Energy Loss at “NLO” in eXtremely hot QCD

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Outline: Energy loss and transport in weakly coupled plasmas at “NLO”

1. Philosophy of weakly coupled calculations – there is only one right answer . . .
   
   (a) Collisional vs. radiative loss
   
   (b) Corrections to collinear formalism
   
   (c) Relation between drag and radiative loss
   
   (d) Can work with enough kicking

2. Energy loss of light quarks and gluons

3. Thermal photons
Three mechanisms for energy loss and transport at LO in QGP

1. Hard Collisions: $2 \leftrightarrow 2$

2. Drag: collisions with soft random classical field

\[
\frac{dp}{dt} = -\eta \hat{v}
\]
3. Brem: $1 \leftrightarrow 2$

- random walk induces collinear bremsstrhalung

- The probability of a transverse kick of momentum $\mathbf{q}_\perp$ from soft fields:

$$\hat{C}_{LO}[\mathbf{q}_\perp] = \frac{Tm_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)}$$

NLO involves corrections to these processes and the relation between them.

Same processes determine the shear viscosity of QCD in high temperature plasma!
Three rates for energy loss at leading order:

1. Hard Collisions – a $2 \leftrightarrow 2$ processes

\[
P \sim E \\
[\partial_t + v_k \cdot \partial_x] f_k = C_{2\leftrightarrow2}[\mu_\perp]
\]

Total $2 \leftrightarrow 2$ scattering rate depends logarithmically on the cutoff
2. Drag and long-diffusion: A longitudinal force-force correlator along the light cone

\[ [\partial_t + v_k \cdot \partial_x] f_k = \eta(\mu) v \cdot \frac{\partial f_k}{\partial k} \]

- Evaluate **longitudinal** force-force with hard thermal loops + sum-rules

\[
\eta(\mu) \propto g^2 C_A \int_0^\mu \frac{d^2 p_T}{(2\pi)^2} \int \frac{dp_+ dp_0}{(2\pi)^4} \langle F_{z+}(P) F_{z+} \rangle 2\pi \delta(p_+) \\
\propto g^2 C_A \int_0^\mu \frac{d^2 p_T}{(2\pi)^2} \frac{m_\infty^2}{p_T^2 + m_\infty^2} \\
\propto g^2 C_A \frac{m_\infty^2}{4\pi} \log(\mu^2/m_\infty^2)
\]

The \( \mu \)-dependence of the drag cancels against \( \mu \)-dependence of the \( 2 \rightarrow 2 \) rate
3. Collinear Bremsstrhalung – a $1 \leftrightarrow 2$ processes

The bremsstrhalung rate is proportional to the rate of transverse momentum kicks, $\hat{C}_{LO}[q_\perp]$:

$$\hat{C}_{LO}[q_\perp] = \text{in medium scattering rate with momentum } q_\perp$$

- Need to compute transverse force-force correlators along the light cone.

$$q_\perp^2 \hat{C}_{LO}[q_\perp] = \int \frac{dq_+dq_0}{(2\pi)^2} \left\langle F_{i+}(Q)F_{i+} \right\rangle 2\pi \delta(q_+)$$

evaluate with sum rule at $q_0 = 0$

$$= \frac{Tm_D^2}{q_\perp^2 + m_D^2}$$
Summary – the full LO Boltzmann equation:

\[ \partial_t + \mathbf{v}_k \cdot \nabla_x f_k = \eta(\mu) \mathbf{v}_k \cdot \frac{\partial f_k}{\partial \mathbf{k}} + C_{2\leftrightarrow2}[\mu] + C_{1\leftrightarrow2} \]

1) The cutoff dependence of the drag cancels against the $2 \rightarrow 2$ rate!

2) Soft sector enters in just a few places.

3) Light cone sum rules.
Use the Boltzmann equation for energy loss or shear viscosity:

$$\frac{dE}{dx} \propto g^2 T^2 \left[ O(g^2 \log) + O(g^2) \right] + O(g^3 \log) + O(g^3) \quad + \ldots$$

NLO, from soft $gT$ gluons, $n_B \sim \frac{T}{\omega} \sim \frac{1}{g}$
\(O(g)\) Corrections to Hard Collisions, Drag, Brems:

1. No corrections to Hard Collisions:

2. Corrections to Drag:

   - Nonlinear interactions of soft classical fields changes the force-force correlator
   - Doable because of HTL sum rules (light cone causality)
3. Corrections to Bremsstrahlung:

(a) Small angle bremsstrahlung. Corrections to AMY coll. kernel. (Caron-Huot)

\[ Q = (q^+, q^-, q_\perp) = (gT, g^2T, gT) \]

\( \theta \sim \frac{m_D}{E} \)

\[ \hat{C}_{LO}[q_\perp] = \frac{T g^2 m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)} \rightarrow \text{A complicated but analytic formula} \]

(b) Large angle bremsstrahlung and collisions with plasmons.

- Include collisions with energy exchange, \( q^- \sim gT \).

\[ Q = (q^+, q^-, q_\perp) = (gT, gT, gT) \]

\( \theta \sim \sqrt{m_D/E} \)

The large-angle (semi-collinear radiation) interpolates collisional and rad. loss.
The NLO Boltzmann equation – a preview:

\[
[\partial_t + v_k \cdot \partial_x] f_k = (\eta(\mu) + \delta \eta(\mu)) \ v_k \cdot \frac{\partial f_k}{\partial k} + C_{2\leftrightarrow2}[\mu] \\
C_{1\leftrightarrow2} + \delta C_{1\leftrightarrow2} + C_{\text{semi-coll}}[\mu]
\]

Cutoff dependence cancels

The \( \mu \)-dependence of the drag at NLO cancels the \( \mu \)-dependence of semi-collinear radiation.
Semi-collinear radiation – a new kinematic window

2 → 2 processes

semi-collinear radiation
collinear radiation

The semi-collinear regime interpolates between brem and collisions
Matching collisions to brem

- When the gluon becomes soft (a plasmon), the $2 \leftrightarrow 2$ collision:

  \[ \theta \sim \sqrt{g} \]

  \[ q^- \sim gT \]

  is not physically distinct from the wide angle brem

  \[ \theta \sim \sqrt{g} \]

Need both processes

- For harder gluons, $q^- \to T$, bremm becomes a normal $2 \to 2$ process.
- For softer gluons, $q^- \to g^2T$, wide angle bremm matches onto collinear limit.
Brem and collisions at wider angles (but still small!)

- Semi-collinear emission:

\[ p_{\text{out}} \equiv z p_{\text{in}} \]

\[ p_{\text{in}} \]

\[ p_{\text{out}} \]

\[ q^{-} = \delta E = \Delta p^{-} \sim gT \]

- The matrix element is:

\[ |\mathcal{M}|^2 \left( 2\pi \right)^4 \delta^4(P_{\text{tot}}) \propto \frac{1 + z^2}{z} \int_{Q} \frac{1}{(q^{-})^2} \left\langle F_{i+}(Q) F_{i+} \right\rangle 2\pi \delta(q^{-} - \delta E) \]

QCD splitting fcn

scattering-center

All of the dynamics of the scattering center in a single matrix element \( \left\langle F_{i+}(Q) F_{i+} \right\rangle \)
The scattering center:

\[ \hat{C}[\mathbf{q}_\perp, \delta E] = \int_Q \frac{1}{(q^-)^2} \langle F_{i+}(Q) F_{i+}^\dagger \rangle 2\pi \delta(q^- - \delta E) \]

1. Soft-correlator has wide angle brem = cut

2. And plasmon scattering = poles
Finite energy transfer sum-rule

\[ q_{\perp} \sim gT \]

\[ \theta \sim \sqrt{g} \]

- The small angle bremsstrahlung rate involves:

\[
q_{\perp}^2 \hat{C}_{LO}[q_{\perp}] = \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} \langle F_{i+} F_{i+}(Q) \rangle |_{q_+ = 0} = \frac{T m_D^2}{q_T^2 + m_D^2}
\]

Rate of transverse kicks of \( q_{\perp} \)

- The wide angle bremsstrahlung rate involves a finite \( q^- = \delta E \) generalization:

\[
\int_{-\infty}^{\infty} \frac{dq_0}{2\pi} \langle F_{i+} F_{i+}(Q) \rangle |_{q_+ = -\delta E} = T \left[ \frac{2(\delta E)^2 (\delta E^2 + q_{\perp}^2 + m_D^2) + m_D^2 q_{\perp}^2}{(\delta E^2 + q_{\perp}^2 + m_D^2)(\delta E^2 + q_{\perp}^2)} \right]
\]

Rate of transverse kicks of \( q_{\perp} \) and energy transfer \( \delta E \)

almost involves the replacement, \( q_{\perp}^2 \rightarrow q_{\perp}^2 + \delta E^2 \)
Matching between brem and drag

\[ 2 \to 2 \text{ processes} \]

semi-collinear radiation

collinear radiation

What happens when the final gluon is soft?

- The semi-collinear emission rate diverges logarithmically when the gluon gets soft

\[ \Gamma_{\text{semi-coll}} \sim g^2 C_A \frac{\delta m^2}{4\pi} \log \left( \frac{2Tm_D}{\mu} \right) \]

When the gluon becomes soft need to relate radiation and drag.
Matching between semi-collinear brem and drag

- When the final gluon line becomes soft, the brem process:

\[ P K \approx P \mu \sim zP \]

is not physically distinct from the drag process:

\[ P K \approx P Q \sim gT \]

but represents a higher order correction to drag.

Separately both processes depend on the separation scale, \( \mu \sim gT \), but . . .

the \( \mu \) dep. cancels when both rates are included
Computing the NLO drag:

\[ \eta(\mu) \propto g^2 C_A \int^\mu \frac{d^2 p_T}{(2\pi)^2} \frac{m_\infty^2 + \delta m_\infty^2}{p_T^2 + m_\infty^2 + \delta m_\infty^2} \]

\[ \propto \text{leading order} + g^2 C_A \frac{\delta m_\infty^2}{4\pi} \left[ \log \left( \frac{\mu^2}{m_\infty^2} \right) - 1 \right] \]

NLO correction to drag

The cutoff dependence of the drag cancels against the semi-collinear emission rate

- Evaluate NLO longitudinal force-force with hard thermal loops + sum-rules
- Only change relative to LO is the replacement \( m_\infty^2 \rightarrow m_\infty^2 + \delta m_\infty^2 \)
The NLO Boltzmann equation review:

\[
[\partial_t + v_k \cdot \partial_x] f_k = (\eta(\mu) + \delta \eta(\mu)) \mathbf{v}_k \cdot \frac{\partial f_k}{\partial k} + C_{2\leftrightarrow 2}[\mu] \\
C_{1\leftrightarrow 2} + \delta C_{1\leftrightarrow 2} + C_{\text{semi-coll}}[\mu]
\]

Lessons from weak coupling

- Tight relation between drag, wide angle emissions, quasi-particle mass shift.
  - Closely related to dimensional reduction.
- The wide angle emission kernel \( \hat{C}[\mathbf{q}_\perp, \delta E] \) is closely related to \( \hat{C}[\mathbf{q}_\perp] \), almost:
  \[
  \mathbf{q}_\perp^2 \to \mathbf{q}_\perp^2 + \delta E^2
  \]
  - Closely related to dimensional reduction.
- Understand in detail the transition from radiative to collisional loss

Currently being implemented into e-loss models, e.g. MARTINI
Simulation of a 20 GeV gluon

\[
\frac{dN_{\text{glue}}}{dp} \ (\text{GeV}^{-1})
\]

- \( t = 1.3 \text{ fm} \)
- \( t = 2.6 \text{ fm} \)
- \( t = 3.9 \text{ fm} \)
- \( t = 5.2 \text{ fm} \)

\( T = 300 \text{ MeV} \) \( \alpha_s = 0.3 \)

\( N_f = 0 \)
Hot QGP

\[ 2k(2\pi)^3 \frac{d\Gamma}{d^3k} = \text{Photon emission rate per phase-space} \]

Same techniques can be used for thermal photon production:

- The rate is function of the coupling constant and \( k/T \):

\[
2k(2\pi)^3 \frac{d\Gamma}{d^3k} \propto e^2 T^2 \left[ O(g^2 \log) + O(g^2) + O(g^3 \log) + O(g^3) \right] + \ldots
\]

From soft \( gT \) gluons, \( n_B \approx \frac{T}{\omega} \approx \frac{1}{g} \)

\( O(g^3) \) is closely related to \( O(g^3) \) in energy loss:
NLO Results:

- In Figure 19, the functions $C(k/T)$ for $N_c=3$, $N_f=3$ as in Fig. 18, but for $\alpha_s=0.05$.

- Figures 19 and 20 show the differential rate $d/dk$ relative to the leading order rate as a function of $k/T$ (or equivalently $C_{LO+NLO}/C_{LO}$). The full next to leading order rate (LO+NLO) is a sum of the leading order rate (LO), a collinear correction (coll), and a soft+semi-collinear correction (soft+sc). The dashed curve labeled LO+coll shows the ratio of rates when only the collinear correction is included, with the analogous notation for the LO + soft+sc curve. The difference between the dashed curves provides a uncertainty estimate for the NLO calculation.

- In Fig. 19 we plot $C_{LO+NLO}(k/T)$ and $C_{LO}(k/T)$ for $\alpha_s=0.05$, and $N_c=3$, $N_f=3$.

- For the smaller coupling constant the NLO correction is always negative and rather flat, and the magnitude of the two largely canceling contributions is also significantly smaller than in the previous case.

- NLO corrections are modest and roughly $k$ independent.
The different contributions at NLO (conversions are not numerically important)

large-\(\theta\) radiation suppressed at NLO

small-\(\theta\) radiation enhanced at NLO

\(\alpha_s = 0.15\)

- small-\(\theta\) radiation only
- large-\(\theta\) radiation only
- full result
Conclusion:

- NLO corrections to collinear processes seem to be modest.
- All of the soft sector buried into a few coefficients, e.g. $\hat{C}[q_\perp, \delta E]$ and $\delta m_\infty^2$
  
  - Can we compute these non-perturbatively with dimensional reduction?
  - First start: Marco Panero, Kari Rummukainen, Adreas Schafer arxiv:1307.5850
  - Use these non-perturbative parameters to compute $\eta/s$

A program for computing QGP transport perturbatively with non-perturbative inputs for the Debye and magnetic sector