Deuteron–induced reactions

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We present a formalism for inclusive deuteron–induced reactions. We thus want to describe within the same framework:

- Direct neutron transfer: should be compatible with existing theories.
- Elastic deuteron breakup: “transfer” to continuum states.
- Neutron capture and compound nucleus formation: absorption above and below neutron emission threshold.
- Important application in surrogate reactions: obtain spin–parity distributions, get rid of Weisskopf–Ewing approximation (see J. Escher’s talk).
Historical background

breakup-fusion reactions

Kerman and McVoy, Ann. Phys. 122 (1979) 197


Last paper: Mastroleo, Udagawa, Mustafa Phys. Rev. C 42 (1990) 683

Controversy between Udagawa and Austern formalism left somehow unresolved.

Britt and Quinton, Phys. Rev. 124 (1961) 877

protons and α yields
bombarding $^{209}$Bi with $^{12}$C and $^{16}$O
Inclusive \((d, p)\) reaction

let’s concentrate in the reaction \(A + d \rightarrow B(=A + n) + p\)

we are interested in the inclusive cross section, \(i.e.,\) we will sum over all final states \(\phi_B^c\).
the double differential cross section with respect to the proton energy and angle for the population of a specific final $\phi_B^c$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d} \rho(E_p) \left| \left\langle \chi_p \phi_B^c | V | \psi(+) \right\rangle \right|^2.$$ 

Sum over all channels, with the approximation $\psi(+) \approx \chi_d \phi_d \phi_A$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2\pi}{\hbar v_d} \rho(E_p)$$

$$\times \sum_c \left\langle \chi_d \phi_d \phi_A | V | \chi_p \phi_B^c \right\rangle \delta(E - E_p - E_B^c) \left\langle \phi_B^c \chi_p | V | \phi_A \chi_d \phi_d \right\rangle$$

$\chi_d \rightarrow$ deuteron incoming wave, $\phi_d \rightarrow$ deuteron wavefunction,

$\chi_p \rightarrow$ proton outgoing wave $\phi_A \rightarrow$ target core ground state.
the imaginary part of the Green’s function $G$ is an operator representation of the $\delta$–function,

$$\pi \delta(E - E_p - E^c_B) = \lim_{\epsilon \to 0} \Im \sum_c \frac{\langle \phi^c_B | \phi^c_B \rangle}{E - E_p - H_B + i\epsilon} = \Im G$$

$$\frac{d^2 \sigma}{d\Omega_p dE_p} = -\frac{2}{\hbar v_d} \rho(E_p) \Im \langle \chi_d \phi_d \phi_A | V | \chi_p \rangle G \langle \chi_p | V | \phi_A \chi_d \phi_d \rangle$$

- We got rid of the (infinite) sum over final states,
- but $G$ is an extremely complex object!
- We still need to deal with that.
Optical reduction of $G$

If the interaction $V$ do not act on $\phi_A$

$$\langle \chi_d \phi_d \phi_A | V | \chi_p \rangle G \langle \chi_p | V | \phi_A \chi_d \phi_d \rangle$$

$$= \langle \chi_d \phi_d | V | \chi_p \rangle G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle,$$

where $G_{opt}$ is the optical reduction of $G$

$$G_{opt} = \lim_{\epsilon \to 0} \frac{1}{E - E_p - T_n - U_{An}(r_{An}) + i\epsilon},$$

now $U_{An}(r_{An}) = V_{An}(r_{An}) + iW_{An}(r_{An})$ and thus $G_{opt}$ are single–particle, tractable operators.

The effective neutron–target interaction $U_{An}(r_{An})$, a.k.a. optical potential, a.k.a. self–energy can be provided by structure calculations (previous talks by W. Dickhoff, C. Barbieri, P. Navratil, G. Hagen, J. Rotureau, J. Holt...)

the imaginary part of $G_{opt}$ splits in two terms

$$\Im G_{opt} = -\pi \sum_{k_n} |\chi_n\rangle \delta \left(E - E_p - \frac{k_n^2}{2m_n}\right) \langle \chi_n| + G_{opt}^\dagger W_{An} G_{opt},$$

we define the neutron wavefunction $|\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d\rangle$

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**cross sections for neutron capture and elastic breakup**

$$\frac{d^2\sigma}{d\Omega_p dE_p} \left|^{\text{capture}}\right. = -\frac{2}{\hbar \nu_d} \rho(E_p) \langle \psi_n | W_{An} | \psi_n\rangle,$$

$$\frac{d^2\sigma}{d\Omega_p dE_p} \left|^{\text{breakup}}\right. = -\frac{2}{\hbar \nu_d} \rho(E_p) \rho(E_n) |\langle \chi_n \chi_p | V | \chi_d \phi_d\rangle|^2,$$
2–step process

**step1** breakup

\[ \langle \chi_p | V | \phi_{AX} d \phi_d \rangle \]

\[ \begin{array}{c}
\text{d} \\
A \\
\end{array} \rightarrow \begin{array}{c}
p \\
\text{n} \\
A \\
\end{array} \]

**step2** propagation of n in the field of A

\[ \begin{array}{c}
p \\
\text{transfer, capture} \\
\end{array} \]

\[ \begin{array}{c}
p \\
elastic breakup \\
\end{array} \]

\[ \begin{array}{c}
G \\
B^* \\
\end{array} + \begin{array}{c}
G \\
A \\
\end{array} \]

to detector
The interaction $V$ can be taken either in the $\textit{prior}$ or the $\textit{post}$ representation,

- Austern (post) $\rightarrow V \equiv V_{\text{post}} \sim V_{pn}(r_{pn})$
- Udagawa (prior) $\rightarrow V \equiv V_{\text{prior}} \sim V_{An}(r_{An}, \xi_{An})$

in the prior representation, $V$ can act on $\phi_A \rightarrow$ the optical reduction gives rise to new terms:

$$\frac{d^2 \sigma}{d\Omega_p dE_p} \bigg|^{\text{post}} = -\frac{2}{\hbar v_d} \rho(E_p) \left[ \Im \langle \psi_{n}^{\text{prior}} | W_{An} | \psi_{n}^{\text{prior}} \rangle + 2 \Re \langle \psi_{n}^{\text{NON}} | W_{An} | \psi_{n}^{\text{NON}} \rangle \right],$$

where $\psi_{n}^{\text{NON}} = \langle \chi_p | \chi_d \phi_d \rangle.$
Neutron states in nuclei

E$-\text{E}_F$ scattering states
weakly bound states
deeply bound states
W (MeV)

E$-\text{E}_F$ (MeV)

Imaginary part of optical potential

narrow single-particle scattering and resonances

broad single-particle scattering states
weakly bound states
deeply bound states

Mahaux, Bortignon, Broglia and Dasso Phys. Rep. 120 (1985) 1
neutron wavefunctions

the neutron wavefunctions

\[ |\psi_n \rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle \]

can be computed for any neutron energy

these wavefunctions are not eigenfunctions of the Hamiltonian

\[ H_{An} = T_n + \Re(U_{An}) \]
neutron transfer limit (isolated–resonance, first–order approximation)

Let’s consider the limit $W_{An} \to 0$ (single–particle width $\Gamma \to 0$). For an energy $E$ such that $|E - E_n| \ll D$, (isolated resonance)

$$G_{opt} \approx \lim_{W_{An} \to 0} \frac{|\phi_n\rangle\langle\phi_n|}{E - E_p - E_n - i\langle\phi_n|W_{An}|\phi_n\rangle};$$

with $|\phi_n\rangle$ eigenstate of $H_{An} = T_n + \Re(U_{An})$

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim \lim_{W_{An} \to 0} \langle\chi_d\phi_d| V |\chi_p\rangle \times \frac{|\phi_n\rangle\langle\phi_n|W_{An}|\phi_n\rangle\langle\phi_n|}{(E - E_p - E_n)^2 + \langle\phi_n|W_{An}|\phi_n\rangle^2} \langle\chi_p| V |\chi_d\phi_d\rangle,$$

we get the direct transfer cross section:

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim |\langle\chi_p\phi_n| V |\chi_d\phi_d\rangle|^2 \delta(E - E_p - E_n)$$
Validity of first order approximation

For $W_{An}$ small, we can apply first order perturbation theory,

$$\frac{d^2\sigma}{d\Omega_p dE_p} (E, \Omega) \bigg|_{\text{capture}} \approx \frac{1}{\pi} \frac{\langle \phi_n | W_{An} | \phi_n \rangle}{(E_n - E)^2 + \langle \phi_n | W_{An} | \phi_n \rangle^2} \frac{d\sigma_n}{d\Omega} (\Omega) \bigg|_{\text{transfer}}$$

we compare the complete calculation with the isolated–resonance, first–order approximation for $W_{An} = 0.5$ MeV, $W_{An} = 0.5$ MeV and $W_{An} = 0.5$ MeV.
elastic breakup and capture cross sections as a function of the proton energy. The Koning–Delaroche global optical potential has been used as the $U_{An}$ interaction (Koning and Delaroche, Nucl. Phys. A 713 (2003) 231).
Observables: angular differential cross sections (neutron bound states)

- capture at resonant energies compared with direct transfer (FRESCO) calculations,
- capture cross sections rescaled by a factor $\langle \phi_n | W_{An} | \phi_n \rangle \pi$.

**double proton differential cross section**

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d} \rho(E_p) \sum_{l,m,l_p} \int \left| \varphi_{lml_p}(r_{Bn}; k_p) Y_{-m}^{l_p}(\theta_p) \right|^2 W(r_{An}) \, dr_{Bn}. $$
Observables: angular differential cross sections (above neutron–emission threshold)

$^{93}\text{Nb} \,(d,p) \ @ \ 15 \text{ MeV}$

$E_p = 9 \text{ MeV}$
Spin distribution of compound nucleus

\[
\frac{d\sigma_l}{dE_p} = \frac{2\pi}{\hbar v_d} \rho(E_p) \sum_{l_p,m} \int |\varphi_{lml_p}(r_{Bn}; k_p)|^2 W(r_{An}) \, dr_{Bn}.
\]
Desired reaction: neutron induced fission, gamma emission and neutron emission.

The surrogate method consists in producing the same compound nucleus $B^*$ by bombarding a deuteron target with a radioactive beam of the nuclear species $A$.

A theoretical reaction formalism that describes the production of all open channels $B^*$ is needed.
Surrogate reactions

Weisskopf–Ewing approximation:
\[ P(d, nx) = \sigma(E) G(E, x) \]

- inaccurate for \( x = \gamma \) and for \( x = f \) in the low–energy regime
- can be replaced by \( P(d, nx) = \sum_{J, \pi} \sigma(E, J, \pi) G(E, J, \pi, x) \) if \( \sigma(E, J, \pi) \) can be predicted.

Younes and Britt, PRC 68(2003)034610

(see J. Escher talk)
Preliminary comparison with experiment

We show very preliminary results for the $^{93}\text{Nb}(d,p)$ reaction with a 15 MeV deuteron beam (Mastroleo et al., Phys. Rev. C 42 (1990) 683)

- we have used the Koning–Delaroche optical potential
- the real part of the optical potential has been shifted to reproduce the position of the $L = 3$ resonance
- the experimental results seem to be sensitive to the position and strength of a modest number of resonances
Summary, conclusions and some prospectives

- Reaction formalism for inclusive deuteron–induced reaction.
- Final neutron states from Fermi energy → to scattering states
- 2–step reaction mechanism → breakup + absorption
- Probe of nuclear structure over a wide energy range
- Need for optical potentials
- Useful for surrogate reactions
- Transfer to individual resonances?
- Extend for \((p, d)\) reactions (hole states)?
The 3–body model

From $H$ to $H_{3B}$

- $H = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + V_{An}(r_{An}, \xi_A) + V_{Ap}(r_{Ap}, \xi_A)$
- $H_{3B} = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + U_{An}(r_{An}) + U_{Ap}(r_{Ap})$