Microscopic nucleon-nucleus optical potentials for neutron-rich systems

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Department of Physics
University of Washington

INT workshop: Reactions and structure of exotic nuclei, 03/05/2015
Physics motivations

- Neutron-capture rates in r-process nucleosynthesis
- Neutron star structure (inner crust)
- Charged-current weak reactions in newly formed neutron stars

Nucleon self energy in homogeneous matter

- Improvements in theory: **perturbative** chiral nuclear forces that **reproduce saturation**
- Benchmark to phenomenological potentials close to valley of stability
- Corrections to the Lane parametrization of the isospin asymmetry dependence
CHALLENGE 1: R-PROCESS NUCLEOSYNTHESIS

Astrophysical site?

Core-collapse supernovae

Neutron-star mergers

http://www.csm.ornl.gov

http://numrel.aei.mpg.de

Microscopic optical potential for neutron-rich systems

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Masses of neutron-rich nuclei

- Determine elemental abundance patterns along isotopic chains during equilibrium

\[
\frac{Y(Z, A + 1)}{Y(Z, A)} \sim \exp \left[ \frac{S_n(Z, A + 1) - S_n^0(T, \rho_n)}{kT} \right]
\]

Beta-decay lifetimes

- Set timescale for formation of heavy elements from seed nuclei
- Partly responsible for peaks at \( A = 130 \) and \( A = 195 \)

Neutron-capture rates

- Relevant during late-time freeze-out phase of the \( r \)-process
- Sensitivity studies vary capture rates over 1–2 orders of magnitude

**INPUTS FROM NUCLEAR STRUCTURE**

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Outer crust is a lattice of nuclei with gas of electrons

Inner crust contains lattice of neutron-rich nuclei together with “dripped” neutrons

Neutron drip density: \[ \rho_{\text{drip}} = 4 \times 10^{11} \text{g/cm}^3 \]
\[ U(r, E) = -V_V(r, E) - iW_V(r, E) - iW_D(r, E) \]
\[ + V_{SO}(r, E) \cdot l \cdot \sigma + iW_{SO}(r, E) \cdot l \cdot \sigma + V_C(r). \]

\[ V_V(r, E) = V_V(E) f(r, R_V, a_V), \]
\[ W_V(r, E) = W_V(E) f(r, R_V, a_V), \]
\[ W_D(r, E) = -4a_D W_D(E) \frac{d}{dr} f(r, R_D, a_D), \]
\[ V_{SO}(r, E) = V_{SO}(E) \left( \frac{\hbar}{m_\pi c} \right)^2 \frac{1}{r} \frac{d}{dr} f(r, R_{SO}, a_{SO}), \]
\[ W_{SO}(r, E) = W_{SO}(E) \left( \frac{\hbar}{m_\pi c} \right)^2 \frac{1}{r} \frac{d}{dr} f(r, R_{SO}, a_{SO}). \]

Koning & Delaroche, NPA (2003)

\[ V_V(E) = v_1 \left[ 1 - v_2 (E - E_f) + v_3 (E - E_f)^2 - v_4 (E - E_f)^3 \right] \]
\[ W_V(E) = w_1 \frac{(E - E_f)^2}{(E - E_f)^2 + (w_2)^2}, \]

Energy dependence

\[ f(r, R_i, a_i) = \left( 1 + \exp \left[ (r - R_i)/a_i \right] \right)^{-1} \]
Isoscalar part of optical potential linear in the isospin asymmetry

\[ U = U_0 + U_I = U_0 + \bar{U}_I \tau_z \delta_{np} \]

\[ \delta_{np} = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} \]

Model predictions

Much less is known/predicted about isovector imaginary part
**MICROSCOPIC OPTICAL POTENTIALS (HOMOGENEOUS MATTER)**

- Identified with the on-shell nucleon self-energy $\Sigma(\vec{r}_1, \vec{r}_2, \omega)$ [Bell and Squires, PRL (1959)]

- Hartree-Fock contribution (real, energy-independent):

  \[ \Sigma_{2N}^{(1)}(q; k_f) = \sum_1 \langle \vec{q} \hbar_1 ss_1 tt_1 | \bar{V}_{2N} | \vec{q} \hbar_1 ss_1 tt_1 \rangle n_1 \]

- Second-order perturbative contributions (complex, energy-dependent):

  \[ \Sigma_{2N}^{(2a)}(q, \omega; k_f) = \frac{1}{2} \sum_{123} \frac{|\langle \vec{p}_1 \vec{p}_3 ss_1 s_3 t_1 t_3 | \bar{V} | \vec{q} \hbar_2 ss_2 tt_2 \rangle|^2}{\omega + \epsilon_2 - \epsilon_1 - \epsilon_3 + i\eta} n_1 n_2 n_3 (2\pi)^3 \delta(\vec{p}_1 + \vec{p}_3 - \vec{q} - \hbar_2) \]

**Benchmarks:**

- Depth and energy dependence of phenomenological volume parts (including isospin dependence)
Quark/gluon (high energy) dynamics

\[ \mathcal{L} = -\frac{1}{4} G^{\alpha}_{\mu\nu} G_{\alpha}^{\mu\nu} + \bar{q}_L i \gamma_\mu D^\mu q_L + \bar{q}_R i \gamma_\mu D^\mu q_R - \bar{q} \mathcal{M} q \]

- Approximate chiral symmetry (left- and right-handed quarks transform independently)

Nucleon/pion (low energy) dynamics

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(0)}_{NN} + \mathcal{L}^{(2)}_{NN} + \cdots \]

- Compatible with explicit and spontaneous chiral symmetry breaking
NUCLEAR FORCES IN CHIRAL EFFECTIVE FIELD THEORY

**SEPARATION OF SCALES + SYMMETRIES**

Energy

- Heavy mesons ($\rho, \omega$)
- Nucleon momenta
- Pion mass

$q = \left(\frac{q}{\Lambda}\right)$

QCD chiral symmetry

- Left handed
- Right handed

Pions weakly-coupled at low momenta!

CHIRAL EFFECTIVE FIELD THEORY

Low-energy theory of nucleons and pions

<table>
<thead>
<tr>
<th>2N force</th>
<th>3N force</th>
<th>4N force</th>
</tr>
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<tbody>
<tr>
<td>$q/\Lambda^0$</td>
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Systematic expansion

Microscopic optical potential for neutron-rich systems

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SEPARATION OF SCALES + SYMMETRIES

NUCLEAR FORCES IN CHIRAL EFFECTIVE FIELD THEORY

CHIRAL EFFECTIVE FIELD THEORY

Low-energy theory of nucleons and pions

Heavy mesons \((\rho, \omega)\) \(\Lambda\)

Nucleon momenta

Pion mass

Energy

\(q\)

QCD chiral symmetry

Quarks

Left handed

Right handed

Pions weakly-coupled at low momenta!

Systematic expansion

Fit to NN scattering (future: lattice QCD?)

\[(q/\Lambda)^0\]

\[(q/\Lambda)^2\]

\[(q/\Lambda)^3\]

\[(q/\Lambda)^4\]

2N force

3N force

4N force

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NUCLEAR FORCES IN CHIRAL EFFECTIVE FIELD THEORY

SEPARATION OF SCALES + SYMMETRIES

Energy
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Consistent vertices in 2N & 3N forces (also $\pi N$)

Systematic expansion

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<td><img src="https://example.com/diagram.png" alt="Diagram" /></td>
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Pions weakly-coupled at low momenta!

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NUCLEAR FORCES IN CHIRAL EFFECTIVE FIELD THEORY

SEPARATION OF SCALES + SYMMETRIES

Energy
- Heavy mesons ($\rho, \omega$) \( \Lambda \)
- Nucleon momenta \( q \)
- Pion mass

QCD chiral symmetry
- Quarks
  - Left handed
  - Right handed

Pions weakly-coupled at low momenta!

CHIRAL EFFECTIVE FIELD THEORY
Low-energy theory of nucleons and pions

Systematic expansion

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<td>Systematic expansion</td>
</tr>
<tr>
<td>((q/\Lambda)^2)</td>
<td></td>
<td>Fit to (^3!H) binding energy and lifetime</td>
</tr>
<tr>
<td>((q/\Lambda)^3)</td>
<td>(\ldots)</td>
<td></td>
</tr>
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Nuclear Forces in Chiral Effective Field Theory

Separation of Scales + Symmetries

Energy

Heavy mesons ($\rho$, $\omega$) \( \Lambda \)

Nucleon momenta \( q \)

Pion mass

QCD chiral symmetry

Quarks

Left handed

Right handed

Pions weakly-coupled at low momenta!

Chiral Effective Field Theory

Low-energy theory of nucleons and pions

Systematic expansion

Frontier in nuclear structure

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Microscopic optical potential for neutron-rich systems

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Regulating function

\[ \exp\left[-\left(\frac{p}{\Lambda}\right)^{2n} - \left(\frac{p'}{\Lambda}\right)^{2n}\right] \langle \vec{p}' | V | \vec{p} \rangle \]

sets resolution scale

Variations in regulator

- Estimate of theoretical uncertainty

\[ \begin{cases} \Lambda = 414 \text{ MeV}, \ n = 10 \\
\Lambda = 450 \text{ MeV}, \ n = 3 \\
\Lambda = 500 \text{ MeV}, \ n = 2 \end{cases} \]

Coraggio, Holt, Itaco, Machleidt & Sammarruca, PRC (2013)
Low-momentum potentials: improved perturbative properties

Microscopic optical potential for neutron-rich systems

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Saturation energy: \( E/A = -15.5 - 15.8 \text{ MeV} \)

Saturation density: \( \rho = 0.16 - 0.17 \text{ fm}^{-3} \)

Asymmetry energy: \( \beta = 31 - 33 \text{ MeV} \)

Compressibility: \( \kappa = 220 - 240 \text{ MeV} \)

Nontrivial without extra tuning
LIQUID-GAS PHASE TRANSITION and THE CRITICAL POINT (CP)

Predicted critical endpoint

- Critical temperature:
  \[ T_c = 17.2 - 19.1 \text{ MeV} \]

- Critical density:
  \[ \rho_c = 0.064 - 0.072 \text{ fm}^{-3} \]

- Critical pressure:
  \[ P_c = 0.3 - 0.4 \text{ MeV fm}^{-3} \]

Experiment (compound nucleus & multifragmentation) [J. B. Elliott et al., PRC (2013)]

\[ T_c = 17.9 \pm 0.4 \text{ MeV} \quad \rho_c = 0.06 \pm 0.02 \text{ fm}^{-3} \quad P_c = 0.31 \pm 0.07 \text{ MeV fm}^{-3} \]
Microscopic optical potential for neutron-rich systems

Holt, Kaiser, Miller & Weise, PRC (2013)
Nearly all momentum dependence comes from the two-pion-exchange 3NF.
BENCHMARK: PHENOMENOLOGICAL OPTICAL POTENTIALS

Holt, Kaiser, Miller & Weise, PRC (2013)

Typical phenomenological potential: $^{56}$Fe

Self consistency: $E_q = \frac{q^2}{2M} + \text{Re} \Sigma(q, E_q)$

Microscopic optical potential for neutron-rich systems

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Microscopic optical potential for neutron-rich systems

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REAL AND IMAGINARY PROTON/NEUTRON POTENTIALS

Microscopic optical potential for neutron-rich systems

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Microscopic optical potential for neutron-rich systems

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Microscopic optical potential for neutron-rich systems

Holt, Kaiser & Miller, in prep.

$W_I^n = 18.5 \delta_{np} - 14.4 \delta_{np}^2$

$W_I^p = -16.1 \delta_{np} - 0.05 \delta_{np}^2$

$n = 0.13 \text{ fm}^{-3}$, $E = 10 \text{ MeV}$
R-process from SNe requires large number of neutrons per seed nucleus

Proton fraction of outflow set by competing charged-current reactions

\[ \nu_e + n \leftrightarrow e^- + p \]
\[ \bar{\nu}_e + p \leftrightarrow e^+ + n \]

Robust r-process nucleosynthesis:

\[ N_p \lesssim 0.4 \]
\[ \langle E_{\bar{\nu}_e} \rangle - \langle E_{\nu_e} \rangle > 4(m_n - m_p) \]

(Anti-)neutrino decoupling region sensitive to nuclear physics inputs: especially nucleon single-particle energies in the neutrinosphere
WHERE DO (ANTI-)NEUTRINOS DECOUPLE?

**Neutrino opacity**

- Charged-current
  \[ \nu_e + n \leftrightarrow e^- + p \]

**Anti-neutrino opacity**

- Neutral-current
  \[ \bar{\nu}_e + n \rightarrow \bar{\nu}_e + n \]

- Charged-current
  \[ \bar{\nu}_e + p \leftrightarrow e^+ + n \]

---

**Neutrino reactions**

- Charged-current
  - \( \nu_e n \rightarrow \nu_e n \)
  - \( \nu_e p \rightarrow \nu_e p \)
  - \( \nu_e n \rightarrow e^- p \)

- Neutral-current
  - \( \nu_e \bar{\nu}_e \rightarrow e^- e^+ \)
  - \( \nu_e \nu_e NN \rightarrow NN \)
  - \( \nu_e e^\pm \rightarrow \nu_e e^\pm \)

**Anti-neutrino reactions**

- Charged-current
  - \( \bar{\nu}_e n \rightarrow \bar{\nu}_e n \)
  - \( \bar{\nu}_e p \rightarrow e^+ n \)
  - \( \nu_e \bar{\nu}_e \rightarrow e^- e^+ \)
  - \( \nu_e \nu_e NN \rightarrow NN \)
  - \( \bar{\nu}_e e^\pm \rightarrow \bar{\nu}_e e^\pm \)

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Martinez-Pinedo et al., J Phys G (2014)

Microscopic optical potential for neutron-rich systems

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Supernova simulations treat protons and neutrons as quasiparticles in the mean-field approximation.

Nuclear interactions are attractive at low momenta and

\[ |\langle np|V_{NN}|np\rangle| > |\langle nn|V_{NN}|nn\rangle| \]

Mean field effects further widen the energy gap between protons and neutrons.

Q-value for (anti-)neutrino absorption changes significantly.
Charged-current reactions ($\nu_e n \rightarrow e^- p$) with $E_\nu = 10$ MeV, $p_n = 100$ MeV

$$E_e = \sqrt{E_\nu^2 - 2E_\nu q \cos \theta + q^2 + m_e^2}$$  \text{lepton}$$
$$E_e = E_\nu + (E_n - E_p) = E_\nu - \frac{1}{2M} (q^2 + 2p_N q \cos \theta) + (M_n - M_p)$$  \text{nucleon}$$

Microscopic optical potential for neutron-rich systems

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(1) Chiral NN potential at mean-field level

(2) Pseudo-potential (reproduces **exact energy shift** when used at the mean field level)

\[
\langle p | V_{lSJ}^{\text{pseudo}} | p \rangle = -\frac{\delta_{lSJ}(p)}{pM_N}
\]

Fumi (1955), Fukuda & Newton (1956)

Nucleon energies: \( E_N(k) = \frac{k^2}{2M} + \Sigma_N(k) \simeq \frac{k^2}{2M^*} - U_N \)

Nuclear equation of state for astrophysical simulations

- Clustering at low densities, match to virial EoS
- Extrapolate to high-density, high-temperature regime

Optical potentials for neutron-rich nuclei

- Derive spin-orbit terms
- Fold with theoretical/empirical density distributions

Neutrino reactions in proto-neutron stars

- Develop consistent equation of state
- Merge with numerical simulations of core-collapse supernovae