Inducing Resonant Interactions with a Modulated Magnetic Field

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Outline

1. Motivation and Background
2. Tuning interactions: MFR, OFR, rf/mwFR
3. Modulated Magnetic FR
   A. Toy model
   B. Matching to a physical system
4. Universal results
5. Experimental application
6. Conclusion
Motivation and background

• Control the interaction strength between particles
• Desire access to all regimes
  ◦ Attractive / Repulsive
  ◦ Strong (UFG, UBG?) / Weak (BCS, BEC)
• Parametrize interactions by s-wave scattering length \( a \)
  \[
  \sigma \propto a^2
  \]
• Related to s-wave phase shift:
  \[
  1/a = -k \cot \delta_0
  \]
• Control \( a \) by resonantly coupling scattering state to bound state.
Magnetic Feshbach resonance (MFR)

- Resonantly couple to molecule in closed hyperfine channel
- Tune relative energy with DC magnetic field
- Some limitations:
  - No control over resonance properties
  - 3-body losses

\[
\frac{1}{a(B)} = \frac{1}{a_{bg}} \frac{B - B_0}{B - B_0 - \Delta} + i\gamma
\]
Optical Feshbach resonance (OFR)

- Use laser to couple to electronically excited p-wave molecule
- **Good**: tune interactions with laser detuning and intensity
- **Bad**: spontaneous decay leads to losses


η: scattering phase shift
mw/rf Feshbach resonance (mw/rfFR)

- Use rf/mw fields to couple to molecule in closed hyperfine channel
- **Good:**
  - tune interactions with detuning and intensity
  - controllable losses
- **Bad:** Induced coupling is weak, making it difficult to significantly enhance $\alpha$.

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Modulated-magnetic Feshbach resonance (MMFR)

Wiggle the magnetic field: \[ B(t) = \bar{B} + \tilde{B} \cos(\omega t) \]

Similar to wiggle spectroscopy:

Loss spectrum for Cs.

Lange et al., PRA 79, 013622 (2009)
Simple parametrization of the scattering length:

\[
\frac{1}{a} = \frac{1}{\bar{a}} \frac{\omega - \omega_0}{\omega - \omega_0 - \delta} + i\gamma
\]

If bound state is a shallow dimer in scattering channel, dimensionless resonance parameters

\[
\frac{\Delta\omega_0}{\omega_B} = \frac{\omega_0 - \omega_B}{\omega_B}, \quad \frac{\delta}{\omega_B}, \quad \gamma\bar{a}
\]

are universal numbers multiplied by

\[
\left[ \frac{a'(\tilde{B})}{a(\tilde{B})} \tilde{B} \right]^2 \sim \left[ \frac{\tilde{B}}{\tilde{B} - B_0} \right]^2
\]

In this regime, \(a\) can be tuned without introducing dramatic loss.
Square well with oscillating depth

\[ V(r, t) = - \left[ \bar{V} + \tilde{V} \cos(\omega t) \right] \times \Theta(r_0 - r) \]

Wiggle depth \(\rightarrow\) wiggle binding energy
Scattering described by the time-dependent Schrödinger equation:

\[ i \frac{d}{dt} u(r, t) = -\frac{1}{m} \frac{\partial^2}{\partial r^2} u(r, t) - \left[ \tilde{V} + \tilde{V} \cos(\omega t) \right] \theta(r_0 - r) u(r, t). \]

Solved analytically using Floquet’s theorem:

\[ u(r, t) = e^{iE_F t} \phi(r, t) \quad \phi(r, t) = \phi(r, t + 2\pi / \omega) \]

\( E_F \), the “Floquet eigenvalue”, is determined by the asymptotic boundary condition.

The full solution is:

\[ u(r, t) = \sum_{n=-\infty}^{\infty} \left\{ 2ia_n \sin(q_n r) \exp \left[ -i(k_n^2/m)t + i\tilde{V} \sin(\omega t) / \omega \right] \right\} \quad r < r_0, \]

\[ (A_n^{\text{out}} e^{ik_n r} + A_n^{\text{in}} e^{-ik_n r}) \exp \left[ -i(k_n^2/m)t \right] \quad r \geq r_0, \]

\[ k_n = [k^2 + mn\omega]^{1/2} \quad q_n = [k^2 + m(\tilde{V} + n\omega)]^{1/2} \]
Toy model

The S-matrix relates the amplitudes of incoming and outgoing states:

\[ A_{n}^{\text{out}} = \sum_{j} S_{n,j} A_{j}^{\text{in}} \]

\[ S_{n,j} = \sum_{l} (M_{-})_{nl} (M_{+})_{lj}^{-1} \]

\[ (M_{\pm})_{jn} = \frac{e^{\pm ik_{j}r_{0}}}{k_{j}} \left[ (k_{j} \mp q_{n})e^{iq_{n}r_{0}} - (k_{j} \pm q_{n})e^{-iq_{n}r_{0}} \right] J_{j-n}(\tilde{V}/\omega) \]

The inverse scattering length is:

\[ \frac{1}{a} = - \lim_{k \to 0} \frac{k \cot \left( -\frac{i}{2} \ln S_{00} \right)} \]
Matching to physical system

• Goal: use toy model to make quantitative predictions for MMFR where the molecule is a shallow dimer from MFR.

• Logic: for shallow dimer, physics insensitive to details of potential.

• Matching conditions:
  1. Determine \( \bar{V} \): \( \bar{a}/r_0 \gg 1 \)
  2. Determine (small) \( \tilde{V} \):

\[
E(\bar{V} + \tilde{V}) - E(\bar{V}) = \omega_B(\bar{B} + \tilde{B}) - \omega_B(\bar{B})
\]

\[
\tilde{V} = -2\omega_B(\bar{B}) \frac{1}{E'(\bar{V})} \frac{a'(\bar{B})}{a(\bar{B})} \tilde{B}
\]

\[
\underbrace{\tilde{b}}_{\tilde{b}}
\]
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Universal results

Scattering length as function of frequency for $\tilde{b} = 0.05$

Fit with

$$\frac{1}{a} = \frac{1}{\tilde{a} \omega - \omega_0 - \delta} + i\gamma$$

for

$$\frac{\Delta\omega_0}{\omega_B} = \frac{\omega_0 - \omega_B}{\omega_B}, \quad \frac{\delta}{\omega_B}, \quad \gamma\tilde{a}$$
For small amplitudes: \[ \frac{\Delta \omega_0}{\omega_B}, \quad \frac{\delta}{\omega_B}, \quad \gamma \bar{a} \propto \tilde{b}^2 \]
Convergence to universality

\[ \frac{\Delta \omega_0}{\omega_B}, \frac{\delta}{\omega_B}, \gamma \bar{a} \propto \bar{b}^2 \]

Plot the proportionality constants.
Summary of universal results

Scattering length near resonance:

\[
\frac{1}{a} = \frac{1}{\bar{a}} \frac{\omega - \omega_0}{\omega - \omega_0 - \delta} + i\gamma
\]

Resonance parameters are controllable and have universal form:

\[
(\omega_0 - \omega_B)/\omega_B = 0.69 \tilde{b}^2
\]
\[
\delta/\omega_B = 0.50 \tilde{b}^2
\]
\[
\gamma\bar{a} = 0.13 \tilde{b}^2
\]

\[
\tilde{b} = \frac{a'(\tilde{B})}{a(\tilde{B})} \tilde{B}
\]
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Experimental application

Can the scattering length be tuned over a significant range without introducing dramatic loss in the universal regime?

Scaling behavior is favorable:

\[
\frac{\text{Re} \alpha}{\text{Im} \alpha} = \frac{1}{\gamma \bar{a}} \frac{\omega_0 - \omega}{\omega_0 - \omega + \delta} \propto \frac{1}{\tilde{b}^2}
\]

Decreasing amplitude helps and hurts, but it helps more!

Look at wiggle spectroscopy experiment with \(^7\)Li near resonance at 738 G: Dyke et al., PRA 88, 023625 (2013).
Experimental application

\[ \tilde{B} = 0.57 \text{ G} \]

From universal relations:
\[ \delta = 7.5 \text{ kHz} \]
\[ \Delta \omega_0 = 10 \text{ kHz} \]
\[ \gamma = \frac{1}{(2.6 \times 10^5 \ a_0)} \]

\[ \text{Re}a \approx 3\tilde{a} \quad (\text{Im}a = -0.04\tilde{a}) \]
Conclusion

• S-wave interactions can be resonantly enhanced by applying an oscillating magnetic field with frequency near the transition to a molecular state.

• Molecule formation leads to atom loss.

• For a shallow dimer in the scattering channel, the dimensionless resonance parameters have universal forms.

• The scattering length can be significantly enhanced without introducing dramatic atom loss.