Quantum fields <-> Correlation functions

On the Green's functions of quantized fields
J. Schwinger PNAS (1951)

✧ Solving a quantum many-body problem is equivalent to knowing all its correlation functions.

✧ In practice, an observer can only measure a finite number of correlations describing the propagation and scattering of excitations.

✧ To solve a problem one need to find degrees of freedom where only few (low order) correlation functions are relevant.

✧ If one finds the degrees of freedom (basis) where the correlation functions factorize, this is equivalent to diagonalization of the many body Hamiltonian.
**System under investigation**

Weakly interacting 1d Bose gas

All energies $\mu, k_B T \ll \hbar \omega$

quasi-condensate

uniform density fluctuating phase

thermally populated

**Lieb-Liniger model**
- Exactly solvable integrable theory

low energy effective field theory:
**Luttinger-liquid**

$$H = \frac{c}{2} \int dx \left[ \frac{K}{\pi} \left( \nabla \phi \right)^3 + \frac{\pi}{K} \phi^2 \right]$$

- excitations are soundwaves (phonons)
- linear dispersion relation

coupled 1d systems:
**Sine-Gordon model**

$$H_{SG} = \frac{bc}{2} \int_{-L/2}^{L/2} dz \left[ \frac{\pi}{K} \phi'(z) + \frac{K}{\pi} \left( \frac{\partial}{\partial z} \phi(z) \right)^2 \right] - 2n_{1D} \int_{-L/2}^{L/2} dz \cos(\sqrt{2} \phi(z))$$

Model for interacting many body systems which can be described by a field theory with long lived excitations.

**The longitudinal phase fluctuations are key for our experiments**

J. Schmiedmayer: High order correlation functions probing many body physics

**interference of phase fluctuating 1D condensates**

Study the dynamics of excitations on a quantum field

create a copy by splitting
quantum connected

create two independent samples
classically separated

J. Schmiedmayer: High order correlation functions probing many body physics
Correlation functions
- fields <-> phase <-> excitations

Characterizing the pre-thermalized state
- Generalized Gibbs ensemble

High order correlation functions
- Quantifying factorization
- Sine-Gordon model
- Quench to a free system

Outlook
- entanglement and spin squeezing
- quantum state tomography
- relaxation in SG model

1000-10000 Rb atoms
T = 10-100 nK
\( \omega_R \approx 2\pi \times 2 \) - 3 kHz
\( \omega_L \approx 2\pi \times 5 \) - 10 Hz
\( \mu, k_B T \ll \hbar \omega_R \)
Correlation functions

fields <-> phase

experiments in a trap
-> non translation invariant correlation functions

\[ C(z_1, z_2) = \frac{\langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_2^\dagger(z_2)\Psi_2(z_2) \rangle}{\langle |\Psi_1(z_1)|^2 \rangle \langle |\Psi_2(z_2)|^2 \rangle} \]

with

\[ \Psi(z) = e^{i\theta(z)} \sqrt{\rho_0(z) + \delta n(z)} \]
\[ \varphi(z) = \theta_1(z) - \theta_2(z) \]

neglecting \( \delta n(z) \)

\[
C(z_1, z_2) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2)] \rangle
\]

4th order:

\[ C(z_1, z_2, z_3, z_4) = \frac{\langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_2^\dagger(z_2)\Psi_2(z_2)\Psi_1(z_3)\Psi_2^\dagger(z_3)\Psi_2^\dagger(z_4)\Psi_2(z_4) \rangle}{\langle |\Psi_1(z_1)|^2 \rangle \langle |\Psi_1(z_3)|^2 \rangle \langle |\Psi_2(z_2)|^2 \rangle \langle |\Psi_2(z_4)|^2 \rangle} \]

\[ C(z_1, z_2, z_3, z_4) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2) + i\varphi(z_3) - i\varphi(z_4)] \rangle \]

Correlation functions

excitations <-> phase

in experiment we measure the phase \( \varphi(z) \) directly
-> look at phase correlators

\[ C^{(2)}(z_1, z_2) = \langle [\varphi(z_1) - \varphi(z_2)]^2 \rangle = \langle [\Delta \varphi(z_1, z_2)]^2 \rangle \]

with \( \Delta \varphi(z_1, z_2) = \varphi(z_1) - \varphi(z_2) \)

Note: \( \Delta \varphi \) is NOT restricted to \( 2\pi \)

using

\[ \varphi(z) = \frac{1}{\sqrt{K}} \sum_{k \neq 0} (-i) \sqrt{\frac{\pi}{|k|}} (b_{k}^\dagger - b_{-k}^\dagger) e^{ikz} \]

\[ \langle [\varphi(z_1) - \varphi(z_2)]^2 \rangle = \sum_{k_1, k_2} \frac{\pi}{K \sqrt{|k_1| |k_2|}} b_{k_1}^\dagger b_{-k_2}^\dagger e^{ik_1 z_1 + ik_2 z_2} + \ldots \]

-> phase correlators are related to the quasi particles

4th order

\[ C^{(4)}(z_1, z_2, z_3, z_4) = \langle [\varphi(z_1) - \varphi(z_2)]^2 [\varphi(z_3) - \varphi(z_4)]^2 \rangle \]
\[ \propto b_{k_1}^\dagger b_{k_2}^\dagger b_{-k_3} b_{-k_4} + \ldots \]

-> quasi particle scattering
Probing dynamics after quench (de) coherence

Experiment: M. Gring, M. Kuhnert, T. Langen et al. (VCQ, Vienna)
Theory: T. Kitagawa, E. Demler (Harvard)
I. Mazets (VCQ, Vienna)

Relaxation in a nearly integrable quantum system

Non-equilibrium state
relaxation in more than one timescales
rapid establishment of quasi-steady state

Quench:
\[ H_0 \rightarrow H_1 \]
\[ \Psi_0 \rightarrow \Psi(t) \]

Study using a model system:
1D Bose gas
isolated & controllable

new Thermal equilibrium
isolated quantum many-body system

Thermal equilibrium
Experimental Procedure

Decay of the mean contrast

Gring et al., Science 337, 1318 (2012)

J. Schmiedmayer: High order correlation functions probing many body physics
1D Bose gas is a (nearly) integrable system
→ many conserved quantities
    inhibit thermalization

Conjecture:
Quantum system to relax to maximum entropy state
described by a Generalized Gibbs Ensemble:

\[ \hat{\rho} = \frac{1}{Z} \exp \left( - \sum_m \lambda_m \hat{I}_m \right) \]

partition function
Lagrange multiplier
\[ \lambda_m \rightarrow \beta_m = 1/k_B T_m \]
conserved quantities: mode occupations

striking feature: a temperature for every mode!

Non-Translation Invariant Correlation Functions

\[ C(z_1, z_2) = \langle e^{i(\varphi(z_1) - \varphi(z_2))} \rangle \]

2d phase correlation function for 'Light Cone'

**instantaneous Quench**

Choose different starting points to evaluate the phase correlation function \( C(z_1, z_2) \)

\[ C(z_1, z_2) = \langle e^{i(\varphi(z_1) - \varphi(z_2))} \rangle \]

Observation: the decay of phase correlation function is independent on starting point \( z_1 \)

Data is described by a model with a single temperatures for phonon modes in the anti-symmetric state.

**Light-Cone dynamics in the decay of coherence**

T. Langen et al NatPhys 9, 460 (2013)


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**Time evolution of the phase correlation function**

\[ C(z' = z - z') = \langle e^{i(\phi(z) - \phi(z'))} \rangle \]

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**Light-Cone dynamics in the decay of coherence**

T. Langen et al NatPhys 9, 460 (2013)


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**Linear dispersion relation -> Light-Cone dynamics**

The region with the final form of the phase correlation function expands with sound velocity
Generalized Gibbs Ensemble

**Slow-fast Quench**

Observation: For specific splitting procedures, the decay of phase correlation function depends on starting point $z_1$ and shows 'revivals' of coherence:

$$C(z_1, z_2) = \langle e^{i(\varphi(z_1) - \varphi(z_2))} \rangle$$

Data is better described by a model with different temperatures for even phonon modes and odd phonon modes in the anti-symmetric state.

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**How many Parameters are needed for the GGE**


The figure shows the mode occupation $n_m \cdot \hbar \omega/mu$ for different modes $m$ and reduced $\xi$.

In the inset, the significance for different $n$ values is plotted.
Correlations outside the 'Light-cone', imprinted by the quench

8 temperature model describes the relaxed state

number of parameters limited by experimental resolution + occupation numbers

a single temperature model (Gibbs ensemble) shows very large deviations

8 temperature model describes approximately the evolution to the state

Conjecture: Differences due to the initial phase of the excitations (in the model we assumed zero phase, as in prethermlisation)

→ path to reconstruct the initial state
Higher order phase correlation functions

\[ C(z_1, z_2) \approx \exp[i\varphi(z_1) - i\varphi(z_2)] \],
\[ C(z_1, z_2, z_3, z_4) \approx \exp[i\varphi(z_1) - i\varphi(z_2) + i\varphi(z_3) - i\varphi(z_4)] \],

Regular fast Quench (1 temperature)
Do higher-order correlation functions factorize?

The Luttinger Liquid Hamiltonian is quadratic: Correlations factorize into 2-point functions

When do higher-order correlation functions factorize?

Exp: T. Schweiger, et al. (Vienna)
Theory: S. Erne, V. Kasper et al. (HD)

www.AtomChip.org
Sine-Gordon physics
tunable tunnel coupling J in double-well

Quantum Sine-Gordon model:

\[ \hat{H}_{\text{SG}} = \frac{\hbar c}{2} \int_{-L/2}^{L/2} dz \left[ \frac{\pi}{K} n^2(z) + \frac{K}{\pi} \left( \frac{\partial}{\partial z} \theta(z) \right)^2 \right] - 2n_1 J \int_{-L/2}^{L/2} dz \cos[\sqrt{2} \theta(z)] \]

that's what we have seen so far ...
“uncoupled harmonic oscillators”

anharmonic, non-gaussian, gapped, universality?

J. Schmiedmayer: High order correlation functions probing many body physics

Characterising the factorisation

experiments probe the phase

-> look at the 'connected part' of the phase correlation function

\[ \langle (\Delta \varphi)^2 \rangle_c = \langle (\Delta \varphi)^2 \rangle \]
\[ \langle (\Delta \varphi)^4 \rangle_c = \langle (\Delta \varphi)^4 \rangle - 3 \langle (\Delta \varphi)^2 \rangle^2 \]
\[ \langle (\Delta \varphi)^6 \rangle_c = \langle (\Delta \varphi)^6 \rangle - 15 \langle (\Delta \varphi)^4 \rangle \langle (\Delta \varphi)^2 \rangle^2 + 30 \langle (\Delta \varphi)^4 \rangle^2 \]
\[ \langle (\Delta \varphi)^8 \rangle_c = \langle (\Delta \varphi)^8 \rangle + 420 \langle (\Delta \varphi)^4 \rangle^2 \langle (\Delta \varphi)^2 \rangle^2 - 630 \langle (\Delta \varphi)^4 \rangle^4 - 35 \langle (\Delta \varphi)^4 \rangle^2 - 28 \langle (\Delta \varphi)^6 \rangle \langle (\Delta \varphi)^2 \rangle^2 = 0 \]

characterized by 'Kurtosis'

\[ \gamma_2 = \frac{\langle (\Delta \varphi)^4 \rangle}{3 \langle (\Delta \varphi)^2 \rangle^2} - 1 \]
\[ \gamma_3 = \frac{\langle (\Delta \varphi)^6 \rangle}{15 \langle (\Delta \varphi)^4 \rangle \langle (\Delta \varphi)^2 \rangle^2 - 30 \langle (\Delta \varphi)^4 \rangle^2} - 1 \]
\[ \gamma_4 = \frac{\langle (\Delta \varphi)^8 \rangle}{630 \langle (\Delta \varphi)^2 \rangle^4 + 35 \langle (\Delta \varphi)^4 \rangle^2 + 28 \langle (\Delta \varphi)^6 \rangle \langle (\Delta \varphi)^2 \rangle^2 - 420 \langle (\Delta \varphi)^4 \rangle^4 - 28 \langle (\Delta \varphi)^6 \rangle \langle (\Delta \varphi)^2 \rangle^2} - 1 = 0 \]
correlation functions for the fields:

\[
C(z_1, z_2) = \frac{\langle \Psi_1(z_1) \Psi_2^\dagger(z_1) \Psi_1^\dagger(z_2) \Psi_2(z_2) \rangle}{\langle |\Psi_1(z_1)|^2 \rangle \langle |\Psi_2(z_2)|^2 \rangle}
\]

\[
C(z_1, z_2) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2)] \rangle
\]

\(C(z_1, z_2)\) contains all orders of connected parts

\[
C(z_1, z_2) = \exp \left[ \sum_{k=1}^{\infty} (-1)^k \frac{\langle (\Delta\varphi)^{2k} \rangle_c}{(2k)!} \right]
\]

for Gaussian fluctuations

\[
C(z_1, z_2) = \exp \left[ -\frac{1}{2} \langle (\Delta\varphi)^2 \rangle \right]
\]

Observable and non-gauss measure

to study factorization of correlation functions we look at

\[
C^{(2)}(z_1, z_3) = \langle [q(z_1) - q(z_3)]^2 \rangle = \langle [\Delta q(z_1, z_3)]^2 \rangle
\]

\[
C^{(4)}(z_1, z_2, z_3, z_4) = \langle [q(z_1) - q(z_2)]^2 [q(z_3) - q(z_4)]^2 \rangle = \langle [\Delta q(z_1, z_2)]^2 [\Delta q(z_3, z_4)]^2 \rangle
\]

\(\Delta q(z_j, z_j) = q(z_j) - q(z_j)\)
\(\Delta \varphi\) is NOT restricted to \(2\pi\)

\[
C^{(4)}(z_1, z_2, -15, 15)
\]

\[
\begin{array}{cccc}
\text{no coupling} & \text{intermediate coupling} & \text{strong coupling} \\
\text{full Wick deviation} & & \\
\end{array}
\]
Characterising non-Gaussian phase fluctuations

Characterising the factorisation by the connected part:
\[ \langle (\Delta \varphi)^4 \rangle_c = \langle (\Delta \varphi)^4 \rangle - 3 \langle (\Delta \varphi)^2 \rangle^2 \]

excess Kurtosis
\[ \gamma_2 = \frac{\langle (\Delta \varphi)^4 \rangle}{3 \langle (\Delta \varphi)^2 \rangle^2} - 1 \]

Experimental data, thermal state in a double well

- plasma-freq. = 0 Hz
- 70 Hz
- 160 Hz
- > 220 Hz

Quantifying factorization of correlation functions

- the breakdown of factorization is evident in the full distribution functions of the phase by new peaks at multiples of \(2\pi\)
- caused by the \(2\pi\) periodic SG Hamiltonian \(\rightarrow 2\pi\) phase jumps, 'kinks', SG solitons
- \(\text{SG Solitons}\) are topological excitations
- Phase fluctuations around \(\text{topologically different Vacua}\)
4-point phase correlators

\[ \omega_p \quad 0 \text{ Hz} \quad 70 \text{ Hz} \quad 160 \text{ Hz} \quad \text{big} \]

- full
- Wick factorization
- difference
- lower limit
- upper limit

6-point Phase correlators

\[ \omega_p \quad 0 \text{ Hz} \quad 70 \text{ Hz} \quad 160 \text{ Hz} \quad \text{big} \]

- full
- Wick factorization
- difference
- lower limit
- upper limit
6-point phase correlators, connected part

- $\omega_p$: 0 Hz, 70 Hz, 160 Hz, big
- Full
- Disconnected part
- Connected part
- Lower limit
- Upper limit

Remove Solitons
Strongly coupled $\omega_p > 500$ Hz

4-point correlator does not factorize:

Without Solitons:
Remove Solitons
intermediate coupling $\omega_p = 160$ Hz

4-point correlator does not factorize:

without Solitons:

phase distribution:

different sectors:

without solitons:

Quench from J>0 to J=0

Initial state non-Gaussian, dynamics Gaussian

very preliminary

[Images of graphs and plots]
Quantum state tomography

Use 2- and 4-point functions to reconstruct higher-order functions via continuous matrix product states with bond length 2

State reconstruction with very weak assumptions

Quantum state tomography

Use 2- and 4-point functions to reconstruct higher-order functions via continuous matrix product states with bond length 2

State reconstruction gets worse with time
C-MPS with bond length 2 have finite entanglement

Question: Can one build a measure for entanglement growth after the quench?

Squeezing

\[ N = 1200 \text{ atoms, } \mu \approx 0.5 \text{ kHz, } T \approx 25 \text{ nK (0.5 kHz)} \]

RMS fluctuations of the number difference

\[ n \equiv N_L - N_R \]
\[ \Delta n = 14(3) \text{ atoms} \]

Whereas \[ \sqrt{N} = 35 \]
Spin squeezing:
\[ \xi_S^2 = \frac{\xi_N^2}{\langle \cos \phi \rangle^2} = -7.7 \text{ dB} \]
Imples that \( \approx 150 \) atoms are entangled!

RMS fluctuations of the phase
\[ \Delta \phi = 0.168(8) \text{ rad} \]

Whereas \[ 1/\sqrt{N} = 0.03 \]

when correcting for measurement noise:
\[ \Delta n \Delta \phi = 2.3 \ (7) \]
\[ \Delta n \Delta \phi \sim 1 \]
Evolution of $\xi^2 \sim -8dB$ 1d gas

Tunnel Coupled $\omega_p=14Hz$

Separated

Relaxation in coupled superfluids

re-coupling starts SG model with a specific phase

$\rightarrow$ study phase locking

$\epsilon$

time

phase

number imbalance

number imbalance

phase locking as a fix-point of the evolution
What have we learned

- Relaxation in quantum systems does not proceed through a simple path: 'prethermalization'
- Relaxed state emerges locally and spreads throughout the system in a light cone like fashion
- Prethermalized state is associated with a Generalized Gibbs Ensemble
- Higher order correlation functions and the question if they factorize (full distribution functions) gives insight in the effective theories describing the many body system
- Experiments allow to probe how classical statistical properties emerge from microscopic quantum evolution through de-phasing of many body eigenstates.

Gring et al., Science 337, 1318 (2012)
Kuhnert et al., PRL 110, 090405 (2013)
Smith et al., NJP 15, 075011 (2013)
Langen et al., Nature Physics 9, 460 (2013)
R. Geiger et al. NJP 16 053034 (2014)