I. Motivation: impact of viscosity on fluctuations and correlations

II. Hydrodynamics modes: fluctuations and dissipation
   a. Viscous diffusion of transverse shear modes
   b. 1st and 2nd order hydrodynamics

III. Contributions to correlation measurements

Work in progress!
small variations in transverse flow in each event

viscous friction as fluid elements flow past one another

**shear viscosity drives velocity toward the average**

\[
T_{zr} = -\eta \frac{\partial v_r}{\partial z}
\]

damping of transverse flow fluctuations $\Rightarrow$ viscosity

**viscosity:**  
SG & Abdel-Aziz, PRL 97 (2006) 162302

**baryon diffusion:**  
SG & Abdel-Aziz, PR C70 (2004) 034905

**$\phi$ correlations (CME):**  
Pratt, Schlichting, SG, PR C84 (2011) 024909
Momentum in Fluctuating Hydrodynamics

**Momentum current** – small fluctuations

\[ M_i \equiv T_{0i} - \langle T_{0i} \rangle \approx (e + p)v_i \approx sTv_i \]

momentum conservation – linearized Navier-Stokes

\[ \partial_t M_i + \nabla_i p = \frac{\eta/3 + \zeta}{sT} \nabla_i (\nabla \cdot \vec{M}) + \frac{\eta}{sT} \nabla^2 M_i \]

Helmholtz decomposition:

\[ \vec{M} = \vec{g}_L + \vec{g} \]

“longitudinal” mode: \[ \nabla \times \vec{g}_L = 0 \]

“transverse” modes: \[ \nabla \cdot \vec{g} = 0 \]
transverse modes: **viscous diffusion**

\[ \partial_t \vec{g} = \nu \nabla^2 \vec{g}, \quad \nu = \frac{\eta}{T_s} \]

- no transverse ‘sound waves’
- vorticity \( \vec{\omega} \propto \nabla \times \vec{g} \)

longitudinal modes

\[ \partial_t \vec{g}_L + \nabla p = \frac{4}{3} \frac{\eta + \zeta}{sT} \nabla (\nabla \cdot \vec{g}_L) \]

longitudinal modes + energy and baryon conservation imply:

**sound waves** – compression waves, damped by viscosity

**thermal diffusion** – heat flow relative to baryons
Transverse Flow Fluctuations

Transverse velocity fluctuations $\rightarrow$ vorticity $\rightarrow$ "transverse" shear modes

\[ T_{0i} - \langle T_{0i} \rangle \approx \gamma_i \]

\[ T_{ji}^{\text{diss}} \approx -\eta \nabla_j v_i = -v \nabla_j g_i + \text{Langevin noise} \]

**diffusion equation** for momentum current

\[ \frac{\partial}{\partial t} g_r = \nu \nabla^2 (g_r + \text{noise}) \]

**correlation function** measures deviation of fluctuations from mean

\[ r = \langle g_r(x_1)g_r(x_2) \rangle - \langle g_r(x_1) \rangle \langle g_r(x_2) \rangle \]

SG & Abdel-Aziz, PRL 97 (2006) 162302
Rapidity Dependence of Transverse Momentum Correlations

**momentum flux density correlation function**

\[ r = \langle g_r(x_1)g_r(x_2) \rangle - \langle g_r(x_1) \rangle \langle g_r(x_2) \rangle \]

\[ \Delta r = r - r_{eq} \text{ satisfies diffusion equation} \]

Gardiner, Handbook of Stochastic Methods, (Springer, 2002)

**fluctuations diffuse through volume, driving**

width in relative spatial rapidity grows \( y = \sinh^{-1} \frac{z}{\tau} \)

from initial value \( \sigma_0 \)

\[ \sigma^2 = \sigma_0^2 + 4 \frac{\eta}{Ts} \left( \frac{1}{\tau_0} - \frac{1}{\tau} \right) \]

 spatial rapidity  

\( r_{eq} \)
Diffusion vs. Wave Motion

**Diffusion (1\textsuperscript{st} Order)**

- Gaussian peak spreads
- tails violate causality

**Wave propagation – e.g. sound waves**

- peak splits into left and right traveling pulses
- propagation speed $c_s$
2nd Order Viscous Diffusion

causal transport equation:

- transverse modes
- derived from linearized Israel-Stewart hydro equations

coordinate space:

- wave-fronts traveling at speed \( \frac{\nu}{\tau_{\pi}} \)^{1/2}
- diffusion-like behavior in between
- no peak at \( \Delta z = 0 \)

\[ \Delta r = r - r_{eq} \]

\[ \tau_{\pi} \frac{d^2}{dt^2} + \frac{d}{dt} - \nu \left( \nabla^2_1 + \nabla^2_2 \right) \Delta r = 0 \]

relaxation time \( \tau_{\pi} \sim \) (mean free path)/(thermal speed)
spatial rapidity

- rapidity separation of fronts saturates
  \[ \Delta \eta \sim \Delta z / \tau \]

- profile depends on initial width \( \sigma_0 \)
2\textsuperscript{nd} Order Viscous Diffusion in Rapidity

\[ \left( \tau \frac{\partial^2}{\partial \tau^2} + \frac{\partial}{\partial \tau} - \frac{v}{\tau^2} \left( \frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2} \right) \right) \Delta r = 0 \]

spatial rapidity

- rapidity separation of fronts saturates
  \[ \Delta \eta \sim \Delta z / \tau \]
- profile depends on initial width \( \sigma_0 \)
Measuring the Correlations

correlation function

\[ r = \langle g_t(x_1)g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle \]

observable:

\[ C = \frac{1}{\langle N \rangle^2} \left\langle \sum_{\text{pairs}} p_{ij} p_{ij} \right\rangle - \left\langle p_t \right\rangle^2 = \frac{1}{\langle N \rangle^2} \int (r - r_{eq}) \, dx_1 \, dx_2 \]

$p_t$ Covariance Measured

**measured:** rapidity width of near side peak

- fit peak + constant offset
- offset is ridge, i.e., long range rapidity correlations
- report rms width of the peak

**find:** width increases in central collisions

$\sigma_{\text{central}} = 1.0 \pm 0.2$

$\sigma_{\text{peripheral}} = 0.54 \pm 0.02$
Rapidity Width Increases in Central Collisions

Central vs. peripheral increase consistent with $\eta/s = 0.17 \pm 0.08$

NeXSPheRIO calculations **fail**


ideal fluctuating hydro doesn’t explain measured growth of width
2\textsuperscript{nd} Order Viscous Diffusion

**causal transport equation:**

\[
\left( \tau_\pi \frac{\partial^2}{\partial \tau^2} + \frac{\partial}{\partial \tau} - \frac{\eta}{sT} (\nabla_1^2 + \nabla_2^2) \right) \Delta r_g = 0
\]

relaxation time \( \tau_\pi \sim \) (mean free path)/(thermal speed)

kinetic theory \( \tau_\pi = \beta (\eta / sT) \) \( \beta \approx 5 \)

**temperature vs time:**

entropy production:
\[
\frac{dT dS}{dt} = \text{viscous heating}
\]

\[
\frac{ds}{d\tau} + \frac{s}{\tau} = \frac{\Phi}{T \tau}
\]

relaxation equation: causality
 delays heating
\[
\frac{d\Phi}{d\tau} = -\frac{1}{\tau_\pi} \left( \Phi - \frac{4 \eta}{3 \tau} \right) - \left[ \frac{1}{\tau} + \frac{\eta T}{\tau_\pi} \frac{d}{d\tau} \left( \frac{\tau_\pi}{\eta T} \right) \right] \frac{\Phi}{2}
\]

Pokharel, Moschelli, S.G. in preparation
Minimum Viscosity Near $T_c$

**sQGP viscosity**  Hirano & Gyulassy

- pQCD at high $T$; hadron gas at low $T$
- limit at $T = T_c$: $\eta / s = 1 / 4\pi$

**EOS-I** – Niemi, Denicol et al.
- Lattice – HotQCD Collaboration
- Lattice viscosity $T > T_C$ – Nakamura & Sakai
- Hagadorn HG – Noronha-Hostler et al.

**EOS-II** – Hirano & Gyulassy
- Bag Model EOS
- QGP viscosity $T > T_C$ – Danielewicz & Gyulassy
- Pion gas $T < T_C$ – Gavin
Time Dependence of Correlation Profile

Compute contribution from early time diffusion in rapidity

\[
\left( \tau \frac{\partial^2}{\partial \tau^2} + \frac{\partial}{\partial \tau} - \nu \left( \frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2} \right) \right) \Delta r = 0
\]

Gaussian initial profile, fixed width

\[
\sigma_0 \approx \sigma_{\text{peripheral}}
\]

Separate peaks?
depends on EOS through \(\nu(\tau)\)

\[C (\text{GeV}^2)\]
**Rapidity Dependence of Covariance vs. Centrality**

C. Pruneau, M. Sharma (STAR)
private communications

freeze out time

$$\tau_F - \tau_0 \propto (R - R_0)^2$$

2nd Order Diffusion, EOS I

$$d\Delta r/d\tau|_0 = 0, \tau_0 = 0.91 \text{ fm}$$

$$\tau_F(b=0) = 12.7 \text{ fm}$$

$$T_C = 155 \text{ MeV}$$

$$T_F = 143 \text{ MeV}$$

$$T_0(b=0) = 209 \text{ MeV}$$

$$\sigma_0 = 0.50$$

**Important:** tails inflate extracted widths
Rapidity Width of Momentum Covariance

freeze out time
\[ \tau_F - \tau_0 \propto (R - R_0)^2 \]

2\textsuperscript{nd} Order Diffusion, EOS I
\[ d\Delta r/d\tau|_{\tau_0} = 0, \quad \tau_0 = 0.91 \text{ fm} \]
\[ \tau_F(b=0) = 12.7 \text{ fm} \]
\[ T_C = 155 \text{ MeV} \]
\[ T_F = 143 \text{ MeV} \]
\[ T_0(b=0) = 209 \text{ MeV} \]
\[ \sigma_0 = 0.50 \]

Important: reported widths include tails

Pokharel, Moschelli, S.G. in preparation

$t_0 = 0.6667 \text{ fm}$

$T_C = 0.155 \text{ GeV}$

$T_F = 0.1434 \text{ GeV}$

$t_0 = 0.7 \text{ fm}$

$t_{fc} = 9 \text{ fm}$

$T_C = 0.155 \text{ GeV}$

$T_F = 0.150 \text{ GeV}$

1st Order Diffusion CAN’T Describe Rapidity Shape
**1st Order Diffusion: Gaussian Doesn’t Describe Shape**

<table>
<thead>
<tr>
<th>Region</th>
<th>Solid EOS I</th>
<th>Dashed EOS II</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5%</td>
<td>t₀ = 0.6667 fm, t_{fc} = 9 fm, T_C = 0.155 GeV, T_F = 0.1434 GeV</td>
<td>t₀ = 0.7 fm, t_{fc} = 9 fm, T_C = 0.155 GeV, T_F = 0.150 GeV</td>
</tr>
<tr>
<td>5-10%</td>
<td></td>
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<tr>
<td>10-20%</td>
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<tr>
<td>20-30%</td>
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<tr>
<td>30-40%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40-50%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Gaussian** Doesn’t Describe Shape.
Gaussian initial distribution $\Delta r$ at $\tau = \tau_0$, width $\sigma_0$

Initial derivative possibilities:

**Near equilibrium**

\[
\left. \frac{\partial \Delta r}{\partial \tau} \right|_{\tau=\tau_0} = \frac{\nu}{\tau^2} \left( \frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2} \right) \Delta r
\]

- fits valley better

**Nonequilibrium:**

\[
\left. \frac{\partial \Delta r}{\partial \tau} \right|_{\tau=\tau_0} = 0
\]

- better overall shape

Moschelli, Pokharel, S.G. in preparation
Near Equilibrium Initial Conditions Enhance Valley

Solid Black
EOS I
2nd Order Diffusion
nonequilibrium IC
τ₀ = 0.911 fm
τ_{Fc} = 12.667 fm
T_F = 0.1434 GeV
T₀ central = 0.209 GeV

Solid Green
EOS I
2nd Order Diffusion
near equilibrium IC
τ₀ = 1.1 fm
τ_{Fc} = 9 fm
T_F = 0.1434 GeV
T₀ central = 0.191 GeV
$\tau_0 = 1.1 \text{ fm}$
$\tau_{Fc} = 9 \text{ fm}$
$T_C = 0.155 \text{ GeV}$
$T_F = 0.1434 \text{ GeV}$
$T_0 \text{ central} = 0.191 \text{ GeV}$

$\tau_0 = 1.1 \text{ fm}$
$\tau_{Fc} = 9 \text{ fm}$
$T_C = 0.155 \text{ GeV}$
$T_F = 0.150 \text{ GeV}$
$s_0 = 0.49874$
Prediction: Valley Disappears at Low Energy

Beam Energy Scan prediction – expect less prominent valley at lower energy

Solid Green
200 GeV
$t_0 = 1.1 \text{ fm}$
$t_{Fc} = 9 \text{ fm}$
$T_c = 0.1434 \text{ GeV}$

Dashed Green
19.6 GeV, EOS I
$2^{nd}$ Order Diffusion
$t_0 = 2 \text{ fm}$
$t_{Fc} = 8 \text{ fm}$
$T_c = 0.130 \text{ GeV}$

$C [\text{GeV}^2]$ vs. $\Delta \eta$ for different beam energies and collision percentages.
Hydro formulation: longitudinal and transverse modes

- Sound waves, shear modes, and heat modes
- Diffusive transverse shear modes important for rapidity dependence of $p_t$ correlations
- 1st and 2nd order viscous fluctuating hydro description of shear modes

Causality shapes the rapidity dependence of correlations

Open Questions

- Influence of sound and heat modes on observables
- Charge balancing, resonances, jets, HBT
Covariance Measures Momentum Flux

covariance
\[
C = \frac{1}{\langle N \rangle^2} \left\langle \sum_{\text{pairs } i \neq j} p_{ti}p_{tj} \right\rangle - \langle p_t \rangle^2
\]

unrestricted sum:
\[
\sum_{\text{all } i,j} p_{ti}p_{tj} = \int p_{t1}p_{t2}dn_1dn_2
\]

\[
dx_1dx_2 = \int dp_1p_{t1}f_1(\int dp_2p_{t2}f_2) = \langle N \rangle^2 \langle p_t \rangle^2 + \int g(x_1)g(x_2)dx_1dx_2
\]

correlation function:
\[
r_g = \langle g_t(x_1)g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle
\]

\[
\int r_gdx_1dx_2 = \langle \sum p_{ti}p_{tj} \rangle - \langle N \rangle^2 \langle p_t \rangle^2 = \langle \sum p_{ti}^2 \rangle + \langle N \rangle^2 C
\]

\[
C = 0 \text{ in equilibrium}
\]

\[
C = \frac{1}{\langle N \rangle^2} \int (r_g - r_{g,eq})dx_1dx_2
\]
Experimental Fit

![Experimental Fit](image-url)

Legend:
- **Numerical**
- **Numerical FO**
- **STAR**

Y-axis: Value range from 0 to 0.005

X-axis: Value range from 0 to 350

N_{part}