Chiral Bose and Fermi phases in optical lattices

W. Vincent Liu

University of Pittsburgh, Western Pennsylvania
This talk is based on work:

- Nature Physics 7, 101 (2011) [Background and perspective (news & views), with M. Lewenstein]
- Nature Communications 5:3205 (2014a)
- Nature Communications 5:5064 (2014b)

Acknowledgement

People

Xiaopeng Li
Bo Liu

External collaborator

Arun Paramekanti (Toronto)
Biao WU (Peking U)
Exp: A. Hemmerich (Hamburg)

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1. Highlights of recent research work – to stimulate discussion in the week

2. Some New Progress in Orbital Optical Lattices
   ✦ Introduction
   ✦ Boson: Chiral Bose liquid
   ✦ Fermion: p-wave pair superfluidity without p-wave interaction

3. Conclusion
Selected recent results by our group


3. Chiral superfluidity with p-wave symmetry from an interacting s-wave atomic Fermi gas: Bo LIU, X. Li, Biao WU, and WVL*, *Nature Communications* 5:5064 (Sep 2014) [*corresponding author] This talk!


Outline

1. Highlights of recent research work – to stimulate discussion in the week

2. Some New Progress in Orbital Optical Lattices
   ✧ Introduction
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3. Conclusion
Orbital degrees of freedom in solids
(skip all early studies of orbital physics, but focus on recent trends)

-iron-based superconductors

-LAO-STO oxide heterostructures

Michael R. Norman, Physics 1, 21 (2008)

J. Kroha, PRL viewpoint, Physics (2011)

X. Li, WVL and Leon Balents, PRL (2014)
Orbital degrees of freedom in optical lattices

Early theoretical work on \textit{p-band} boson

- A. Isacsson and S. M. Girvin, PRA 72, 053604 (2005);
- WVL and C. Wu, Phys. Rev. A 74, 013607 (2006);

- This talk

Chiral Bose $p+i\mu phase driven by interaction
[WVL and C. Wu, PRA (2006)]

- Repulsive interaction favors spontaneous rotation order
  
  \[ H_{int} = \frac{1}{2} U \sum_r \left[ n_r^2 - \frac{1}{3} L_r^2 \right] \]

  
  Density field operator: \( n_r = \sum_\mu b^\dagger_{\mu r} b_{\mu r} \)

  Angular momentum operator: \( L_{\mu r} = -i \sum_{\nu, \lambda} \epsilon_{\mu \nu \lambda} b^\dagger_{\nu r} b_{\lambda r} \)

\( \mu, \nu = x, y, z \quad \text{or} \quad p_x, p_y, p_z \)

... leads to
- \( p_x + i p_y \) angular momentum ordered BEC (breaks T-symmetry)

**Recall:** Condense at Finite linear momentum
Experiment of p- and f-band bosons – double well lattices

Hamburg/ A. Hemmerich group
First observation of p-band BEC with C4 symmetry and hence orbital degeneracy

- Even earlier p-band fermion observed in Feshbach crossing “accidentally” M. Köhl et al, PRL 94, 080403 (2005)

“P-band superfluidity+orbital order in chequerboard (double well) lattice”, long life time [G. Wirth, M. Olschlager, A. Hemmerich, Nature Physics 2011]
...“Observing Chiral Superfluid Order by Interference” [Kock, Mathey, Hemmerich et al, PRL, March 2015]
Hamburg interference experiment: Evidence of p+ip order firmed up

Optical barrier of $2E_R$ splits the system into two sub-gases → Young’s double-slit

4 points in k-space:

1, 3 = $|p_x\rangle$

2, 4 = $|p_y\rangle$

• Two classes of interference (I) vs (II)
• Evidence of $\pm\pi/2$ phase difference between px and py components, i.e., px ±ipy

Kock, Mathey, Hemmerich et al, PRL 114, 115301 (March 2015)
Part 2A: Chiral Bose non-superfluid phase at finite temperature
Main finding: Chiral Bose liquid

Our collaborative Work:
Xiaopeng Li (Pitt student -> postdoc in JQI Maryland)
Arun Paramekanti (U Toronto)
Andreas Hemmerich (U Hamburg)
WVL (U of Pitt)

Nature Communications 5:3205 (2014)
Experiments Revisited: finite temperature

momentum distribution

Note thermal background

A. Hemmerich et al., Nat. Phys (2011)
Experiments revisited.
Open questions at Finite temperature

- Exotic features
  - staggered px+ipy order: TRS breaking, condensed at finite momentum.
  - superfluid: U(1) symmetry breaking

WVL, C. Wu, PRA (2006)
X. Li, E. Zhao, WVL, PRA (2011)
Z Cai, C. Wu, PRA (2011)

- Questions and Challenges
  - How does this superfluid state melt under thermal fluctuations?
  - Go beyond mean field (ground state): Orbital excitations …? Topological configurations (other than vortices)?

A. Hemmerich et al., Nature Physiics (2011)
s+p-band model for Hamburg checkerboard lattice

- double wells, mixed s and p orbitals

\[ H = H_{\text{tun}} + H_{\text{loc}} \]

\[ H_{\text{tun}} = -\frac{t}{\sqrt{2}} \sum_{\mathbf{r}} \left\{ [b_{x}^{\dagger}(\mathbf{r}) + b_{y}^{\dagger}(\mathbf{r})] [b_{s}(\mathbf{r}_1) - b_{s}(\mathbf{r}_2)] \\
+ [b_{y}^{\dagger}(\mathbf{r}) - b_{x}^{\dagger}(\mathbf{r})] [b_{s}(\mathbf{r}_3) - b_{s}(\mathbf{r}_4)] + h.c. \right\} \quad (1) \]

\[ H_{\text{loc}} = -\sum_{\mathbf{r}} [\mu_p n_p(\mathbf{r}) + \mu_s n_s(\mathbf{r}_1)] \\
+ \sum_{\mathbf{r}} \frac{U_p}{2} \left\{ n_p(\mathbf{r}) \left[ n_p(\mathbf{r}) - \frac{2}{3} \right] - \frac{1}{3} \mathcal{L}_z^2(\mathbf{r}) \right\} \\
+ \sum_{\mathbf{r}} \frac{U_s}{2} n_s(\mathbf{r}_1) [n_s(\mathbf{r}_1) - 1]. \quad (2) \]

\[ \mathbf{r}_\alpha = \mathbf{r} \pm \frac{\hat{a}_x \pm \hat{a}_y}{2} , \quad \alpha = 1, \ldots, 4 \]

[Xiaopeng Li, Arun Paramekanti, Andreas Hemmerich & WVL, Nature Communications 2014]
Strong coupling & integer fillings: $p_x + ip_y$ Mott insulator
(simple/easy/clean case in theory: s-band raised higher than p; filling $n \geq 2$)

Effective model reduced to 2D (classical) Ising:

$$H_{\text{loc}} = -\sum_r [\mu_p n_p(r) + \mu_s n_s(r_1)] + \sum_r \frac{U_p}{2} \left\{ n_p(r) \left[ n_p(r) - \frac{2}{3} \right] - \frac{1}{3} \mathcal{L}_z^2(r) \right\} + \sum_r \frac{U_s}{2} n_s(r_1) [n_s(r_1) - 1].$$

Hund’s rule $\rightarrow$ two degenerate states of maximum angular momentum $|L_z|

$$\mathcal{L}_z(r) \equiv \sigma_z(r) |\mathcal{L}_z(r)|$$

$$H_{\text{Ising}}^{\text{eff}} = \sum_{\langle r, r' \rangle} J \sigma_z(r) \sigma_z(r'), \quad J = \frac{3n^2(n+2)}{2(n+1)} \frac{t_{||} t_{\perp}}{U} > 0.$$

$$\left( \begin{array}{c} |p_x + ip_y\rangle \\ |p_x - ip_y\rangle \end{array} \right) \rightarrow \sigma_z = \left( \begin{array}{c} + \\ - \end{array} \right)$$

Results mapped from 2D Ising model:

- **T=0** (ground state) and **T<T_{\text{Ising}}**: long range order with staggered $L_z$ order for integer filling $n \geq 2$
- **T>T_{\text{Ising}}**: Ising transition to a symmetry restored phase
- Critical $T$: $k_B T_I \approx 2.27 J$ \cite{Onsager1944}
Effects of thermal fluctuations ---strong interaction regime

-super-exchange interaction in Mott states (filling $\geq 2$)

\[
H_{\text{eff}} = J \sum_{\langle r, r' \rangle} \sigma_y(r)\sigma_y(r') \quad J > 0
\]

Anti-Ferromagnetic \hspace{2cm} Paramagnetic

Ising \hspace{2cm} Temperature

*exact solution from Onsager

[Xiaopeng Li, Arun Paramekanti, Andreas Hemmerich & WVL, Nature Communications 2014]
Weak coupling, Finite temperature – three phases found: Kosterlitz-Thouless superfluid, Chiral Bose liquid, and normal

Theory: U(1)×U(1) Phase model with interaction:

\[ H_{\text{phase}}^{\text{eff}} = \sum_{\mathbf{r}} \left\{ 2J_{\parallel} \cos(\Delta_x \theta_x(\mathbf{r})) - 2J_{\perp} \cos(\Delta_y \theta_x(\mathbf{r})) \right\} + \{x \leftrightarrow y\] - U \sum_{\mathbf{r}} \sin^2(\theta_x(\mathbf{r}) - \theta_y(\mathbf{r})) \]

\[ b_{x,y}^\dagger \sim \sqrt{\rho/2} e^{i\theta_{x,y}} \quad (p_x, p_y) \text{ components, coherent} \]

Solve by Monte Carlo simulations (Arun Paramekanti) … next slides
Weak interaction regime

Monte Carlo results: finite size scaling, two stage transition

\[ \rho_s(T_{\text{BKT}}, L) = \rho_s(T_{\text{BKT}}, \infty) \left(1 + \frac{1}{2 \log L + c}\right) \]

\[ B_L(L_z) = 1 - \frac{\langle M^4 \rangle}{3 \langle M^2 \rangle^2} \]

*Xiaopeng Li, Arun Paramekanti, Andreas Hemmerich & WVL, Nature Communications 2014*
Exotic “orbital” phases of bosons: Chiral “normal” liquid prediction for Hamburg checkerboard lattice p-band experiments

[X Li, A. Paramekanti, A. Hemmerich and WVL, *Nature Communications* (2014)]

Part 2B:

Chiral superfluidity with p-wave symmetry from an interacting s-wave atomic Fermi gas

Crucial difference: neither direct nor induced p-wave interaction needed

Our collaborative Work:

Bo Liu (Pitt Postdoc)
Xiaopeng Li (Pitt student -> postdoc in JQI Maryland)
Biao WU (Peking Univ, China)
WVL (U of Pitt)

Nature Communications 5:5064 (Sep 2014)
Next ...

Basic idea:
Concept of s+p cross-orbital pairing ---

gives topological superconductivity;
does not require Spin-Orbital coupling, nor any form of induced p-wave interaction.

How?
Chiral superfluidity with p-wave symmetry from an interacting s-wave atomic Fermi gas

[B. Liu, X. Li, B. Wu, & WVL, Nature Comm 5:5064 (30 Sep 2014)]

Highlights

- Center-of-mass $p+ip$ topological superfluid phase discovered.
- Pure s-wave interaction, no SOC (spin-orbital coupling).
- **High Tc for a p-wave**, potentially of order of $E_F$ (fermi energy) as in s-wave Feshbach resonant atomic gases.
The idea
---special geometry optical lattice

Spin dependent
Lattice potential

Spin imbalanced
Chemical potentials

Tight binding model

A sites: \(\uparrow + \downarrow\)
B sites: only \(\downarrow\) fermions

\[ N_{\downarrow} > N_{\uparrow} \]
Fermi surface matched

\(| \uparrow, s \rangle \)

\((t_2/t_0 = 0, t_3/t_0 = 0)\)

\(| \downarrow, p_{x,y} \rangle \)

Diamond = Brillouin zone for up
Momentum Units: \( \frac{\pi}{2a} \)

Whole square (biggest Square)
= Brillouin zone for down

• Two pairing order parameters \( \rightarrow (p_x, p_y) \) Cooper pairs \( = p \) molecules (bosons)

\[ \langle C^{\uparrow s} C^{\downarrow p_x} \rangle = \Delta_x \quad \langle C^{\uparrow s} C^{\downarrow p_y} \rangle = \Delta_y \]
Our model

- No p-wave resonance;
- No long-range interaction;
- No spin-orbit coupling;
- but spin imbalance and possibly a spin-dependent optical lattice of novel geometry.

* The special lattice geometry is to make cooper paring favorable even with weak interactions. With strong interactions as in the resonance regime, such special lattice is expected to be unnecessary.
Chiral center-of-mass $p$-wave pairing of fermions

$s+p_x$ and $s+p_y$ Cooper pairs condense!  

“$p+ip$” superconductor. 


-our center of mass topological $p+ip$
Zero-temperature phase diagram

Obtained by Feynman diagram expansion + symmetry based Ginzburg-Landau theory

There is a wide parameter region supporting $p_x \pm ip_y$ SF.

NG = normal state
SF = superfluid
What is next? --- Work in progress with R. Hulet group

Rice/R. Hulet group experiment achieved (to the best estimate)

- Spin polarized Fermi gas (Li-6 atoms), in 3D optical lattices
- Already very cool, down to Antiferromagnetic transition $T$, $t^2/U$; we only need low $T$ down to a fraction of tunneling energy scale enough
- $s$-wave Feshbach resonance

Theoretical proposal:

- Tune lattice depths to make quasi-1D [so to have just one $p$-band, e.g., keep $p_{y,z}$ bands significantly higher than $p_x$ band]. (Rice/Hulet group has quasi-1D tubes of atoms already – 1D Fermi gases – see Xiwen GUAN talk)
- Further tune spin polarization, so to make the two spins in $s$ and $p$ bands, respectively
Conclusion---Orbital Optical Lattice Physics

Chiral Bose liquid at finite T

Center-of-mass topological $p$-wave superconductivity

next two weeks at INT --- Discussion about Other recent work


A Domain Wall connecting two TRS pairs

Ferromagnetic domain formation is common in Ferro-Magnetic (FM) solid state materials [many references …]

Example: Ferromagnetic phase domains in momentum space seen in atomic gases

Dynamic Ferromagnetic transition
Shown in k-space. About 25,000 Cs atoms. TOF observes condensate on one of the minima of the double wells.

[C. Parker, Cheng Chin,, et al., Nature Physics (2013)]
A Domain Wall connecting two TRS pairs

Ferromagnetic domain formation is common in Ferro-Magnetic (FM) solid state materials [many references …]

... when Orbital Angular momentum shows a kink!

A domain wall decorated superconducting background
Time-Reversal-Symmetry breaking, Ferro-orbital order with superfluidity → Chiral topological SF

Ferro-orbital order coexists with superfluid

Nonzero Chern number

\[ p_x + ip_y \rightarrow C_+ = 1 \]
\[ p_x - ip_y \rightarrow C_- = -1 \]

Bogoliubov-de Gennes Bloch bands

Chern number

\[ C = \sum_{n, \text{occupied}} \int_{BZ} \frac{d^2 \vec{k}}{\pi} \text{Im} \langle \partial_{k_x} \phi_n(\vec{k}) | \partial_{k_y} \phi_n(\vec{k}) \rangle \]
Gapless chiral fermions

- Two chiral modes on the domain wall
- At $k_x a = 0$ or $\pi$, zero energy Majorana fermions

Number of right- (left-) moving edge-modes = $|C_1 - C_2|$

In our case: $|\Delta C| = 2$


Topological nature:

The in-gap states can be probed by radio-frequency spectroscopy.

**Green** = domain wall fermions

**Red** = edge states (boundary of the system)
Key features of the s+p band pairing

- Center-of-mass p-wave
- Topological and chiral
- High critical Tc: as high as s-wave Feshbach resonant BCS-BEC crossover superfluid
- Alternative to the mechanisms of p-wave Feshbach resonance, or induced one (e.g., by spin-orbital coupling SOC)
- Predicted for spin polarized Fermi gas on lattice:

\[ N_\downarrow < N_\uparrow \]