On 'Higgs' modes and the optical conductivity in O(2) models in condensed matter physics

Lode Pollet

in collaboration with:
Kun Chen, Longxiang Liu, Youjin Deng, Nikolay Prokof’ev

UMass Amherst, MA, USA
USTC Hefei, China

Ref: PRL 2012, PRL 2013, PRL 2014
Mexican hat potential

order parameter: \[ \Psi(r, t) = |\Psi(r, t)| e^{i\phi(r, t)} \]

models: \( \text{O}(N) \)

\( \text{O}(2) : \) superfluids
\( \text{O}(3) : \) antiferromagnets

fluctuations of the modulus of order parameter = scalar

hence amplitude mode is hard to couple to

necessary condition: explicit/
emergent Lorentz invariance

decomposition of fluctuations of order parameter into:
- longitudinal & transverse
- radial & tangential

this help understanding behavior of different correlation functions
Consider a relativistic quantum field theory with mass m, and a complex scalar field

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \frac{1}{2} \lambda (\phi^* \phi)^2$$

or, for negative mass,

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + m^2 \phi^* \phi - \frac{1}{2} \lambda (\phi^* \phi)^2$$

The Lagrangian has a global U(1) symmetry

$$\phi(x) \rightarrow \phi(x) e^{i\theta}$$

In terms of the Mexican hat potential,

$$V(\phi) = -\frac{1}{2} \lambda \nu \phi^* \phi + \frac{1}{2} \lambda (\phi^* \phi)^2$$

$$\nu = \frac{-2m^2}{\lambda}$$

the minimum occurs for

$$|\phi|^2 = \frac{\nu^2}{2}$$

courtesy of I. Bloch
We pick one of the minima and expand around it,

$$
\phi = \frac{1}{\sqrt{2}} (\nu + \varphi_1 + i\varphi_2)
$$

The low-energy Lagrangian is then

$$
\mathcal{L} = \frac{1}{2} \left[ (\partial_\mu \varphi_1)^2 + (\partial_\mu \varphi_2)^2 \right] - \frac{1}{2} \lambda \nu^2 \varphi_1^2 + \ldots
$$

where we see a massless Goldstone mode and a massive Higgs mode.
Consider now the case of coupling to a gauge field and local gauge invariance,

\[
\theta \to \theta(x)
\]
\[
A_\mu \to A_\mu - \frac{1}{e} \partial_\mu \theta(x)
\]
\[
D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi
\]
\[
\mathcal{L} = D_\mu \phi^* D^\mu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi)
\]

Breaking the symmetry now leads to

\[
\mathcal{L} = \frac{1}{2} \left[ (\partial_\mu \varphi_1)^2 + (\partial_\mu \varphi_2 + e\nu A_\mu)^2 \right] - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \lambda \nu^2 \varphi_1^2 + \ldots
\]

exactly same terms as for global gauge invariance
O(N) field theories

\[ Z_Q = \int D\phi_\alpha(x,\tau) \exp\left(-\int d^d x \int_0^{1/T} d\tau \mathcal{L}_Q\right) \]

\[ \mathcal{L}_Q = \frac{1}{2} \left[ \frac{1}{c^2} (\partial_\tau \phi_\alpha)^2 + (\nabla_x \phi_\alpha)^2 + (r_c + r) \phi_\alpha^2 \right] + \frac{u}{4!} (\phi_\alpha^2)^2. \]

\[ \chi^0(q) = \frac{u}{q^2 + (r_c + r)} \]

mean-field pole at amplitude mass

\[ \Pi^0(q) \sim \int \frac{1}{k^2 (k + q)^2} \frac{d^{d+1}k}{(2\pi)^{d+1}} \]

IR divergence

\[ d>3 : u \text{ is irrelevant} \]

(Gaussian free field theory)

\[ \chi^0(q) = \frac{u}{q^2 + (r_c + r)} \]

mean-field pole at amplitude mass

\[ \Pi^0(q) \sim \int \frac{1}{k^2 (k + q)^2} \frac{d^{d+1}k}{(2\pi)^{d+1}} \]

IR divergence

\[ d=2, n=1,2 \]

fig from S. Sachdev
usually the low effective field theory is of the form

\[ Z = \int \mathcal{D}\Psi^*(x, \tau) \mathcal{D}\Psi(x, \tau) \exp \left( -\int d^d x \int d\tau \mathcal{L}[\Psi^*, \Psi] \right) \]

\[ \mathcal{L}[\Psi^*, \Psi] = \frac{\partial r \Psi^* \frac{\partial \Psi}{\partial \tau}}{\partial \mu} \quad \text{It is hard to couple to the Higgs mode:} \]

ph - symmetry needed for it to vanish (superconductors, not metals nor superfluids)

Raman spectra of NbSe2

S. Sachdev, Quantum Phase Transitions, 1999

(3d quantum antiferromagnet)

other systems:
- pump-probe experiments
- He3 (p-wave)
- Raman spectrum of LCO?

FIG. 3 (color online). Summary of INS results for the gaps of all three triplet excitations as functions of pressure at $T = 1.85$ K. Data for $T_N(p)$ from Ref. [5]. Modes $L$ and $T_1$ are degenerate within experimental resolution at $p < p_c$. Red symbols show the longitudinal mode $L$ at $p > p_c$. Solid and dashed lines are theoretical fits.
longitudinal susceptibility has branch cut no pole-like structure at a frequency of order $\rho_s(0)$
Anomalous Fluctuations in Phases with a Broken Continuous Symmetry

W. Zwerger

Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria
(Received 7 April 2003; published 16 January 2004)

derived same formula’s, and used them in the dynamic structure factor:

\[ S(q, \omega) = 2m_s^2 \xi J \frac{N-1}{N} \left[ \frac{\pi}{2q} \delta(\omega - cq) \right. \]

\[ + \frac{\xi J \theta(\omega - cq)}{16 \sqrt{\omega^2 - c^2 q^2}} \left. \right] \]

``The longitudinal fluctuations of the Neel order thus lead to a critical continuum above the spin wave pole at \( \omega \sim cq \), which decays only algebraically. The continuum results from the decay of a normally massive amplitude mode with momentum \( p \) into a pair of spin waves with momenta \( q \) and \( p-q \), which is possible for any \( \omega > cq \), with a singular cross section because of the large phase space. The amplitude mode is thus completely overdamped in two dimensions.”
Scalar and longitudinal susceptibility

\[ \chi_{\sigma\sigma}'' \sim \omega^{-1} \]
\[ \chi_{\rho\rho}'' \sim \omega^3 \]

Chubukov, Sachdev, Ye '93
Podolsky, Auerbach, Arovas '11
S. Huber, G. Blatter, E. Altman
Universal scaling predictions

\[ \chi_{\rho \rho}^{''} (\omega) \propto \Delta^{3-2/\nu} F(\omega / \Delta) \]
\[ \Delta \propto (U_C - U)^\nu, \quad \nu = 0.6717 \]

Chubukov, Sachdev, Ye '93
Sachdev '99

Podolsky et al.
MISSING SPECTRAL DENSITY

Podolsky et al.
Two has more than three

“The model I came up with in 1964 is just the invention of a rather strange sort of medium that looks the same in all directions and produces a kind of refraction that is a little bit more complicated than that of light in glass or water“ — P. Higgs

\[ d = 3 + 1 \]

Longitudinal response: finite width peak

\[ \chi''_L \]

\[ \Omega_H \]

Gaussian fixed point

Higgs peak is critically well defined

\[ \frac{\Gamma}{\omega_H} \sim \frac{1}{\ln |g - g_c|} \]

Energy ratio: \( \frac{\omega_H}{\Delta} = \sqrt{2} \)

Affleck & Wellman, PRB 92

\[ d = 2 + 1 \]

Longitudinal response IR divergent

universal scaling function

**Strongly coupled fixed point**

Higgs peak is marginally defined

\[ \frac{\Gamma}{\omega_H} \to \text{const} \]

Energy ratio: \( \frac{\omega_H}{\Delta} \neq \sqrt{2} \)

D. Podolsky
Physics of Bose-Hubbard in a nutshell

\[ H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) - \sum_i \mu_i n_i \]

- **U(1) symmetry**
- **global** \( b_i \rightarrow b_i e^{i\phi} \)
- **decoupling approximation** (mean-field)
- \( b_i^\dagger b_j = \psi (b_i^\dagger + b_j) - \psi^2 \)
- \( \psi = \langle b_i \rangle = \langle b_i^\dagger \rangle \)

**Mott phase:**
- Integer density
- Zero compressibility
- Gap
- Insulating

\[ Z = \int \mathcal{D}\Psi^*(x, \tau) \mathcal{D}\Psi(x, \tau) \exp \left( -\int d^d x \int d\tau \mathcal{L} [\Psi^*, \Psi] \right) \]

\[ \mathcal{L} [\Psi^*, \Psi] = -\frac{\partial r}{\partial \mu} \Psi^* \frac{\partial \Psi}{\partial \tau} + |\partial_\tau \Psi|^2 + c^2 |\nabla \Psi|^2 + r |\Psi|^2 + \frac{u}{2} |\Psi|^4 + \ldots \]

must vanish!

M. P. A. Fisher et al, PRB 1989

S. Sachdev, Quantum Phase Transitions, 1999
Bogoliubov regime (Galilean invariance)

one gapless mode (sound)

two gapless modes at QCP phase(sound) and amplitude(Higgs)

particle and hole condense

one gapless mode (phase -- sound)

particle condenses; hole remains gaped

Mott insulator

\[
Z = \int \mathcal{D}\Psi^*(x, \tau) \mathcal{D}\Psi(x, \tau) \exp \left( - \int d^d x \int d\tau \mathcal{L} [\Psi^*, \Psi] \right)
\]

\[
\mathcal{L} [\Psi^*, \Psi] = -\frac{\partial r}{\partial \mu} \Psi^* \frac{\partial \Psi}{\partial \tau} + |\partial \Psi|^2 + c^2 |\nabla \Psi|^2 + r |\Psi|^2 + \frac{u}{2} |\Psi|^4 + \cdots
\]

M. P. A. Fisher et al, PRB 1989

S. Sachdev, Quantum Phase Transitions, 1999

must vanish!
Our response

This involves analytic continuation of the kinetic energy correlation function

\[ S(\omega) \]

\[ \omega_H \]

\[ U=16 \]
Long Monte Carlo simulations (LMC)
The ‘Higgs’ Amplitude Mode at the Two-Dimensional Superfluid-Mott Insulator Transition

Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schauß, Christian Gross, Eugene Demler, Stefan Kuhl, and Immanuel Bloch

Nature 2012

|\chi_\omega| = \langle K(\tau) K(0) \rangle_\omega + \langle K \rangle

Energy dissipation rate: \omega \text{ Im} \chi_\omega

Total energy absorbed: \Delta E = \omega \text{ Im} \chi_\omega \left( \frac{2\pi M}{\omega} \right) \propto \text{ Im} \chi_\omega
The experimental results
The experimental results

softening of onset of spectral weight on approach to the critical point
Attempt to compare signals (amplitude adjusted)

Take a realistic temperature and trapping parameters into account

$U/J = 14 \rightarrow j/j_c \approx 1.2$
universal scaling function
results by Podolsky et al

\[ S_E = \frac{1}{g} \left[ - \sum_{\langle i,j \rangle} \vec{\phi}_i \cdot \vec{\phi}_j - \mu \sum_i |\vec{\phi}_i|^2 + \sum_i (|\vec{\phi}_i|^2)^2 \right] \]

S. Gazit et al, arXiv:1212.3759, PRL

on SF side:
\[ \omega_H = 2.1(3) \Delta \]

compare to ours:
\[ \omega_H = 3.2(8) \Delta \]
conclusion and future work

- conditions under which amplitude/Higgs mode can be seen as a sharp and universal peak in correlation functions
- strongly interacting fixed point in 2d; also conductivity accurately computed (cf AdS/CFT correspondence)
- further experiments would be welcome though challenging
- universal scaling function determined; explicit demonstration of Lorentz symmetry under way
- what about (artificial) graphene (Gross-Neveu criticality)? what about 1d?

Special thanks:
Kun Chen, Longxiang Liu, Youjin Deng, Nikolay Prokof’ev
W. Witczak-Krempa. E. Sorensen, S. Sachdev, D. Pekker, M. Endres, I. Bloch
W. Zwerger, D. Manske, M. Dressel