Review on nucleon charges from lattice QCD

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QCD for New Physics at the Precision Frontier

September 28th, 2015
OUTLINE

A Motivation

B Nucleon on the Lattice
   • Introduction
   • Systematics

C Nucleon Charges
   • Axial Charge
   • Scalar & Tensor Charges

D Spin Structure of the Nucleon
   • Quark contributions
   • Gluon contributions

E Conclusions & Perspectives
A MOTIVATION
Lattice QCD meets Nature

Why Lattice QCD?

★ Well-established non-perturbative approach to QCD
★ Makes contact with well determined experimental measurements
★ Provides input for quantities not easily accessible in experiments
★ Interpretation of experimental data
★ Tests of SM and New Physics searches
Rich experimental activities in major facilities

- Investigation of baryon and meson structure
- Origin of mass and spin
- New physics searches: scalar/tensor interactions, \((g-2)_\mu\), dark photon, EDMs
- proton radius puzzle
- the list is long...

JPARC  
RHIC  
BES III  

JLAB  
ALICE  
COMPASS  

PSI  
MAMI
12GeV Upgrade at JLab

Continuous Electron Beam Accelerator Facility

“...to employ new methods for studying the basic properties of the building blocks of the universe, how they are formed, how they interact and the forces that mediate these interactions.” ... “expanding our knowledge of nuclear and particle physics well beyond its current level.”

Physics Program for CLAS12 (Selected Experiments)

- Spin/Flavor Structure of the Nucleon
- Nucleon Resonance Studies with CLAS12
- Origins of quark confinement
- High Precision Measurement of the Proton Charge Radius
- Scalar and Tensor interactions
- The Transverse Structure of the Hadrons
Proton Radius Puzzle

$\langle r_p^2 \rangle$ from muonic hydrogen $\mu p$ is $1.7\sigma$ smaller than hydrogen spectroscopy.

CREMA Collaboration

- Measured energy difference between the 2P and 2S states of muonic hydrogen
- $\mu p$: 10 times more accurate than other measurements
- Very sensitive to the proton size
- No obvious way to connect with other measurements (4% diff)

[R. Pohl et al. Nature 466, 213 (2010)]
[J. Bernauer et al. (2010), arXiv:1007.5076]
NUCLEONS ON THE LATTICE
Lattice formulation of QCD

- Space-time discretization on a finite-sized 4-D lattice
  \( L: \text{lattice size, } a: \text{lattice spacing} \)
  - \( \Psi(x), \bar{\Psi}(x) \): Quark fields on lattice points
  - \( U_\mu(x) \): Gauge fields (gluons) on links (Wilson lines)

- Finite degrees of freedom

- Construction of an action \( S = S_{\text{fermions}} + S_{\text{gluons}} \) (with correct continuum limit)

- Numerical simulations and perturbative lattice calculations
Improved fermion action

- **Clover improved Wilson**
  - ✓ computationally fast
  - ✗ broken chiral symmetry & requires operator improvement
  - ★ Employed by: ALPHA, BMW, CLS, LHPC, NPQCD, PACS-CS, QCDSF, RQCD

- **Twisted Mass**
  - ✓ computationally fast & automatic improvement
  - ✗ broken chiral symmetry & violation of isospin
  - ★ Employed by: ETMC

- **Staggered**
  - ✓ computationally fast &
  - ✗ 4 doublers & difficult contractions
  - ★ Employed by: MILC, LHPC

- **Overlap**
  - ✓ exact chiral symmetry
  - ✗ computationally expensive
  - ★ Employed by: JLQCD

- **Domain Wall**
  - ✓ improved chiral symmetry
  - ✗ computationally demanding & requires tuning
  - ★ Employed by: RBC-UKQCD

★ Fermion actions: $O(a)$-improved
★ Gluon actions: $O(a^2)$-improved
Probing Nucleon Structure

- Generalized Parton Distributions (GPDs)
  
  Introduced late '90s
  
  Deep inelastic scattering (DIS)
  
  Comprehensive description of hadron structure from first principles

- Parametrization of off-forward matrix of a bilocal quark operator (light-like)

\[
F_{\Gamma}(x, \xi, q^2) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p' | \bar{\psi}(-\lambda n/2) \mathcal{O} Pe^{-\lambda/2} \psi(\lambda n/2) | p \rangle
\]

- \( q = p' - p, \bar{P} = (p' + p)/2 \)
- \( n: \) light-cone vector \( (\bar{P} \cdot n = 1) \)
- \( \xi = -n \cdot \Delta/2 \)
- Choices of operators in LQCD:
  towers of local twist-2 operators

- Rely on OPE to extract moments

含信息于:
★ Form factors and parton distributions
★ quark orbital angular momentum
★ spin structure of the nucleon

A Unpolarized

\[ O^{\mu_1 \ldots \mu_n} = \bar{q} \gamma^{\mu} iD^{\mu_1} \ldots iD^{\mu_{n-1}} q \]

DIS, Drell-Yan, W-asymmetry, \( \gamma^+ \) jet, ...

B Helicity (polarized)

\[ \tilde{O}^{\mu_1 \ldots \mu_n} = \bar{q} \gamma_5 \gamma^{\mu} iD^{\mu_1} \ldots iD^{\mu_{n-1}} q \]

polarized DIS, SIDIS, pp collisions, photo/electro production, ...

C Transversity

\[ O^{\mu_1 \ldots \mu_{n-1}} = \bar{q} \sigma^{\mu} iD^{\mu_1} \ldots iD^{\mu_{n-1}} q \]

single-spin asymmetry in SIDIS, ...
1. **Diagrams:**

   ![Diagram 1](slide.png)
   
   ![Diagram 2](slide.png)

   **Connected**

   **Disconnected**

2. **Two-pt and three-pt functions:**

   **2pt:**
   \[
   G(\vec{q}, t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f \cdot \vec{q}} \Gamma^0_{\beta\alpha} \left\langle J_\alpha(\vec{x}_f, t_f) \bar{J}_\beta(0) \right\rangle
   \]

   **3pt:**
   \[
   G^G(\Gamma^\kappa, \vec{q}, t) = \sum_{\vec{x}_f, \vec{x}} e^{i\vec{x} \cdot \vec{q}} e^{-i\vec{x}_f \cdot \vec{p}} \Gamma^\kappa_{\beta\alpha} \left\langle J_\alpha(\vec{x}_f, t_f) \mathcal{O}(\vec{x}, t) \bar{J}_\beta(0) \right\rangle
   \]

   \[
   \Gamma^0 \equiv \frac{1}{4} (1 + \gamma_0)
   \]

   \[
   \Gamma^2 \equiv \Gamma^0 \cdot \gamma_5 \cdot \gamma_i
   \]

   and other variations
3. Optimized ratio:

\[ R_\mathcal{O}(\Gamma, \vec{q}, t) = \frac{G_\mathcal{O}(\Gamma, \vec{q}, t)}{G(\vec{0}, t_f)} \times \sqrt{\frac{G(-\vec{q}, t_f-t)G(\vec{0}, t)G(\vec{0}, t_f)}{G(\vec{0}, t_f-t)G(-\vec{q}, t)G(-\vec{q}, t_f)}} \]

\[ t_f \rightarrow \infty \quad \Pi(\Gamma, \vec{q}) \text{ (Plateau Method)} \]

4. Renormalization: connection to experiments

\[ \Pi^R(\Gamma, \vec{q}) = Z_\mathcal{O} \Pi(\Gamma, \vec{q}) \]

5. Extraction of form factors

e.g. Axial current:

\[ A_\mu^3 \equiv \bar{\psi} \gamma_\mu \gamma_5 \frac{\tau^3}{2} \psi \Rightarrow \bar{u}_N(p') \begin{pmatrix} G_A(q^2) \gamma_\mu \gamma_5 + G_\rho(q^2) \frac{q_\mu \gamma_5}{2 m_N} \end{pmatrix} u_N(p) \]

- Isovector Combination
  - disconnected contributions cancel out
  - Simpler renormalization

- Isoscalar Combination
  - disconnected contributions
  - operator mixing
Systematic uncertainties: Challenges & Progress

1. Cut-off Effects: finite lattice spacing
   - Continuum limit \( a \to 0 \)
   - Simulations with fine lattices (\( a < 0.1 \text{ fm} \))
   - Improve actions, algorithmic improvements

2. Finite Volume Effects
   - Infinite volume limit \( L \to \infty \)
   - Simulating hadrons in large volumes (Rule of thumb: \( L m_\pi > 3.5 \))

3. Contamination from other hadron states
   - Various methods for extracting information from lattice data

4. Not simulating the physical world
   - Chiral extrapolation
   - Simulations at physical parameters are now feasible

5. Renormalization and mixing
   - Subtraction of lattice artifacts, utilize perturbation theory
RENORMALIZATION: Lattice artifacts: important!

Synergy of perturbative and non-perturbative results

- Lattice artifacts computed perturbatively
- Subtraction from non-perturbative estimates

**Usage of momentum-source method:**
- Dirac equation solved with momentum source
- # of inversion depends on # of momenta considered
- Application of any operator
- High statistical accuracy

**Control of lattice artifacts (Lorentz non-invariant):**

\[
\frac{\sum_{\rho} p_{\rho}^4}{\left(\sum_{\rho} p_{\rho}^2\right)^2} < 0.4 \quad \text{(empirically)}
\]
C

NUCLEON CHARGES
**AXIAL CHARGE**

Nucleon Axial current:

\[ A^3_{\mu} \equiv \bar{\psi} \gamma_{\mu} \gamma_5 \frac{\tau^3}{2} \psi \]

\[
\langle N(p') O^a_A N(p) \rangle = \bar{u}_N(p') \left[ G_A(q^2) \gamma_{\mu} \gamma_5 + G_P(q^2) \frac{q_{\mu} \gamma_5}{2m_N} \right] u_N(p)
\]

\[ g_A \equiv \left. \langle N(p') O^a_A N(p) \rangle \right|_{q^2=0} = G_A(0) \]

- governs the rate of \( \beta \)-decay (Well-determined!)

- related to the fraction of the nucleon spin carried by the quarks
- On the lattice: requires the lowest moment and zero momentum
- determined directly from lattice data (no fit necessary)
Lattice data from 'plateau' methods
Latest achievement: lattice results at physical $m_\pi$
No necessity of chiral extrapolation
Different strategies for addressing systematic uncertainties

$g^\text{exp}_A = 1.2701(25)$ [PDG'12]

$m_\pi > 200\text{MeV}$: lattice results below exp: $\sim 10\%-15\%$

Systematic uncertainties
Excited States

$m_\pi = 310\text{ MeV}$

$m_\pi = 135\text{ MeV}$

[AAbdel-Rehim et al. (ETMC), arXiv:1507.04936]
Cut-off effects

Continuum extrapolation requires 3 lattice spacings

-C. Alexandrou et al. (ETMC), arXiv:1012.0857
-G. Bali et al. (RQCD), 2014
-R. Gupta et al. (PNDME), 2014

\( a < 0.1 \text{ fm is sufficient} \)

Finite Volume Effects

-128 \( \text{MeV} \leq m_\pi \leq 300 \text{ MeV} \)

![Graph showing lattice data for plateau method and finite volume effects](image)

Lattice data for plateau method
No volume corrections

Volume effects not fully understood
SCALAR & TENSOR CHARGES

★ Non $V - A$ structure of weak interaction

★ Small contributions of scalar/tensor interactions in SM
  ($10^{-3}$)

★ $\epsilon_S, \epsilon_T$: low-energy couplings

$$H_{eff} = G_F \left( J_{VA}^l \times J_{VA}^q + \sum_i \epsilon_i O_i^l \times O_i^q \right)$$

▶ related to masses of new TeV-scale particles
▶ require knowledge of $g_S$: $\langle p|\bar{u}d|n\rangle$, $g_T$: $\langle p|\bar{u}\sigma^{\mu\nu}d|n\rangle$

★ scalar interactions: $0^+ \rightarrow 0^+$ nucleon decays

★ tensor interactions: radiative pion decay $\pi \rightarrow e\nu\gamma$

★ Upcoming experiments (TeV scale) that probe small signals:
  UCNB & UCNb at LANL, Nab at ORNL, ATLAS at LHC
SCALAR CHARGE

- sensitivity of $m_N$ to $m_q$: \[ \langle N|\bar{q}q|N \rangle = \frac{\partial m_N}{\partial m_q} \]
- no direct experimental measurements
- Indirect measurements: meson-nucleon scattering amplitudes (large system.)
- related to nucleon \( \sigma \) - terms:
  \[ \sigma_l = \frac{1}{2} (m_u + m_d) \langle N|\bar{u}u + \bar{d}d|N \rangle \]
  \[ \sigma_s = m_s \langle N|\bar{s}s|N \rangle \]
  nucleon mass generated by the quarks via spontaneous chiral symmetry breaking

Strange quark content of nucleon:
\[ y_N = \frac{2 \langle N|\bar{s}s|N \rangle}{\langle N|\bar{u}u + \bar{d}d|N \rangle} = 1 - \frac{\sigma_0}{\sigma_l} \]

- important for direct search of dark matter
  large coupling of strange quarks to candidate dark matter

[J. Ellis et al., arXiv:0801.3656]

Lattice calculations

Direct Method
\[ \langle N|\bar{q}q|N \rangle \text{ (3pt CI & DI)} \]
discussed in this talk

Spectrum Method
Feynman-Hellmann on \[ \frac{\partial m_N}{\partial m_q} \]
R. Young, [arXiv:1301.1765]
SCALAR CHARGE: The Squiggly One

\[ g_S \equiv \langle N|\bar{u}u - \bar{d}d|N\rangle|_{Q^2=0} \]

Challenging calculation:
- smallest signal-to-noise ratio
- systematics are not well-controlled
- disconnected contributions not negligible
- requires vacuum subtraction

★ Severe contamination of excited states

TMF & Clover: \( N_f = 2 \) \( m_\pi = 135 \text{MeV} \)
- Increasing trend for plateau value for large \( T_{\text{sink}} \)

[L.A. Abdel-Rehim et al. (ETMC), arXiv:1507.04936]
\( \sigma - \text{terms} \)

\[ m_\pi = 135 \text{MeV} \]

[AA Abdel-Rehim et al. (ETMC), preliminary]

**Phenomenology:**

\( \sigma_l = 45 \pm 8 \text{MeV} \)

[J. Gasser et al., PLB 253(1991) 252]

\( \sigma_l = 64 \pm 7 \text{MeV} \)

[M. Pavan et al., hep-ph/0111066]

\( \sigma_l = 59 \pm 7 \text{MeV} \)

[J. Alarcon et al., arXiv:1110.3797]

\[ \sigma_l (\text{MeV}) \]

[R. Young, Lattice 2012]

[A. Vaquero et al., Lattice 2015]
Tensor Charge

\[ g_T \equiv \langle N | \sigma^{\mu\nu} | N \rangle |_{Q^2=0} \]

**SIDIS results (HERMES, COMPASS) and BELLE e+ e- analysis**

- \( g_T^{\exp} (0.8 \text{GeV}^2) = 0.77^{+0.13}_{-0.27} \)
  
  [M.Anselmino et al., arXiv:0812.4366]

**Agreement among most lattice points**

**Mild \( m_\pi \) dependence**

**\( g_T^{IV} \) input in analysis of neutron \( \beta \)-decay**

**Strong scale-dependence**
Investigation of Systematics

\[ m_\pi = 135 \text{MeV}, \ a = 0.093 \text{fm}, \ T_{\text{sink}}: 0.93 - 1.31 \text{fm} \]

\[ m_\pi = 290 \text{MeV}, \ a = 0.071 \text{fm}, \ T_{\text{sink}}: 0.5 - 1.2 \text{fm} \]

\[ T_{\text{sink}} > 1 \text{ fm is safe} \]

Little sensitivity to \( m_\pi, \ a, \ T_{\text{sink}} \)
Implication of $g_T$ to New Physics Searches

- **New interaction at TeV scale:**
  - source of $C\,P$ violation
  - may give rise to nEDM (quark-photon coupling)

- $g_T$ related to the quark contributions to the nEDM:
  
  \[ d_n = d_u g_T^u + d_d g_T^d + d_s g_T^s \]

  - LQCD may constrain the low-energy effective couplings $d_u, d_d, d_s$
  - individual quark contributions $\Rightarrow$ disconnected contributions
  - current best exp. upper limit:
    \[ |d_n| < 2.9 \times 10^{-26} \text{ e cm (90\% C.L.)} \]

  \[ (\text{ILL Grenoble}) \]

  \underline{90\% confidence interval bounds of $d_u$, $d_n$ (Assumption: $g_T^s = 0$)}

\[
\begin{align*}
\chi_{\text{PT}} \text{ to 135MeV, } a \to 0, \ L \to \infty \\
g_T^u &= +0.774(66) \\
g_T^d &= -0.233(28) \\
\text{Include DI}
\end{align*}
\]

\[ [T. Bhattacharya et al. (PNDME), arXiv:1506.06411] \]

Talks by G. Schierholz & T. Bhattacharya on Thu
Consequences in split SUSY models


★ all scalars much heavier than electroweak scale (except one Higgs doublet)
★ preserves gauge coupling unification
★ there is a dark matter candidate
★ avoids constraints related to flavor & CP problem
★ qEDMs leading contributions
★ gaugino ($M_2$) and Higgsino $\mu$ masses, their relative phase $\phi$ and Higgs vacuum expectation value $\tan(\beta)$

\[ d_e = 8.7 \times 10^{-29} \text{ e cm (90\% CL)} \] (ACME)

Not overlapping bands due to precision of lattice results!
\[ d_n < 4 \times 10^{-28} \text{ e cm} \] for split-SUSY to hold

Talks by V. Cirigliano & T. Bhattacharya on Thursday

Thanks to V. Cirigliano for material
SPIN STRUCTURE
Understanding of nucleon spin has evolved:

- Simple parton model
  \[ \frac{1}{2} (\Delta u_v + \Delta d_v) = \frac{1}{2} \]

- Sea quarks & Gluons are polarized
  \[ \frac{1}{2} (\Delta q + \Delta \bar{q}) + \Delta G = \frac{1}{2} \]

- Parton orbital angular momentum
  \[ \frac{1}{2} (\Delta q + \Delta \bar{q}) + \Delta G + L_z = \frac{1}{2} \]

Where does the nucleon spin come from? Exper. Status

- Quark Contributions
  - Spin 20% - 30% (DIS)
  - Orbital angular momentum (Upcoming experiments of GPDs and TMDs)

- Gluon Contributions
  - Spin 40% (STAR, PHENIX, COMPASS)
  - Orbital angular momentum zero

[S. Brodsky et al., hep-ph/0608219]

There is a need to find the missing contributions to the spin!
**NUCLEON SPIN**

**Input from Lattice QCD**

**Spin Sum Rule:**
\[
\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left( L^q + \frac{1}{2} \Delta \Sigma^q \right) + J^G
\]

**Extraction from LQCD:**
\[
J^q = \frac{1}{2} (A_{20}^q + B_{20}^q) , \quad L^q = J^q - \Sigma^q , \quad \Sigma^q = g_A^q
\]

**Status of Lattice Calculations**

- **Quark Contributions**
  - Quark Spin (Connected) \( \sim 40\% - 50\% \)
  - Light Quark Spin (Disconnected) \( \sim 5\% - 7\% \)
  - Strange Quark Spin (Disconnected) \( \sim 3\% \)
  - Orbital angular momentum \( L^u + d \sim 0 \)
    \( L^u \sim -L^d \)
  - Total Spin carried almost exclusively by the up quark \( J^d \sim 0 \)
Gluon unpolarized distribution

\[\text{Lattice Calculations}\]

Disconnected diagram is required

- Direct lattice computation of gluon moment \(\langle x \rangle_g\):

Gluon Operator

\[O_{\mu\nu}^g = -\text{Tr} \left[ G_{\mu\rho} G_{\nu\rho} \right]\]

\[
\langle N(p)| O_{44} - \frac{1}{3} \sum_{j=1}^{3} O_{jj} | N(p) \rangle = \left( m_N + \frac{2}{3} E_N \vec{p}^2 \right) \langle x \rangle_g
\]

- Decomposition of Energy-momentum Tensor

\[
J^i_{q,g} = \frac{1}{2} \epsilon^{ijk} \int d^3 x \left( \mathcal{T}_{q,g}^{0k} x^j - \mathcal{T}_{q,g}^{0j} x^k \right)
\]

\[
\tau_{\{4i\}}^{(E)} q = -\frac{i}{4} \sum_f \bar{\psi}_f \left[ \gamma_4 \overset{\hspace{1cm}}{D}_i + \gamma_i \overset{\hspace{1cm}}{D}_4 - \gamma_4 \overset{\hspace{1cm}}{D}_i - \gamma_i \overset{\hspace{1cm}}{D}_4 \right] \psi_f
\]

\[
\tau_{\{4i\}}^{(E)} g = -\frac{i}{2} \sum_{k=1}^{3} 2 \text{Tr}^c \left[ G_{4k} G_{ki} + G_{ik} G_{k4} \right]
\]
Lattice Results

Quenched

Feynman-Hellmann

\[ N_f=0 \text{ Clover, } m_\pi = 314 - 555 \text{ MeV} \]
\[ \langle x \rangle_g = 0.43(7)(5) \]

Energy-Momentum tensor

\[ N_f=0 \text{ Wilson, } m_\pi = 478 - 650 \text{ MeV} \]
\[ \langle x \rangle_g = 0.313(56) \]

Dynamical

\[ N_f=2+1+1 \text{ ETM, } m_\pi = 375 \text{ MeV} \]
\[ \langle x \rangle_g = 0.309(25) \]

\[ N_f=2 \text{ TMF & Clover, } m_\pi = 135 \text{ MeV} \]
\[ \langle x \rangle_g = 0.283(41) \]

Smearing: improves signal
Challenges with Renormalization of Gluon operator

- Mixing with Quark singlet operator:
  \[ O^q \equiv \bar{\psi} \gamma^\mu \overset{\leftrightarrow}{D}^\nu \psi \]
  Unavoidable

- Mixing with other Operators:
  - Gauge invariant
  - BRS-variations
  - vanish by the e.o.m.

Vanish in physical matrix elements

\[ \begin{pmatrix} \langle x \rangle_{g}^{\overline{\text{MS}}} (\mu) \\ \sum_q \langle x \rangle_{q}^{\overline{\text{MS}}} (\mu) \end{pmatrix} = \begin{pmatrix} Z_{gg}^{\overline{\text{MS}}} (\mu) & Z_{gq}^{\overline{\text{MS}}} (\mu) \\ Z_{qg}^{\overline{\text{MS}}} (\mu) & Z_{qq}^{\overline{\text{MS}}} (\mu) \end{pmatrix} \begin{pmatrix} \langle x \rangle_{q} \\ \sum_q \langle x \rangle_{q} \end{pmatrix} \]

\[ \langle x \rangle_{g}^{R} = Z_{gg} \langle x \rangle_{g}^{B} + Z_{gq} \sum_q \langle x \rangle_{q}^{B} \]
and
\[ \sum_q \langle x \rangle_{q}^{R} = Z_{qq} \sum_q \langle x \rangle_{q}^{B} + Z_{qg} \langle x \rangle_{g}^{B} \]

★ Quenched case: \( Z_{qq} = 1 - Z_{qq}, Z_{gq} = 1 - Z_{qq} \)
Perturbative computation

\[ Z_{qq} : \quad \Lambda_{qq} = \langle q|O_q|q \rangle \]

\[ Z_{qg} : \quad \Lambda_{qg} = \langle q|O_q|g \rangle \]

\[ Z_{gq} : \quad \Lambda_{gq} = \langle g|O_q|q \rangle \]

\[ Z_{gg} : \quad \Lambda_{gg} = \langle g|O_g|g \rangle \]

many millions of terms...
- Multiplicative renormalization
- Identification of mixing
- General action parameters
- (wide applications)
- Stout smearing (action & operator)

**Example**

Clover fermions, Iwasaki gluons, 2 stout smearing steps for $O_g$

\[
Z_{gg} = 1 + \frac{g^2}{16\pi^2} \left( 1.0574 N_f + \frac{-13.5627}{N_c} - \frac{2 N_f}{3} \log(a^2 \bar{\mu}^2) \right)
\]

\[
Z_{gq} = 0 + \frac{g^2 C_f}{16\pi^2} \left( 0.8114 + 0.4434 c_{SW} - 0.2074 c_{SW}^2 + \frac{4}{3} \log(a^2 \bar{\mu}^2) \right)
\]

\[
Z_{qq} = 1 + \frac{g^2}{16\pi^2} \left( -1.8557 + 2.9582 c_{SW} + 0.3984 c_{SW}^2 - \frac{8}{3} \log(a^2 \bar{\mu}^2) \right)
\]

\[
Z_{qg} = 0 + \frac{g^2 N_f}{16\pi^2} \left( 0.2164 + 0.4511 c_{SW} + 1.4917 c_{SW}^2 - \frac{4}{3} \log(a^2 \bar{\mu}^2) \right)
\]

[M. Constantinou et al. (Cyprus Group), 2015]

**Application for** $N_f=2$ TMF & clover, $m_{\pi}=135$ MeV:

\[
\langle x \rangle^R_{u+d} = 0.587(18) \quad \langle x \rangle^R_g = 0.283(41) \quad \Rightarrow \quad \langle x \rangle^R_{u+d} + \langle x \rangle^R_{u+d} = 0.870(43)
\]

Missing quark disconnected contributions
CONCLUSIONS
SUMMARY & CHALLENGES

- Simulating the physical world
- New physics BSM
  - Lattice QCD provides predictions
- Dedication of human force and computational resources on:
  - Control of statistical uncertainties $\Rightarrow$ noise reduction techniques crucial
  - Comprehensive study of systematic uncertainties
  - Study of DI at lower masses (Target: physical $m_\pi$!)
    - Challenging task
    - Exploid techniques: AMA, hierarchical probing, others
    - Usage of GPUs
    - Current computations of DI provide bounds
- Nucleon spin: dynamical simulations for gluon angular momentum
  - Becoming feasible
  - Overcoming difficulties with renormalization and mixing
  - Rely on perturbation theory

Stay Tuned!
THANK YOU
BACKUP SLIDES
Computation of Observables

Configuration Generation

Quark Propagators

\[ \langle O \rangle = \frac{1}{Z} \int U \mathcal{O}(D^{-1}, U) \det(D[U]^{Nf}) e^{-S[U]} \]

Contraction of propagators

(Data analysis)

Physics!
RENORMALIZATION

RI' scheme:

\[
Z_q = \frac{1}{12} \text{Tr}\left[\left(S^L(p)\right)^{-1} S^{\text{Born}}(p)\right] \bigg|_{p^2 = \mu^2} \\
Z_q^{-1} \frac{1}{12} \text{Tr}\left[\Gamma^L_{\bar{O}}(p) \left(\Gamma^{\text{Born}}_{\bar{O}}(p)\right)^{-1}\right] \bigg|_{p^2 = \bar{\mu}^2} = 1
\]

☆ Conversion to $\overline{\text{MS}}(\mu = 2\text{GeV})$:

conversion to $\overline{\text{MS}}(\mu = 2\text{GeV})$ under control

Systematics due to conversion to $\overline{\text{MS}}$ under control

Scalar Operator: 3-loop expressions necessary

[M. Constantinou et al. (QCDSF), arXiv:1408.6047]
LHPC: $m_\pi = 149 - 356\text{MeV}$
[J.R. Green et al. (LHPC), arXiv:1206.4527]

$\star m_\pi = 149\text{MeV}: 0.93\text{ fm } \langle T_{\text{sink}} \rangle = 139\text{ fm}$

PNDME: $m_\pi = 220\text{MeV}$
[T. Bhattacharya et al. (PNDME), arXiv:1501.07639]

$\star m_\pi = 149\text{MeV}: 0.9\text{ fm } \langle T_{\text{sink}} \rangle = 126\text{ fm}$

Increasing trend for the plateau value for larger values of $T_{\text{sink}}$

RQCD: $m_\pi = 150\text{MeV}$
[G. Bali et al. (RQCD), arXiv:1412.7336]

$\star 0.64\text{ fm } \langle T_{\text{sink}} \rangle = 107\text{ fm}$

ETMC: $N_f = 2$ & $c_{SW}, m_\pi = 135\text{MeV}$
[A. Abdel-Rehim et al. (ETMC), arXiv:1507.04936]

$\bullet 0.93\text{ fm } \langle T_{\text{sink}} \rangle = 13\text{ fm}$

$\bullet T_{\text{sink}} \geq 15\text{ fm}: \text{agreement with SM}$
**NUCLEON SPIN**

**Input from Lattice QCD**

**Spin Sum Rule:**

\[
\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left( L^q + \frac{1}{2} \Delta \Sigma^q \right) + J^G
\]

**Extraction from LQCD:**

\[
J^q = \frac{1}{2} (A^q_{20} + B^q_{20}), \quad L^q = J^q - \Sigma^q, \quad \Sigma^q = g_A^q
\]

- Individual quark contributions: disconnected insertion contributes
- Status of proper \(Z_0^{singlet} \): Perturbatively, Feynman-Hellmann

We need to compute all-to-all propagator
- extremely difficult to compute
- very noisy and very expensive computationally
- We've come far in development of techniques:
  - Truncated Solver Method
  - One-end-trick
  - All-Mode-Averaging
  - Hierarchical probing
**Agreement of various results**

**DI for \( g_A \) lower the total value**

[T. Bhattacharya et al., arXiv:1503.05975]
Gluon unpolarized distribution

**Experimental Status**

- Spin 40% (STAR, PHENIX, COMPASS)
- Orbital angular momentum zero
- Glue helicity 0.2 (STAR, COMPASS)

**Lattice Calculations**

- Direct lattice computation of gluon moment $\langle x \rangle_g$:

  **Gluon Operator**

  $$ O^g_{\mu
u} = -\text{Tr} [G_{\mu\rho} G_{\nu\rho}] $$

  $$ \langle N(p) | O_{44} - \frac{1}{3} \sum_{j=1}^{3} O_{jj} | N(p) \rangle = \left( m_N + \frac{2}{3 E_N} \vec{p}^2 \right) \langle x \rangle_g $$

- Decomposition of Energy-momentum Tensor

  $$ J^i_{q,g} = \frac{1}{2} \epsilon^{ijk} \int d^3 x \left( \mathcal{T}^{0k}_{q,g} x^j - \mathcal{T}^{0j}_{q,g} x^k \right) $$

  $$ \mathcal{T}^{(E)}_{\{4i\} q} = -\frac{i}{4} \sum_f \bar{\psi}_f \left[ \gamma_4 \vec{D}_i + \gamma_i \vec{D}_A - \gamma_4 \vec{D}_i - \gamma_i \vec{D}_A \right] \psi_f $$

  $$ \mathcal{T}^{(E)}_{\{4i\} g} = -\frac{i}{2} \sum_{k=1}^{3} 2 \text{Tr}^c \left[ G_{4k} G_{ki} + G_{ik} G_{4k} \right] $$