectroscopy
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### Summary

**Baryon Spectroscopy**
- We are still not sure of the spectrum of baryons

**Model Discrimination**
- We need an analogue of a Rayleigh Criterion

**Data Consistency**
- {Work in progress}: Create joint observable PDFs
- Clean or process data for model inference
Collaborators

[In addition to members of the Glasgow group]

- CLAS Collaboration: Meson Photoproduction measurements
- J. Nys and J. Ryckebusch (University of Gent, Belgium): Model Discrimination


