Loop currents and experimental signatures in optical lattices

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[Figure credit: S. Kelley/JQI]
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-collaboration with

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Outline

➢ Experimental observations of loop currents in optical lattices and cuprates

➢ Chiral spin condensate and spin loop currents in a hexagonal lattice
  \[XL, S. Natu, A. Paramekanti, S. Das Sarma, Nature Communications (2014)]

➢ Spontaneous Quantum Hall effect with spinor Bose-Fermi mixture in a triangular lattice
  \[Z.-F. Xu, XL, P. Zoller, W. Vincent Liu, PRL (2015)\]

➢ Loop current order and spontaneous topological superfluids
  \[Bo Liu, XL, Biao Wu, W. Vincent Liu, Nature Communications (2014)\]

➢ Topological density waves with Rydberg dressed fermions
Time-reversal symmetry is spontaneously broken. Electron loop current order is a promising candidate.
What is the loop current pattern?

consistent with Kerr rotation

Loop currents in optical lattices?

-Current in a lattice model

\[ J_{r' \rightarrow r} = -it_{rr'} \psi^\dagger_r \psi_{r'} + h.c. \]

For Bose-Einstein condensates, current means phase modulations in condensate wavefunctions.

*assumed that there is no pair hopping
Measurement is simple when the band minima are not time-reversal invariant points.
2nd band of Checkerboard lattice

G. Wirth, A. Hemmerich et al., Nat Phys (2011)

Band minima at time-reversal invariant points!

[Theory work:
A. Isacsson and S. Girvin, PRA (2005)
W. V. Liu, C. Wu, PRA (2006);
...]

-Condensate wavefunction

\[ \langle \psi_r \rangle = \sqrt{n_s} \left( e^{iK_x \cdot r} \pm i e^{iK_y \cdot r} \right) \]

\[ K_x = (\pi, 0) \quad K_y = (0, \pi) \]

Spinor BEC in a Hexagonal lattice

If we have spin \( S_z \) conservation, this current pattern cannot be dynamically stable. Its life time would be too short (1ms), in contrast to experimental observations (100ms).

\[
\sqrt{n_s} |s\rangle \pm i \sqrt{n_p} |p\rangle
\]

- current flow: not a loop current?


P. Soltan-Panahi K. Sengstock et al., NPHYS 8, 71-75 (2012)

XL, 2013 March meeting presentation
Question: What if some particles are left in the massive Dirac valleys of the 2\textsuperscript{nd} band.
Spinor Bosons in a double-valley band

(a)

(e^{-iK \cdot r}, e^{iK \cdot r})

(b)

Chiral spin superfluid

(c)

Chiral charge superfluid

(d)

\( \varphi_{\uparrow r} \rightarrow \varphi_{\downarrow r}^* \)

\[ E[\varphi_{\uparrow r}, \varphi_{\downarrow r}^*] = E[\varphi_{\uparrow r}, \varphi_{\downarrow r}] \]

Second order perturbation theory

\[ \frac{\Delta E^{(2)}}{N_s} = -\int \frac{d^d k}{(2\pi)^d} \rho \rho \left\{ \frac{|U^{\uparrow \downarrow}(k - K)|^2}{\epsilon(k) + \epsilon(Q - k)} \right\}, \quad Q = 2K \]

\[ \Delta E^{(2)} = E^{(2)}_{\chi_c} - E^{(2)}_{\chi_s} \]

**TRS:** \( T \phi_{\sigma}(k) T^{-1} = \phi_{\sigma}(-k) \)

*an anti-unitary transformation*
Universal quantum “order-by-disorder”

\[
\Delta E^{(2)} = E^{(2)}_{\chi_c} - E^{(2)}_{\chi s}
\]

\[
\Delta E^{(2)} / N_s = -\int \frac{d^d k}{(2\pi)^d} \rho_{\uparrow\downarrow} \left\{ \frac{|U_{\uparrow\downarrow}(k - K)|^2}{\epsilon(k) + \epsilon(Q - k)} \right\}
\]

\[
= -\frac{1}{2} \frac{|U_{\uparrow\downarrow}(k - K)|^2}{\epsilon(k) + \epsilon(-k)} - \frac{1}{2} \frac{|U_{\uparrow\downarrow}(K - k)|^2}{\epsilon(Q - k) + \epsilon(k - Q)}
\]

Chiral spin superfluid with the two spin components condensing at opposite valleys always has lower fluctuation energy. This universal quantum “order by disorder” selection rule only relies on the “Time-reversal” symmetry.

This momentum space antiferromagnetism can be generalized to multi-valley case with crystalline symmetries.
Logarithmic divergence and renormalized theory

In two dimensions, the bare perturbative result has a logarithmic divergence

$$\int d^2k \frac{1}{k^2} \rightarrow \text{infrared log divergence}$$

-renormalized theory

$$\frac{\Delta E^{(2)}}{N_s} = -\frac{1}{2} \rho_\uparrow \rho_\downarrow \int_k g^2(k) \rightarrow \text{effective scattering among quasi-particles}$$

$$\times \left\{ \frac{2}{\epsilon_\uparrow(k,Q-k)+\epsilon_\downarrow(k,Q-k)} \right\} \rightarrow \text{Bogoliubov spectra}$$

$$= \frac{1}{\epsilon_\uparrow(k,Q-k)+\epsilon_\downarrow(-Q+k,-k)+\Delta\epsilon(k,Q-k)-\Delta\epsilon(-Q+k,-k)}$$

$$- \frac{1}{\epsilon_\downarrow(-Q+k,-k)+\epsilon_\uparrow(k,Q-k)+\Delta\epsilon(-Q+k,-k)-\Delta\epsilon(k,Q-k)} \right\}$$
Universal Chiral spin superfluid

With two component bosons loaded into a double-valley band, quantum fluctuations universally select the chiral spin superfluid through a quantum order by disorder mechanism.

\[ \int d^d \mathbf{k} \mathbf{k} \left[ n_{\uparrow}(\mathbf{k}) - n_{\downarrow}(\mathbf{k}) \right] \neq 0 \]

Spin-loop current

[Figure credit: S. Kelley/JQI]
Experimental signatures

- experimental data

\[ \mathcal{N}^\uparrow \quad \mathcal{N}^\downarrow \]

Lattice momentum = 0

- our theory prediction

\[ n^\uparrow(\mathbf{k}) - n^\downarrow(\mathbf{k}) \]

Dirac points

P. Soltan-Panahi K. Sengstock et al., NPHYS 8, 71-75 (2012)

Relevance to other double-valley bands

C. Chin group (Chicago)
[C. Parker et al., Nat Phys (2013)]

T. Esslinger group (ETH)
[L. Tarruell et al., Nature (2012)]

I. Spielman group (JQI/NIST)
[Y.J. Lin et al., Nature (2011)]
Finite temperature perspective

Quantum simulations---Where are we with ultracold atoms?

BEC with spontaneous loop current order

- A generic state for bosons with valley degrees of freedom
- Time reversal symmetry is broken
- Easy to measure if the valley is not at time-reversal invariant point; harder otherwise, but has been measured
- No conclusive evidence for spin loop current yet
- Thermal phase transition, sharing features in cuprate phase diagram
Topological states with loop current order (fermions)

If we have a Cooper pair condensatation of this pattern, the fermionic state then has all ingredients required by topological superfluids.

G. Wirth, A. Hemmerich et al., Nat Phys (2011)
Cooper pairs with p-orbital symmetries

A cooper pair composed of s and p orbital fermions respects p-orbital symmetries.
Topological $p+ip$ topological superfluids

-spin dependent checkerboard lattice

This lattice may have already been realized in JQI by Trey Porto’s group.

Fermions in the current order background form a topological superfluid, featuring protected chiral spin currents on the edges

Cooper pairing fields:

\[
\Delta_x(x) \rightarrow (-1)^{R_x + R_y} U \Psi_{P_x}^\dagger (R) \Psi_{A}^\dagger (R)
\]

\[
\Delta_y(x) \rightarrow (-1)^{R_x + R_y} U \Psi_{P_y}^\dagger (R) \Psi_{A}^\dagger (R)
\]

Bo Liu, XL, Biao Wu, W.V. Liu, Nat Comms 5:5064(2014)
Chiral spin currents on the domain wall
Essential ingredients:

Fermi surface nesting $\rightarrow$ Spontaneous Loop currents $\rightarrow$ effective gauge fields $\rightarrow$ Topological states
Atomic spinor Bose-Fermi mixture

- **Local moments:**
  Spin-1 Rb-87 BEC: ferromagnetic interaction

- **Kondo coupling:**
  Spin-changing collision between spin-1 Rb-87 and spin-1/2 Li-6 atoms

\[
\hat{V}_{bf}(\mathbf{r}_1 - \mathbf{r}_2) = \left( g_{1/2} P_{1/2} + g_{3/2} P_{3/2} \right) \delta(\mathbf{r}_1 - \mathbf{r}_2) \\
= \left( g_d + g_s S \times \mathbf{F} \right) \delta(\mathbf{r}_1 - \mathbf{r}_2)
\]

*Spinor Bose-Fermi mixture is a natural platform to simulate Kondo lattice model (either quantum or classical).*

Triangular lattice

Fermi surface nesting and chiral magnetic ordering

Fermi surface at \( \frac{3}{4} \) filling

\[
H = -t \sum_{<ij>} c_{i\alpha}^{\dagger} c_{j\alpha} - J \sum_i \vec{S}_i \cdot \vec{c}_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}
\]

\[
\langle \vec{S}_i \rangle = \vec{S}_{Q_1} e^{iQ_1 \cdot r_i} + \vec{S}_{Q_2} e^{iQ_2 \cdot r_i} + \vec{S}_{Q_3} e^{iQ_3 \cdot r_i}
\]

Rb-87: topological spin-textures in the ground state

[Z. F. Xu, XL, P. Zoller, and W. V. Liu, PRL (2015)]
Li-6: Quantum Hall state

Rb-87: Chiral superfluid

As a consequence of emergent gauge fields, Bosons condense at finite momenta

Systems to look for topological loop currents

- Multi-band Fermion systems (parity mixing)
- Bose-Fermi mixture
- Fermions with non-local interactions, say by Rydberg dressing

Common feature:

*Fermi surface nesting*->*Spontaneous Loop currents*->*effective gauge fields*->*Topological states*
Rydberg dressing and non-local interactions

J. B. Balewski et al., NJP (2014)
N. Henkel, R. Nath, T. Pohl, PRL (2010)

\[ V_{\text{eff}} (\mathbf{r}) = \frac{V_6}{1 + (|\mathbf{r}|/r_c)^6} \]
Topological density waves

$$H_{\text{BdG}}(\mathbf{k}) = \begin{bmatrix} \epsilon_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta_{\mathbf{k}}^* & -\epsilon_{\mathbf{k}} \end{bmatrix}$$

$$\rho_{\mathbf{k}} = \langle \psi^\dagger(\mathbf{k} + \mathbf{Q})\psi(\mathbf{k}) \rangle$$

$$\Delta_{\mathbf{k}} = \int \frac{d^3q}{(2\pi)^3} \left[ \hat{V}(\mathbf{Q}) - \hat{V}(\mathbf{k} - \mathbf{q}) \right] \rho_q$$

XL, S. Das Sarma, arXiv (2015), accepted to Nature Communications
Summary

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