ROTONS AND STRIPES
IN SPIN-ORBIT COUPLED BECs

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Stimulating discussions with:
Jean Dalibard, Gabiele Ferrari, Giacomo Lamporesi
Two detuned ($\Delta \omega_L$) and polarized laser beams + non linear Zeeman field ($\omega_Z$) provide Raman transitions between two spin states, giving rise to new single particle physics.
New single particle Hamiltonian

\[ h_0 = \frac{1}{2} \left[ (p_x - k_0 \sigma_z)^2 + p_{\perp}^2 \right] + \frac{1}{2} \Omega \sigma_x + \frac{1}{2} \delta \sigma_z \]

- \( p_x \) is canonical momentum
- \( k_0 \) is laser wave vector difference
- \( \Omega \) is strength of Raman coupling
- \( \delta = \Delta \omega_L - \omega_z \) is effective Zeeman field

physical velocity equal to

\[ v_x = (p_x - k_0 \sigma_z) / m \]

Hamiltonian \( h_0 \)
- is translationally invariant despite the presence of the laser fields \( [h_0, p_x] = 0 \)
- Breaks parity, time reversal and Galilean invariance
Symmetry properties of the spin-orbit Hamiltonian

\[ h_0 = \frac{1}{2} \left[ (p_x - k_0 \sigma_z)^2 + p_\perp^2 \right] + \frac{1}{2} \Omega \sigma_x + \frac{1}{2} \delta \sigma_z \]

- **Translational** invariance: uniform ground state configuration, unless crystalline order is formed spontaneously (**stripes, supersolidity**)

- **Violation of parity** and **time** reversal symmetry breaking of symmetry \( \omega(q) = \omega(-q) \) in excitation spectrum (exp: Si-Cong Ji et al. PRL 2015; theory: Martone et al. PRA 2012)

- **Violation of Galilean** invariance: breakdown of Landau criterion for superfluid velocity and emergence of dynamical instabilities in uniform configurations (exp: Zhang et al. PRL 2012, theory: Ozawa et al. (PRA 2013))
Different strategies to realize novel quantum phases

- **First strategy** (Lin et al., Nature 2009).
  Spatially dependent detuning ($\delta(y)$) in strong Raman coupling ($\Omega >> k_0^2$) regime yields position dependent vector potential

$$h_0 = \frac{1}{2m^*} \left( p_x - A_x(y) \right)^2$$

and **effective Lorentz force in neutral atoms**.

This causes the appearance of quantized vortices.

No y-dependence in detuning

Strong y-dependence in detuning
Second strategy (Lin et al. Nature 2011)

- Small detuning ($\delta \approx 0$) and smaller Raman coupling ($\Omega < 2k_0^2$) give rise to the appearance of two degenerate minima which can host a Bose-Einstein condensate.
Key question:

Role of **interactions** in the presence of the novel spin-orbit single particle Hamiltonian.

Rich scenario with **new quantum phases**
Theory of quantum phases in 1D SO coupled $s=1/2$ BECs ($T=0$)

Ho and Zhang (PRL 2011), ... , Yun Li, Pitaevskii, Stringari (PRL 2012)

\[ H = \sum_{i} h_{0}(i) + \sum_{\alpha, \beta} \frac{1}{2} \int d\vec{r} g_{\alpha\beta} n_{\alpha} n_{\beta} \]

- With

\[ h_{0} = \frac{1}{2} \left[ (p_{x} - k_{0}\sigma_{z})^{2} + p_{\perp}^{2} \right] + \frac{1}{2} \Omega \sigma_{x} + \frac{1}{2} \delta \sigma_{z} \]

- We assume $g_{\uparrow\uparrow} g_{\downarrow\downarrow} > g_{\uparrow\downarrow}^{2}$ \(\iff\) phase mixing in the absence of Raman coupling

- Interactions are treated within mean field approximation ($s=1/2$ coupled Gross-Pitaevskii equations)
For small values of $\Omega$ two sp states can host BEC with canonical momentum

$$|k_1| = k_0 \sqrt{1 - \Omega^2 /[2k_0^2 + \text{int}]}^2 \neq k_0$$

**Order parameter in the new phases**

**I) Stripe phase**

$$\Psi = \sqrt{N \over 2V} \left[ \begin{pmatrix} \cos \theta \\ -\sin \theta \end{pmatrix} e^{ik_1x} + \begin{pmatrix} \sin \theta \\ -\cos \theta \end{pmatrix} e^{-ik_1x} \right] + \text{higher harmonics}$$

$$\cos \frac{\theta}{2} = \frac{k_1}{k_0}$$

$$n(x) = n(1 + \frac{\Omega}{2k_0^2} \cos 2k_1x)$$

$$\text{density fringes fixed by } k_1$$

**II) Plane wave phase**

$$\Psi = \sqrt{N \over 2V} \begin{pmatrix} \sin \theta \\ -\cos \theta \end{pmatrix} e^{-ik_1x}$$

$$\langle \sigma_z \rangle = \frac{k_1}{k_0}$$

**II) Zero momentum phase**

$$\Psi = \sqrt{N \over V} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
New quantum phases (T=0)

| $^{87}\text{Rb}$ | $|\uparrow\rangle = |F=1, m_F = 0\rangle$ | $|\uparrow\rangle = |F=1, m_F = -1\rangle$ |
|-----------------|-----------------------------|-----------------------------|
| $a_{\uparrow\uparrow} = 101.41 a_B$ | $a_{\downarrow\downarrow} = a_{\uparrow\uparrow} = 100.94 a_B$ |

**Diagram:**

- **Single-minimum phase III**
- **Plane-wave** (phase separated)
- **Stripe phase I** (phase mixed)

**Parameters:**

- $\Omega/k_0^2$
- $\bar{n}/\bar{n}(c)$
- $k_1/k_0$
Plane wave-single minimum phase transition

Transition is **second order**. It has been observed at the predicted value

\[ \Omega \approx 4E_L = 2k_0^2 \]

of the Raman coupling

Lin et al., Nature 2011

Phase transition is driven by single-particle Hamiltonian. does not require two-body interactions.

Spin polarizability diverges at the transition (G. Martone, Yun Li, S.S. EPL 2012)

\[ \chi(\sigma_z) = \frac{\Omega^2}{k_0^2(4k_0^2 - \Omega^2)} \quad \Omega < 2k_0^2 \]

\[ \chi(\sigma_z) = \frac{2}{\Omega - 2k_0^2} \quad \Omega > 2k_0^2 \]

Zhang et al. PRL 2012
Transition is \textbf{first order}. Critical frequency is \[ \Omega_{cr} = 2k_0^2 \sqrt{\frac{2\gamma}{1 + 2\gamma}} \] where \[ \gamma = \frac{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} - 2g_{\uparrow\downarrow}}{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} + 2g_{\uparrow\downarrow}} \]

A phase transition between a spin mixed and a spin separated phase has been observed at the predicted value of \( \Omega \).

Density modulations are not however visible in the spin mixed phase (too small contrast and too small fringe separation) \textbf{Proof} of coherence is provided by \textit{fringes}, not by mixing.

Lin et al., Nature 2011

Si-Cong Li et al. Nat. Phys., 2014
Spin-orbit coupling has important consequences on the dynamic behavior of BECs.

**Experiments already available**

Quenching of Center of mass frequency in harmonic trap (violation of Kohn’s theorem)
(exp: Zhang et al. 2012; theory: Yun Li et al. EPL 2012)

Emergence of Roton and softed Phonon Modes
(exp: Si-Cong Ji et al.; PRL 2015;
theory Yun Li et al. Martone et al. PRA 2012)

Two Goldstone modes in striped Phase
Theory: Yun Li et al. PRL 2013
Emergence of Rotons in Plane Wave phase

Excitation spectrum exhibits two branches. Due to Raman coupling only one branch is gapless and exhibits a phonon behavior at small $q$.

Exp: Si-Cong Ji et al., PRL 2015
Theory: Martone et al., PRA 2012
At small Raman coupling, a **roton** structure emerges in the lower branch

\[ \omega(q) \neq \omega(-q) \]  

consequence of violation of **parity** and **time reversal** symmetry

Roton gap decreases as Raman coupling is lowered: **onset of crystallization** (striped phase)

Exp: Si-Cong Ji et al., PRL 114, 105301 (2015)  
Theory: Martone et al., PRA 86, 063621 (2012)
Other theoretical proposals

- Liquid Helium (historical)
- Spin-orbit BEC’s (2014)
- BEC’s in shaken lattices (2014)
- Pancake dipolar BEC’s
- BEC’s with soft core repulsive potentials
Why is the **striped phase** of a Bose-Einstein condensate interesting?

Can we make **stripes visible** and **stable**?
Competition between spin and density dependent interactions (for simplicity we assume \( g_{\uparrow\uparrow} \approx g_{\uparrow\downarrow} \equiv g \))

- **Spin** dependent term (proportional to \( (g - g_{\uparrow\downarrow}) \int d\vec{r} (n_{\uparrow} - n_{\downarrow})^2 \)) favours spin mixing and hence, in the presence of SO term and Raman coupling, favours the emergence of density modulations (**stripes**)

- **Density** dependent term (proportional to \( (g + g_{\uparrow\downarrow}) \int d\vec{r} (n_{\uparrow} + n_{\downarrow})^2 \)) favours uniformity (**plane wave phase**).

At small Raman coupling (for \( \Omega < \Omega_{cr} \)) stripes are energetically favoured. Value of \( \Omega_{cr} \) depends only on the ratio \( (g - g_{\uparrow\downarrow})/(g + g_{\uparrow\downarrow}) \).
Striped phase results from **spontaneous** breaking of two continuous symmetries: **gauge** and **translational** symmetries.

**Two Goldstone modes:**

- Double band structure in the striped phase of a SO coupled Bose-Einstein condensate
- Lower phonon branch better excited by spin operator  
  (Yun Li et al. PRL 2013)
Improving visibility and stability of superstripes
(Martone, Yun Li and Stringari, Phys. Rev. A 90, 041604(R) (2014)
In Nist (Lin et al., Nature 2011) and Shanghai (Si-Cong Li Nat. Phys. 2014) experiments stripes are not visible:

- **contrast** is too small. Doubly integrated density from Gross-Pitaevskii simulation in the same condition of 87Rb Nist exp. exhibits small contrast.

- **Separation** between fringes is too small (fraction of micron)

- **Stripes** are fragile against magnetic fluctuations. Tiny magnetic field (corresponding to detuning of 3-5 Hertz) destabilizes the stripes.
How to increase the contrast of density modulations?

Maximum value of Raman coupling compatible with the striped phase is given by

\[ \Omega_{cr} = 2k_0^2 \sqrt{\frac{2\gamma}{1 + 2\gamma}} \]

where \( \gamma = \frac{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} - 2g_{\uparrow\downarrow}}{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} + 2g_{\uparrow\downarrow}} \)

In 87Rb \( \Omega_{cr} = 0.1 k_0^2 \) and hence achievable contrast is very small.

In order to increase \( \Omega_{cr} \) one should reduce \( g_{\uparrow\downarrow} \). HOW?

- Feshbach tuning of interspecies scattering length \( a_{\uparrow\downarrow} \) preserving condition \( a_{\uparrow\uparrow} \approx a_{\downarrow\downarrow} \)

- We propose 2D geometry based on two spin layers separated by distance \( d \). Separation reduces \( g_{\uparrow\downarrow} \) by factor \( \exp(-d^2 / 2a_{z}^2) \) with respect to \( g_{\uparrow\uparrow} \) and \( g_{\downarrow\downarrow} \)

\[ V_{ext}(z) = \frac{1}{2} \omega_z^2 (z - \frac{d}{2} \sigma_z)^2 \]
3D Gross-Pitaevskii simulation for $4 \times 10^4$ $^{87}\text{Rb}$ atoms in harmonic trap

Parameters:

\begin{align*}
(\omega_x, \omega_y, \omega_z) &= 2\pi \times (25, 100, 2500) \text{Hz} \\
a_{\uparrow\uparrow} &= 101.41 a_B, \quad a_{\downarrow\downarrow} = a_{\uparrow\downarrow} = 100.94 a_B \\
E_r &= \frac{k_0^2}{2} = 2\pi \times 1.8 \text{KHz}
\end{align*}

\[ V_{\text{ext}}(\vec{r}) = \frac{1}{2} \omega_x^2 x^2 + \frac{1}{2} \omega_y^2 y^2 + \frac{1}{2} \omega_z^2 \left(z - \frac{d}{2\sigma_z}\right)^2 \]

\[ \Omega = \frac{1}{2} \Omega_{\text{cr}} \]

$\Omega = 170 \text{Hz}$

$\Omega = 0.1 E_r$

\[ d = 0 \]
3D Gross-Pitaevskii simulation for $4 \times 10^4$ $^{87}\text{Rb}$ atoms in harmonic trap

$V_{\text{ext}}(\vec{r}) = \frac{1}{2} \omega_x^2 x^2 + \frac{1}{2} \omega_y^2 y^2 + \frac{1}{2} \omega_z^2 (z - \frac{d}{2} \sigma_z)^2$

**Parameters:**

$$(\omega_x, \omega_y, \omega_z) = 2\pi \times (25, 100, 2500) \text{Hz}$$

$a_{\uparrow\uparrow} = 101.41 \ a_B$; $a_{\downarrow\downarrow} = a_{\uparrow\downarrow} = 100.94 \ a_B$

$E_r = k_0^2 / 2 = 2\pi \times 1.8 \text{KHz}$

$\Omega = \frac{1}{2} \Omega_{cr}$

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**Increasing contrast of fringes**

(a) Increasing contrast of fringes

(b) Increasing contrast of fringes
Increasing wave length of fringes

Apply \( \pi / 2 \) Bragg pulse transferring momentum to the atomic cloud (with \( \varepsilon << k_1 \)). Each component of the condensate function in the striped phase

\[
\Psi_{\downarrow} \propto e^{-i(k_1-k_0)x} + e^{i(k_1+k_0)x}
\]

will be fragmented in 3 terms. Components (a) and (b), after \( \pi / 2 \) pulse, will be able to interfere with wave length \( 2\pi / \varepsilon \)

\[
p_B = 2k_1 - \varepsilon
\]
Density fringes of striped phase after \( \frac{\pi}{2} \) Bragg pulse with momentum \( p_B = 2k_1 - \varepsilon \) and \( \varepsilon = 0.2k_1 \).
Density fringes of striped phase after a Bragg pulse with momentum $p_B = 2k_1 - \varepsilon$ and $\varepsilon = 0.2 k_1$.

Increasing wave length of fringes

Without Bragg

With Bragg
Stability of the striped phase

Reducing interspecies coupling constant enhances robustness of the striped phase.

Chemical potential difference between mixed and demixed phase at $\Omega = 0$ is given by $\Delta \mu = n(g - g_{\uparrow \downarrow})/2$ with $g_{\uparrow \uparrow} \approx g_{\downarrow \downarrow} \equiv g$

Choosing $d = 0$ ($g_{\uparrow \uparrow} \approx g$) critical detuning corresponds to tiny fraction of recoil energy (few Hertz)
Reducing interspecies coupling constant enhances robustness of the striped phase. Chemical potential difference between mixed and demixed phase at $\Omega = 0$ is given by $\Delta \mu = n(g - g^{\uparrow \downarrow})/2$ with $g^{\uparrow \downarrow} \approx g \equiv g$.

Choosing $d = 0$ ($g^{\uparrow \downarrow} \approx g$) critical detuning corresponds to tiny fraction of recoil energy (few Hertz).

Choosing $d = a_z$ ($g^{\uparrow \downarrow} \approx 0.6g$) critical detuning corresponds to $\approx 0.6E_r$ (a few hundred Hertz).

Increasing stability of fringes.
Rotonic structure, static structure factor and static response function in plane wave phase

\[ \Psi = \sqrt{N/V} \begin{pmatrix} \cos \theta \\ -\sin \theta \end{pmatrix} e^{i k_1 x} \]

Apply static perturbation

\[ \mathcal{V}_{ext} = s E_R (2 k_1) e^{i 2 k_1 x} \]

Strong non linear effects caused by large value of static response:

Emergence of stripes

\[ \Omega/k_0^2 = 0.30 \]
- Stripes have small contrast in stripe phase
- No polarization

- stripes have large contrast in PW phase with s=0.05
- spin polarization is strongly reduced

- No stripes in PW phase
- Almost full spin polarization
Main conclusions:

- **Rotonic excitation** in plane wave phase is onset of **crystallization** exhibited by striped phase.

- **Two Goldstone modes** in striped phase.

- **Contrast and stability** of stripes can be significantly **enhanced** creating two separated spin layers.

- **Wave length** of density fringes can be **increased** by applying $\pi/2$ Bragg or rf pulse.

- **Stripes** can be produced in **plane wave phase** by adding small **static perturbation**.
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