Can we control complex species like Er or Dy with dense sets of chaotic overlapping resonances?

A Work in Progress

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Krzysztof Jachymski (U. Warsaw)
Tilman Pfau Group, U. Stuttgart, especially Igor Ferrier
3-body theory by Yujun Wang

http://www.jqi.umd.edu/

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Collisions of atoms and molecules on the microscale (nm) control quantum phenomena on the macroscale (>>μm) and destructive interactions that set the system lifetime ( < ms to >> 1s )

Can we understand and control complex atoms or molecules having collision complexes with dense sets of resonances?

Outline

Review experimental findings—new Dy data, Pfau group

Introduce basic theory of overlapping resonances: Cs example CC and MQDT models

Some thoughts about what is going on in Er or Dy
**Properties of Dysprosium**

<table>
<thead>
<tr>
<th></th>
<th>Dysprosium</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Protons</strong></td>
<td>66</td>
</tr>
<tr>
<td><strong>Stable Isotopes</strong></td>
<td>( ^{160}\text{Dy} (2%), ^{161}\text{Dy} (19%), ^{162}\text{Dy} (26%), ^{163}\text{Dy} (25%), ^{164}\text{Dy} (28%) )</td>
</tr>
<tr>
<td><strong>Electronic structure</strong></td>
<td>[Xe] 4f(^{10}) 6s(^2) (\rightarrow) (^{5})I(_8)</td>
</tr>
<tr>
<td><strong>Angular momentum L</strong></td>
<td>6</td>
</tr>
<tr>
<td><strong>Spin S</strong></td>
<td>2 ( (4\text{ unpaired }e^-) )</td>
</tr>
<tr>
<td><strong>Magnetic moment (\mu)</strong></td>
<td>10 (\mu_B)</td>
</tr>
<tr>
<td><strong>Nuclear spin</strong></td>
<td>0 ( (\text{bosons}), 5/2 ( \text{fermions} )</td>
</tr>
</tbody>
</table>

**Similar Er\(^3\)H\(_6\)**

- Dy\(_2\): 153 potential curves
- Er\(_2\): 91 potential curves

**Anisotropic C\(_6\), C\(_3\)**

Kotochigova and Petrov, PCCP 13, 19165 (2011)
“Quantum Dipolar Gases in Boson or Fermion Flavor”

Ferlaino group
Innsbruck
$^{168}$Er
30000 atoms
order 100nK

Lev group
Stanford
$^{161}$Dy
6000 atoms
64nK $T/T_F=0.2$
Quantum chaos in ultracold collisions of gas-phase erbium atoms

Albert Frisch¹, Michael Mark¹, Kiyotaka Aikawa¹, Francesca Ferlaino¹, John L. Bohn², Constantinos Makrides³, Alexander Petrov⁴,⁵ & Svetlana Kotochigova³

¹¹⁶⁸Er in ground state \(^{3}H_{6}(m=-6)\)
Diagonal potential energy curves for $^{164}$Dy + $^{164}$Dy at $B = 50$G

There are 91 curves for channels $|(j_1 j_2)j_m l_m>$, $m_j + m_l = -16$, $l$ up to 10

From Petrov et al, PRL 109, 103002 (2012)
From Frisch et al (2014)
Dense set of overlapping (interacting) resonances: mixed eigenstates of “random” character

From Frisch et al (2014)
From Mayle, Ruzic, Bohn, Phys. Rev. A 85, 062712 (2012)

Toy Statistical model

Rb + KRb

Energy vs. magnetic field (G)

short-range resonances

incoming / outgoing channel:
rovibrational ground state
$v=n=0$
Recent unpublished Dy data deleted, pending submission of paper
$>10 \text{ THz (1000K)} \quad 10 \text{ Ghz (K)} \quad 10 \text{ kHz (}\mu\text{K})$

**Short-range**
- Chemical Complexity
- Few-body Reactions
- Bridge
  - Sturdy, but simple
  - well-understood

**Long-range**
- Highly controlled
  - $R \gtrsim 100 \text{ nm}$
- Quantum state preparation
- Low entropy
- Free space
- 1D, 2D, 3D confined
- Many-body
- Gauge potentials

$V(R)$

$R_{\text{bond}}$

$\bar{a}$

Fig. 2, PSJ Faraday Disc 142, 361 (2009)
>10 THz (1000K)  10 Ghz (K)  10 kHz (µK)

**Short-range**

- A few parameters
  - Phase ($\alpha$)
  - Feshbach strength ($s_{res}$)
  - Reactivity ($y$)
  - Statistical complexity

**Long-range**

- Not unique
  - More than one way
  - To build a bridge
    (Mies/PSJ/Hutson Greene/Bohn, Gao)
  - Analytic
  - Numerical

**Separated**

- $A,B =$ Atom or molecule
- Highly controlled
  - $R \gtrsim 100$ nm
- Quantum state preparation
- Low entropy
- Free space
  - 1D, 2D, 3D confined
- Many-body
  - Gauge potentials

Fig. 2, PSJ Faraday Disc 142, 361 (2009)
Long-range potential

\[ v(r) = -\frac{1}{rp} + \frac{\ell(\ell + 1)}{r^2} \]

For \( p=6 \), we also use \( R_{vdW} = \frac{1}{2} R_6 \) or \( \bar{a} = 0.478 \ldots R_6 \)
“Size” of vdW potential

\[ E_b = -\frac{\hbar^2}{2\mu a^2} \]

<table>
<thead>
<tr>
<th>( R_{vdw}(a_0) )</th>
<th>( E_{vdw}(mK) )</th>
<th>(MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^6\text{Li} )</td>
<td>31</td>
<td>29</td>
</tr>
<tr>
<td>( ^{40}\text{K} )</td>
<td>65</td>
<td>1.0</td>
</tr>
<tr>
<td>( ^{85}\text{Rb} )</td>
<td>83</td>
<td>0.35</td>
</tr>
<tr>
<td>( ^{133}\text{Cs} )</td>
<td>101</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Feshbach resonances in ultracold gases

Resonance “pole strength”

\[ s_{\text{res}} = \frac{a_{bg}}{\bar{a}} \frac{\Delta \mu_{\text{diff}}}{\bar{E}} \]

\[ a = a_{bg} \left( 1 - \frac{\Delta}{B - B_0} \right) \]

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![Graphical representation of Feshbach resonances](image-url)

![Graph showing the a(B) s basis only and Efimov physics](image)

-1
-2
-3

-400
-800

-7(4)6s(6)
-8(3)6s(6)
-1(4)6s(6)
-2(3)6s(6)

-3(3)6s(6)

E/h (MHz)

magnetic field strength (G)

(a)

(b)
Cs |3,+3> + |3,+3>, 143 resonances L=0,2,4,6,8 up to 1000G

Cs₂ nearest neighbor spacing statistics
Blue: Poisson statistics
Red: Random matrix theory statistics

Data indicated for 3-body loss maxima (solid) and minima (open).

Analytical model of overlapping Feshbach resonances

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\[ a(B) = a_{bg} \prod_{i=1}^{N} \left( 1 - \frac{\tilde{\Delta}_i}{B - B_{i}^{\text{res}}} \right) \]
\[ a(B) = a_{bg} \sum_{i=1}^{N} P_i(B) \]

\[ P_i(B) = \frac{1}{2} \frac{\hat{r}_i}{\delta \mu_i} C^{-2}(E) / k \]

\[ B - B_i - \frac{1}{2} \tan \lambda(E) \left( \frac{\hat{r}_i}{\delta \mu_i} - \sum_{j \neq i} \frac{B - B_i}{B - B_j} \frac{\hat{r}_j}{\delta \mu_j} \right) \]

\[ a(B) = a_{bg} \left( 1 - \sum_i \Delta_i \right) \frac{\delta B_i}{B - B_i - \delta B_i - \sum_{j \neq i} \frac{B - B_i}{B - B_j} \delta B_j} \]

\[ a(B) = a_{bg} \prod_{i=1}^{N} \left( 1 - \frac{\tilde{\Delta}_i}{B - B_i^{\text{res}}} \right) \]

\[ a(\text{near } B_i) = \tilde{a}_{bg,i} \left( 1 - \frac{\tilde{\Delta}_i}{B - B_i^{\text{res}}} \right) \]

\[ \tilde{a}_{bg,i} = a_{bg} \prod_{j \neq i}^{N} \left( 1 - \frac{\tilde{\Delta}_j}{B_i^{\text{res}} - B_j^{\text{res}}} \right) \]

“global” background

Short-range coupling

Interaction shift

“local” background
3-Body recombination of 3 alkali-metal atoms

Computer codes and calculations by Yujun Wang

\[ \text{Cs} + \text{Cs} + \text{Cs} \rightarrow \text{Cs}_2 + \text{Cs} \]

Three-channel Cs + Cs interaction: “Exact” 2-body Feshbach model

2-channel numerical model using \( s_{\text{res}}, \alpha_{\text{bg}} \) for Cs-Cs

6-12 Lennard-Jones potentials
+ short-range coupling

Number of bound states can be varied, \( N = 2 \) to 4.

Given “exact” 2-body model parameterized by known \( s_{\text{res}}, \alpha_{\text{bg}}, m_{\text{dif}} \)

plus 3-body interactions as a sum of 2-body ones,
numerically solve 3B equations in hyperspherical basis
Cs overlapping resonances (multiple $s_{res}$): Jachymski, PSJ, PRA 88, 052701(2013)
Calculations: no adjustable parameters

554G zero crossing (3 spin)

Calculations by Yujun Wang

$L_3 \text{ (cm}^6\text{/s)}$

$a/r_{vdW}$
“Toy” van der Waals system (no dipoles)
The End