From C to Parton Sea
PDFs on the Lattice

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This talk is based on the work
“Flavor Structure of the Nucleon Sea from Lattice QCD”, 1402.1462 [hep-ph]
performed in collaboration with

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Saul Cohen (UW)
Xiangdong Ji (UMD/SJTU/INPAC)
Parton Distribution Functions

§ Quark distribution
- spin-averaged
- unpolarized

§ Helicity distribution
- spin-dependent
- longitudinally polarized

§ Transversity distribution
- spin-dependent
- transversely polarized

most well known

very poorly known
What can LQCD Help?

§ Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories

✔ Ideal tool for studying nonperturbative hadron structure

§ Physical observables are calculated from the path integral

\[ \langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi \ e^{iS(\bar{\psi}, \psi, A)} O(\bar{\psi}, \psi, A) \]

§ Lattice QCD

✔ Impose a UV cutoff
discretize spacetime

✔ Impose an Infrared cutoff
finite volume

Huey-Wen Lin — Studies of 3D Structure of Nucleon
Many lattice calculations of the moments of the PDFs

\[ \langle x^{n-1} \rangle_q = \int_{-1}^{1} dx \, x^{n-1} q(x) \]

\[ \langle x^{n-1} \rangle_{\Delta q} = \int_{-1}^{1} dx \, x^{n-1} \Delta q(x) \]

\[ \langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx \, x^{n-1} \delta q(x) \]

Limited to the lowest few moments

Might provide constraints on models or tests of experiment

Also applies to GPDs: limited to 3\textsuperscript{rd} moment

Most progress made in quark contributions

Very costly to obtain useful gluon signal

Limited by available computational resources
§ Leading moment $\langle x \rangle$, hypercubic decomposition

$4_1 \otimes 4_1 = 1_4 \oplus 3_4 \oplus 6_4 \oplus 6_4$:

Both operators go to same continuum limit

§ No mixing with operators of same or lower dimension

§ To improve to $O(a)$

Consider all irrelevant operators of same symmetry:

§ Higher moments

$4_1: O_{111}$ mixes with $q \sigma_{\mu\nu} D_{[\nu} D_{\rho]} q$

$4_2: O_{\{123\}}$ requires all momentum components to be nonzero

$8_1: O_{\{441\}} - (O_{\{221\}} + O_{\{331\}})/2$ mixes under renormalization

§ For higher spin, all ops mix with lower-dimension ops

$\langle x^n \rangle$ Moments

Symmetry: You Break it, You Buy It.
§ Easiest case: first moment \( \langle x \rangle \), hypercubic decomposition

\[ 4_1 \otimes 4_1 = \begin{array}{c} 1_1 \\ \oplus \\ 3_1 \\ \oplus \\ 6_1 \\ \oplus \\ 6_3 \\ \end{array} \]

\[ 0_{44} = \frac{1}{3} (O_{11} + O_{22} + O_{33}) \]

\[ O_{14} + O_{41}, \text{ (requires } p \neq 0) \]

Both operators go to same continuum limit

No mixing with operators of same or lower dimension

§ To improve to \( O(a) \)

Consider all irrelevant operators of same symmetry:

\[ O_{\{\mu\nu\}} \rightarrow (1 + a m_q c_0) O_{\{\mu\nu\}} + i a c_1 \bar{q} \sigma_{\mu\rho} \begin{array}{c} D_{\nu} \\ \quad \end{array} D_{\rho} q \]

+ \[ a c_2 \bar{q} \begin{array}{c} D_{\mu} \\ \quad \end{array} D_{\nu} q + i a c_3 \partial_{\rho} \bar{q} \sigma_{\mu\rho} \begin{array}{c} D_{\nu} \\ \quad \end{array} q \]

§ Higher moments \( \langle x^2 \rangle \)

\[ 4_1: O_{111} \text{ mixes with } \bar{q} \gamma_1 q \text{ with coefficient } \sim \frac{1}{a^2} \]

\[ 4_2: O_{\{123\}} \text{ requires all momentum components to be nonzero} \]

\[ 8_1: O_{\{441\}} - (O_{\{221\}} + O_{\{331\}})/2 \text{ mixes under renormalization} \]

§ For higher spin, all ops mix with lower-dimension ops
§ For higher spin, all ops mix with lower-dimension ops

-Tricks: subtraction to remove divergent terms, heavy fields, four-point functions... **None is practical enough**

§ Relative error grows in higher moments

- Calculation would be costly and difficult

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Dolgov et al. PRD66, 034506 (2002)
Göckeler et al. PRD71, 114511 (2005)

LHPC (SCRI, SESAM): 2f, Wilson and clover
QCDSF: 0f
§ What can we learn about the $x$-distribution?

Make an ansatz of some smooth form for the distribution and fix the parameters by matching to the lattice moments.

\[ xq(x) = ax^b(1 - x)^c(1 + \epsilon \sqrt{x} + \gamma x) \]

Cannot separate valence-quark contribution from sea

New idea needed to access the sea!

A New Kid in Town
**The Idea**

§ Finite-momentum quark distribution

\[ q(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left< \gamma_z \exp \left( -ig \int_0^z dz' A_z(z') \right) \psi(0) \right| P \right> \]

Nucleon momentum \( P_\mu = \{P_0, 0, 0, P_z\} \)

- In \( P_z \rightarrow \infty \) limit, parton distribution is recovered
- For finite \( P_z \), corrections are needed

\[ x = \frac{k_z}{P_z} \]

Lattice \( z \) coordinate

Product of lattice gauge links

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013); this workshop
**Some Lattice Details**

§ Exploratory study

\( N_f = 2+1+1 \) clover/HISQ lattices (MILC)

\( M_\pi \approx 310 \text{ MeV}, \ a \approx 0.12 \text{ fm} \ (L \approx 2.88 \text{ fm}) \)

Isovector only ("disconnected" suppressed)

  - gives us flavor asymmetry between up and down quark

  - 2 source-sink separation \( (t_{\text{sep}} \approx 0.96 \text{ and } 1.2 \text{ fm}) \) used

§ Properties known on these lattices

  - Lattice \( Z_\Gamma \) for bilinear operator \( \sim 1 \)
    - (with HYP-smearing)

  - \( M_\pi L \approx 4.6 \) large enough to avoid finite-volume effects

§ Feasible with today's computational resources!

  - 8/16 nodes on UW Hyak cluster


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Hyak @ UW

HWL et al, 1402.1463 (submitted to Phys. Rev. Lett.)
Exploratory study

*MILC Collaboration, Phys. Rev. D87, 054505 (2013)*

\[ N_f = 2+1+1 \text{ clover/HISQ lattices (MILC)} \]

\[ M_\pi \approx 310 \text{ MeV}, \ a \approx 0.12 \text{ fm} (M_\pi L \approx 4.5) \]

Warning!

NO SYSTEMATICS YET!

Demonstration that the method works!

Intend to motivate future LQCD work on many quantities
\[ \left\langle P \left| \bar{\psi}(z) \gamma_z \exp\left(-i g \int_0^z d z' A_z(z')\right) \psi(0) \right| P \right\rangle \]

How many links are needed?

Lattice momenta discretized by finite size of volume

\[ P_z \in \{1, 2, 3\} \frac{2\pi}{L} \]
Quark Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \right| \bar{\psi}(z) \gamma_z \exp\left(-i g \int_0^z dz' A_z(z')\right) \psi(0) \left| P \right\rangle$$

$$P_z \in \{1, 2, 3\} \frac{2\pi}{L}$$

Uncorrected bare lattice results

$$x = \frac{k_z}{P_z}$$
Quark Distribution

§ Exploratory study

\[ \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z) \gamma_z \exp\left(-ig \int_0^z dz' A_z(z')\right) \psi(0) \right| P \right\rangle \]

\[ P_z \in \{1, 2, 3\} \frac{2\pi}{L} \]

Distribution gets sharper as \( P_z \) increases

Artifacts due to finite \( P_z \) on the lattice

Improvement?

Work out leading-\( P_z \) corrections
§ Back to the continuum

\[ q(x, \mu) = q_{FP}(x, \mu, P_z) + O(\Lambda_{QCD}^2 / P^2_z) + O(M_N^2 / P^2_z) + O(\alpha_s) \]

\[ P_z \in \{1, 2, 3\}^{2\pi / L} \]

What we want

What we calculate on the lattice

Smaller \( P_z \) correction but complicated higher-twist operator
(extrapolate it away)

J.-H. Zhang, Y. Zhao, J.-W. Chen et al. (in preparation)

Dominant correction (for nucleon); known scaling form

J.-W. Chen et al. (in preparation)

Finite \( P_z \to \infty \)

Estimate \( O(20\%) \) effect

X. Xiong, X. Ji, J. Zhang, 1310.7471 [hep-ph]; this workshop
Exploratory study

Take ratios (partially cancel statistical and systematic errors)

\[ q_{\text{norm}}(x, \mu, P_z) = \frac{q(x, \mu, P_z)}{\int dx \ q(x, \mu, P_z)} \]

Removing \( O(M_N^n/P_z^n) \) errors + \( O(\alpha_s) \)

No significant finite-momentum effect seen for \( P_z > 1 \)
§ Exploratory study

- Take ratios (partially cancel statistical and systematic errors)

\[
q_{\text{norm}}(x, \mu, P_z) = \frac{q(x, \mu, P_z)}{\int dx \, q(x, \mu, P_z)}
\]

Removing \(O(M_N^n/P_z^n)\) errors + \(O(\alpha_s)\)

No significant finite-momentum effect seen for \(P_z > 1\)

Further removing \(O(\Lambda_{\text{QCD}}^2/P_z^2)\) errors
Exploratory study

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MSTW
CJ12
Lattice

A.D. Martin et al.

J.F. Owens et al.
PRD 87, 094012 (2012)
Quark Distribution

§ Compare with experiments

\[ \bar{q}(x) = -q(-x) \]

Compared with E866

Too good to be true?

Lost resolution in small-\(x\) region

Future improvement to have larger lattice volume

\[
\int dx \left( \bar{u}(x) - \bar{d}(x) \right) \approx -0.16(7)
\]

<table>
<thead>
<tr>
<th>Experiment</th>
<th>(x) range</th>
<th>(\int x_0^1 [\bar{d}(x) - \bar{u}(x)]dx)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E866</td>
<td>(0.015 &lt; x &lt; 0.35)</td>
<td>(0.118 \pm 0.012)</td>
</tr>
<tr>
<td>NMC</td>
<td>(0.004 &lt; x &lt; 0.80)</td>
<td>(0.148 \pm 0.039)</td>
</tr>
<tr>
<td>HERMES</td>
<td>(0.020 &lt; x &lt; 0.30)</td>
<td>(0.16 \pm 0.03)</td>
</tr>
</tbody>
</table>

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
§ Exploratory study

\[ \int \frac{dz}{4\pi} e^{-izk_z} \left( \bar{\psi}(z) \gamma_z \gamma_5 \exp\left(-ig \int_0^z d z' A_z(z')\right) \psi(0) \right| P \]

Uncorrected bare lattice results
We found $\Delta \bar{u} > \Delta \bar{d}$ with large sea asymmetry.

\[
\int dx \left( \Delta \bar{u}(x) - \Delta \bar{d}(x) \right) \approx 0.24(6)
\]
§ Model: $\text{large-}N_c$ predicts larger polarized antiquark asymmetry 
chiral quark-soliton model $\int dx (\Delta \bar{u}(x) - \Delta \bar{d}(x)) \approx 0.31$

§ Experimental comparison

D. de Florian et al. PRD 80, 034030 (2009); P. Jimenez-Delgado et al. arXiv:1310.3734

B. Dressler et al, hep-ph/9809487
§ Model: large-$N_c$ predicts larger polarized antiquark asymmetry in chiral quark-soliton model $\int dx \left( \Delta \bar{u}(x) - \Delta \bar{d}(x) \right) \approx 0.31$

B. Dressler et al, hep-ph/9809487

Improved fits with preliminary 2012STAR data

§ Future full data analysis from RHIC will reveal more info

E. Aschenauer et al. (2013), 1304.0079
Transversity Distribution

§ Exploratory study

\[ \int \frac{d\ell}{4\pi} e^{-i\ell \cdot k} \left| P \right| \overline{\psi}(z) \sigma_{xy} \exp\left( -ig \int_0^z dz' A_z(z') \right) \psi(0) \left| P \right| \]

Uncorrected bare lattice results
§ There have only been 2 attempts (still very preliminary)

- Requires more theory input and experimental data
- More assumptions are made to extract the distribution


§ Exploratory study

We found $\delta \bar{u} < \delta \bar{d}$ with large sea asymmetry.

Chiral quark-soliton model

\[ \int dx \left( \delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.26(10) \]

\[ \int dx \left( \delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.082 \]

\[ \delta \bar{q}(x) = -\delta q(-x) \]
Overcoming longstanding obstacle to $x$-distribution

New idea by Ji for studying full $x$-dependence of PDFs

Promising results on unpolarized and polarized sea asymmetry compared with experiments, even at non-physical pion mass

Caveats

Not a precision calculation yet

Need to complete the other $p_z$ corrections (on going; possibly done in a couple weeks)

Systematics due to large momenta (some ideas to improve it)

Need improvement for large-$q$ form factors, hadronic and flavor physics, …

A NEW HOPE

It is a period of war and economic uncertainty.

Turbmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful...
Summary and Outlook

Exciting time for hadron structure on the lattice

§ Overcoming longstanding obstacle to full $x$-distribution
  ✧ Demonstrates Ji proposal can be used for practical calculations

§ First ab-initio approach to study sea asymmetry
  ✧ Promising results on unpolarized and polarized sea asymmetry compared with experiments, even at non-physical pion mass
  ✧ Prediction of transversity sea asymmetry

§ Caveats
  ✧ Not a precision calculation yet, better statistics, improve large-momentum signal, proper renormalization,…

§ Opens doors for future lattice-QCD work
  ✧ Wide variety of light-cone quantities can be computed
Outlook

Lattice Parton Physics Project (LP3) Workshop

https://sites.google.com/a/lbl.gov/lp3dc/

Maryland Center for Fundamental Physics
University of Maryland
College Park, MD
March 31–April 2

Organizers:
Xiangdong Ji (Maryland/Shanghai Jiaotong)
Huey-Wen Lin (University of Washington)
Kostas Orginos (William and Mary/JLab)
Jianwei Qiu (Brookhaven)
Christian Weiss (JLab)
Feng Yuan (LBNL)
Backup Slides
Pion Distribution Amplitude

Exploratory study

\[ \int \frac{dz}{2\pi} \frac{e^{-izk_z}}{1} \left\langle 0 \g | d(z) \gamma_z \gamma_5 \exp \left( -ig \int_0^z dz' A_z(z') \right) u(0) | \pi^+(P) \right\rangle \]

Only leading mass correction applied.

Dominated by \( O(\Lambda_{QCD}^2/P_z^2) \) errors.

\( P_z \in \{1, 2, 3\} \frac{2\pi}{L} \)
Outlook

Exciting time for hadron structure on the lattice

§ Improved calculations planned

- Obtained time at NERSC for physical pion mass ensemble

Achievable with supercomputing

§ More: strange and heavy-quark distributions, gluons, TMD...

- Charm: future EIC (eRHIC?), LHC \( pp \rightarrow \gamma cX \), ...
- Polarized gluon: polarized \( pp \) collisions at RHIC, ...

| 2018 | HP14 (new) | Extract accurate information on spin-dependent and spin-averaged valence quark distributions to momentum fractions \( x \) above 60% of the full nucleon momentum |