Bottomonium Suppression in the QGP

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Heavy Flavor and Electromagnetic Probes in Heavy Ion Collisions
Outline

• Why bottomonium?
• Non-equilibrium plasma dynamics
  \( \rightarrow \) plasma momentum-space anisotropy
• Incorporating anisotropy in the heavy quark potential
• Putting the pieces together
• Results
• What are we doing now?
• Conclusions
Why Bottomonium?

• Bottom quarks \( (m_b \approx 4.2 \text{ GeV}) \) are more massive than charm quarks \( (m_c \approx 1.3 \text{ GeV}) \) and, as a result, the heavy quark effective theories underpinning phenomenological applications are on somewhat surer footing.

• Due to their higher mass, the effects of initial state (IS) nuclear suppression are expected to be smaller than for the charmonium states. At forward/backward rapidities, however, IS effects on bottomonium could still be very important.

• The masses of bottomonium states \( (m_\Upsilon \approx 10 \text{ GeV}) \) are much higher than the temperatures \( (T < 1 \text{ GeV}) \) generated in HICs → bottomonia production will be dominated by initial hard scatterings.

• Since bottom quarks and anti-quarks are relatively rare, the probability for regeneration of bottomonium states through statistical recombination is much smaller than for charm quarks. (Still can be “correlational pairing” though...)
Vacuum Quarkonia Spectra

**Bottomonia**

<table>
<thead>
<tr>
<th>State</th>
<th>Name</th>
<th>Exp. [92]</th>
<th>Model</th>
<th>Rel. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^1S_0$</td>
<td>$\eta_b(1S)$</td>
<td>9.398 GeV</td>
<td>9.398 GeV</td>
<td>0.001%</td>
</tr>
<tr>
<td>$1^3S_1$</td>
<td>$\Upsilon(1S)$</td>
<td>9.461 GeV</td>
<td>9.461 GeV</td>
<td>0.004%</td>
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<tr>
<td>$1^3P_0$</td>
<td>$\chi_b(1P)$</td>
<td>9.859 GeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1^3P_1$</td>
<td>$\chi_b(1P)$</td>
<td>9.893 GeV</td>
<td>9.869 GeV</td>
<td>0.21%</td>
</tr>
<tr>
<td>$1^3P_2$</td>
<td>$\chi_b(2P)$</td>
<td>9.912 GeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1^1P_1$</td>
<td>$h_b(1P)$</td>
<td>9.899 GeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2^1S_0$</td>
<td>$\eta_b(2S)$</td>
<td>9.999 GeV</td>
<td>9.977 GeV</td>
<td>0.22%</td>
</tr>
<tr>
<td>$2^3S_1$</td>
<td>$\Upsilon(2S)$</td>
<td>10.002 GeV</td>
<td>9.999 GeV</td>
<td>0.03%</td>
</tr>
<tr>
<td>$2^3P_0$</td>
<td>$\chi_b(2P)$</td>
<td>10.232 GeV</td>
<td></td>
<td></td>
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<tr>
<td>$2^3P_1$</td>
<td>$\chi_b(2P)$</td>
<td>10.255 GeV</td>
<td>10.246 GeV</td>
<td>0.05%</td>
</tr>
<tr>
<td>$2^3P_2$</td>
<td>$\chi_b(2P)$</td>
<td>10.269 GeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2^1P_1$</td>
<td>$h_b(2P)$</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3^1S_0$</td>
<td>$\eta_b(3S)$</td>
<td>-</td>
<td>10.344 GeV</td>
<td>-</td>
</tr>
<tr>
<td>$3^3S_1$</td>
<td>$\Upsilon(3S)$</td>
<td>10.355 GeV</td>
<td>10.358 GeV</td>
<td>0.03%</td>
</tr>
</tbody>
</table>

Cornell potential + spin-spin interaction fixed to lattice
J. Alford and MS, 1309.3003

**Charmonia**

<table>
<thead>
<tr>
<th>State</th>
<th>Name</th>
<th>Exp. [92]</th>
<th>Model</th>
<th>Rel. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^1S_0$</td>
<td>$\eta_c(1S)$</td>
<td>2.984 GeV</td>
<td>3.048 GeV</td>
<td>2.2%</td>
</tr>
<tr>
<td>$1^3S_1$</td>
<td>$J/\psi(1S)$</td>
<td>3.097 GeV</td>
<td>3.100 GeV</td>
<td>0.11%</td>
</tr>
<tr>
<td>$2^1S_0$</td>
<td>$\eta_c(2S)$</td>
<td>3.639 GeV</td>
<td>3.721 GeV</td>
<td>2.3%</td>
</tr>
<tr>
<td>$2^3S_1$</td>
<td>$J/\psi(2S)$</td>
<td>3.686 GeV</td>
<td>3.748 GeV</td>
<td>1.7%</td>
</tr>
</tbody>
</table>

- With a simple pNRQCD potential model one can describe the known bottomonia state masses with a maximum error of 0.22%
- The situation with charmonia is a bit worse and one has to add relativistic corrections with additional parameters.
LHC Heavy Ion Collision Timescales

- **Hot Hadron Gas**: $5 \leq \tau < 9 \text{ fm/c}$
- **Equilibrium QGP**: $2 \leq \tau < 5 \text{ fm/c}$
- **Non-equilibrium QGP**: $0.3 \leq \tau < 2 \text{ fm/c}$
- **Semi-hard particle production**: $0 \leq \tau < 0.3 \text{ fm/c}$
QGP momentum anisotropy cartoon

Expansion rate is much faster than the interaction time scale \(1/t \gg 1/t_{\text{int}}\)

Expansion rate and isotropization via interactions become comparable

Decreasing shear viscosity

\[ \tau_0 \sim Q_s^{-1} \quad \tau_{\text{hydro}} \quad \log \tau \]

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Estimating Early-time Pressure Anisotropy

• CGC @ leading order predicts negative $\Rightarrow$ approximately zero longitudinal pressure

• QGP scattering + plasma instabilities work to drive the system towards isotropy on the fm/c timescale, but don’t seem to fully restore it

• Viscous hydrodynamics predicts early-time anisotropies $\leq 0.35 \Rightarrow 0.5$ (see next slide)

• AdS-CFT dynamical calculations in the strong coupling limit predict anisotropies of $\leq 0.3$ (discussion in three slides from now)
Estimating Anisotropy – Viscous hydro

- To get a feeling for the magnitude of pressure anisotropies to expect, let’s consider the Navier-Stokes limit

\[
\left( \frac{P_L}{P_T} \right)_{NS} = \frac{P_{eq} + \pi_{NS}^{zz}}{P_{eq} + \pi_{NS}^{xx}} = \frac{3\tau T - 16\tilde{\eta}}{3\tau T + 8\tilde{\eta}}
\]

\[
\tilde{\eta} = \frac{\eta}{S}
\]

\[
\pi_{NS}^{zz} = -2\pi_{NS}^{xx} = -2\pi_{NS}^{yy} = -\frac{4\eta}{3\tau}
\]

- \(P_L/P_T\) decreases with increasing \(\eta/S\)
- \(P_L/P_T\) decreases with decreasing \(T\)
- Assume \(\eta/S = 1/4\pi\) in order to get an upper bound on the anisotropy
- Using RHIC initial conditions (\(T_0 = 400\) MeV @ \(\tau_0 = 0.5\) fm/c) we obtain \(P_L/P_T \leq 0.5\)
- Using LHC initial conditions (\(T_0 = 600\) MeV @ \(\tau_0 = 0.25\) fm/c) we obtain \(P_L/P_T \leq 0.35\)
- Negative \(P_L\) at large \(\eta/S\) or low temperatures!?
Estimating Anisotropy – Viscous hydro

- Navier-Stokes solution is “attractor” for the 2nd order solution
- $\tau_\pi$ sets timescale to approach Navier-Stokes evolution
- $\tau_\pi \sim 5\eta/(TS) \sim 0.1$ fm/c at LHC temperatures
- Assume isotropic LHC initial conditions $T_0 = 600$ MeV @ $\tau_0 = 0.25$ fm/c and solve for the 0+1d viscous hydro dynamics

![Graphs showing anisotropy and viscous hydro dynamics](image-url)
Estimating Anisotropy – AdS/CFT

- In 0+1d case there are now numerical solutions of Einstein’s equations to compare with. [Heller, Janik, and Witaszczyk, 1103.3452]

- They studied a wide variety of initial conditions and found a kind of universal lower bound for the thermalization time.

<table>
<thead>
<tr>
<th>RHIC 200 GeV/nucleon:</th>
<th>LHC 2.76 TeV/nucleon:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0 = 350$ MeV, $\tau_0 &gt; 0.35$ fm/c</td>
<td>$T_0 = 600$ MeV, $\tau_0 &gt; 0.2$ fm/c</td>
</tr>
</tbody>
</table>
N=4 SUSY using AdS/CFT

However, at that time the system is not isotropic and remains anisotropic for the entirety of the evolution.

Other AdS/CFT numerical studies which include transverse expansion reach a similar conclusion [van der Schee et al. 1307.2539]

Pressure Anisotropy

\[ 1 - \frac{3p_L}{\varepsilon} = 12 \frac{F(w)}{w} - 8 \]

\[ P_L/P_T = 0.31 \]

See also J. Casalderrey-Solana et al. arXiv: 1305.4919
Temperature dependence of $\eta/S$

[Hot and Dense QCD Matter, Community Whitepaper 2014]
Anisotropic Hydrodynamics Basics

Viscous Hydrodynamics Expansion

\[ f(\tau, x, p) = f_{eq}(p, T(\tau, x)) + \delta f \]

Anisotropic Hydrodynamics Expansion

\[ f(\tau, x, p) = f_{aniso}(p, \Lambda(\tau, x), \xi(\tau, x)) + \delta f \]

\[ T_{\perp} \]

\[ \text{anisotropy} \]

\[ \rightarrow \text{"Romatschke-Strickland" form in LRF} \]

\[ f_{aniso}^{LRF} = f_{iso} \left( \frac{\sqrt{p^2 + \xi(x, \tau)p_z^2}}{\Lambda(x, \tau)} \right) \]

\[ \xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1 \]

\[ -1 < \xi < 0 \quad \xi = 0 \quad \xi > 0 \]
Why spheroidal form at LO?

• What is special about this form at leading order?
  \[ f_{\text{aniso}}^{LRF} = f_{\text{iso}} \left( \frac{\sqrt{p^2 + \xi(x, \tau)p_z^2}}{\Lambda(x, \tau)} \right) \]

• Gives the ideal hydro limit when \( \xi = 0 \) \( (\Lambda \to T) \)
• For longitudinal \((0+1d)\) free streaming, the LRF distribution function is of spheroidal form; limit emerges naturally in aHydro
  \[ \xi_{FS}(\tau) = (1 + \xi_0) \left( \frac{\tau}{\tau_0} \right)^2 - 1 \]

• Since \( f_{\text{iso}} \geq 0 \), the one-particle distribution function and pressures are \( \geq 0 \) (not guaranteed in viscous hydro)
• Formalism reduces to 2\textsuperscript{nd}-order viscous hydrodynamics in limit of small anisotropies
  \[ \frac{\Pi}{\mathcal{E}_{eq}} = \frac{8}{45} \xi + \mathcal{O}(\xi^2) \]
Hints from Viscous Hydro

\[ \Sigma = \pi^{xx} + \pi^{yy} \]
\[ \Delta = \pi^{xx} - \pi^{yy} \]

Au+Au, b=7 fm
SM-EOS Q
0+1d Pressure Anisotropy

\[ \frac{1}{1+\xi} \partial_\tau \xi - \frac{2}{\tau} - 6 \partial_\tau \log \Lambda = 2\Gamma \left[ 1 - \mathcal{R}^{3/4}(\xi) \sqrt{1+\xi} \right] \]

\[ \frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)} \partial_\tau \xi + 4 \partial_\tau \log \Lambda = \frac{1}{\tau} \left[ \frac{1}{\xi(1+\xi)} \mathcal{R}(\xi) - \frac{1}{\xi} - 1 \right] \]

\[ \frac{\eta}{S} = \frac{1}{4\pi} \]

\[ \frac{\eta}{S} = \frac{2}{4\pi} \]

\[ \frac{\eta}{S} = \frac{4}{4\pi} \]

\[ \frac{\eta}{S} = \frac{8}{4\pi} \]

\[ \frac{\eta}{S} = \frac{16}{4\pi} \]

\[ T_0 = 600 \text{ MeV} @ \tau_0 = 0.25 \text{ fm/c} \]
Including Transverse Dynamics

- Allowing variables to depend on $x$ and $y$, while still assuming boost-invariance, we obtain the “2+1d” dimensional AHYDRO equations
- Conformal system $\rightarrow$ four equations for four variables $u_x, u_y, \xi$, and $\Lambda$.

0\textsuperscript{th} moment

\begin{equation}
Dn + n\theta = J_0.
\end{equation}

\begin{equation}
D \equiv u^\mu \partial_\mu,
\end{equation}

\begin{equation}
\theta \equiv \partial_\mu u^\mu,
\end{equation}

\begin{equation}
u_0 = \sqrt{1 + u_x^2 + u_y^2}.
\end{equation}

1\textsuperscript{st} moment

\begin{equation}
D\mathcal{E} + (\mathcal{E} + \mathcal{P}_\perp)\theta + (\mathcal{P}_L - \mathcal{P}_\perp) \frac{u_0}{\tau} = 0,
\end{equation}

\begin{equation}
(\mathcal{E} + \mathcal{P}_\perp) D u_x + \partial_x \mathcal{P}_\perp + u_x D \mathcal{P}_\perp + (\mathcal{P}_\perp - \mathcal{P}_L) \frac{u_0 u_x}{\tau} = 0,
\end{equation}

\begin{equation}
(\mathcal{E} + \mathcal{P}_\perp) D u_y + \partial_y \mathcal{P}_\perp + u_y D \mathcal{P}_\perp + (\mathcal{P}_\perp - \mathcal{P}_L) \frac{u_0 u_y}{\tau} = 0.
\end{equation}
NLO aHydro

Viscous Hydrodynamics Expansion

\[ f(\tau, x, p) = f_{eq}(p, T(\tau, x)) + \delta f \]

Isotropic in momentum space

Anisotropic Hydrodynamics Expansion

\[ f(\tau, x, p) = f_{aniso}(p, \Lambda(\tau, x), \xi(\tau, x)) + \tilde{\delta}f \]

Now let’s treat this term “perturbatively”

[ D. Bazow, U. Heinz, and MS, 1311.6720]

→ “Romatschke-Strickland” form in LRF

\[ f_{aniso}^{LRF} = f_{iso} \left( \frac{\sqrt{p^2 + \xi(x, \tau)p^2_z}}{\Lambda(x, \tau)} \right) \]

\[ \xi = \frac{\langle p^2_T \rangle}{2\langle p^2_L \rangle} - 1 \]

-1 < \xi < 0 \hspace{2cm} \xi = 0 \hspace{2cm} \xi > 0

prolate \hspace{2cm} \text{oblate}
Example: Entropy Generation

- Number (entropy) production vanishes in two limits: ideal hydrodynamic and free streaming limits.
- In the conformal model which we are testing with, number density is proportional to entropy density.
**Spatiotemporal Evolution**

- Pb-Pb, $b = 7$ fm collision with Monte-Carlo Glauber initial conditions
  
  \[ T_0 = 600 \text{ MeV} \at \tau_0 = 0.25 \text{ fm/c} \]

- Left panel shows effective temperature; right shows pressure anisotropy

\[ \frac{\eta}{S} = \frac{1}{4\pi} \]
Anisotropic Heavy Quark Potential

Using real-time formalism one can express potential in terms of static advanced, retarded, and Feynman propagators

\[
V(\mathbf{r}, \xi) = -g^2 C_F \int \frac{d^3 \mathbf{p}}{(2\pi)^3} (e^{i\mathbf{p} \cdot \mathbf{r}} - 1) \frac{1}{2} \left( D^{*L}_R + D^{*L}_A + D^{*L}_F \right)
\]

Real part can be written as

\[
\text{Re}[V(\mathbf{r}, \xi)] = -g^2 C_F \int \frac{d^3 \mathbf{p}}{(2\pi)^3} e^{i\mathbf{p} \cdot \mathbf{r}} \frac{\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2}{(\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2)(\mathbf{p}^2 + m_\beta^2) - m_\delta^4}
\]

With direction-dependent masses, e.g.

\[
m_\alpha^2 = -\frac{m_D^2}{2p_\perp^2 \sqrt{\xi}} \left( p_z^2 \arctan \sqrt{\xi} - \frac{p_z^2}{\sqrt{p^2 + \xi p_\perp^2}} \arctan \frac{\sqrt{\xi} p_z}{\sqrt{p^2 + \xi p_\perp^2}} \right)
\]

Anisotropic potential calculation: Dumitru, Guo, and MS, 0711.4722 and 0903.4703
Gluon propagator in an anisotropic plasma: Romatschke and MS, hep-ph/0304092

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**Full anisotropic potential**

- Result can be parameterized as a Debye-screened potential with a direction-dependent Debye mass

- The potential also has an imaginary part coming from the Landau damping of the exchanged gluon!

- This imaginary part also exists in the isotropic case \[ \text{[Laine et al hep-ph/0611300]} \]

- Used this as a model for the free energy \((F)\) and also obtained internal energy \((U)\) from this.

\[
V(r, \theta, \xi, p_{\text{hard}}) = -C_F \alpha_s \frac{e^{-\mu(\theta, \xi, p_{\text{hard}})}r}{r}
\]

D Bazow and MS, 1112.2761; MS, 1106.2571.

\[
V_R(r) = -\frac{\alpha}{r} (1 + \mu r) \exp (-\mu r) + \frac{2\sigma}{\mu} [1 - \exp (-\mu r)] - \sigma r \exp(-\mu r) - \frac{0.8 \sigma}{m_Q^2 r}
\]

Dumitru, Guo, Mocsy, and MS, 0901.1998

\[
V_I(r) = -C_F \alpha_s p_{\text{hard}} \left[ \phi(\hat{r}) - \xi (\psi_1(\hat{r}, \theta) + \psi_2(\hat{r}, \theta)) \right]
\]

Dumitru, Guo, and MS, 0711.4722 and 0903.4703
Solve the 3d Schrödinger EQ with complex-valued potential

Obtain real and imaginary parts of the binding energies for the $\Upsilon(1s)$, $\Upsilon(2s)$, $\Upsilon(3s)$, $\chi_{b1}$, $\chi_{b2}$
Results for the $\Upsilon(1s)$ binding energy

\[ n_v(\tau) = \langle \phi_v^*(\tau, x) \phi_v(\tau, x) \rangle, \]
\[ = \langle \phi_v^*(\tau_0, x) \phi_v(\tau_0, x) \rangle e^{2\Im [E](\tau - \tau_0)}, \]
\[ = n_v^0 e^{2\Im [E](\tau - \tau_0)}. \]
Results for the $\chi_{b1}$ binding energy

$\chi_{b1}$

- $\xi = 0$, (Real Part)
- $\xi = 0$, -(Imaginary Part)
- $\xi = 1$, (Real Part)
- $\xi = 1$, -(Imaginary Part)

Binding Energy [GeV]

$\Lambda/T_c$

$\xi = 0$

$\xi = 1$
Spatiotemporal Evolution

- Pb-Pb, $b = 7$ fm collision with Monte-Carlo Glauber initial conditions
  $T_0 = 600$ MeV @ $\tau_0 = 0.25$ fm/c
- Left panel shows effective temperature; right shows pressure anisotropy
The suppression factor

- Resulting decay rate $\Gamma_T \equiv -2 \text{Im}[E_{\text{bind}}]$ is a function of $\tau$, $x_\perp$, and $\varsigma$ (spatial rapidity). First we need to integrate over proper time

$$\bar{\gamma}(x_\perp, p_T, \varsigma, b) \equiv \int_{\max(\tau_{\text{form}}(p_T), \tau_0)}^{\tau_f} d\tau \Gamma_T(\tau, x_\perp, \varsigma, b)$$

- From this we can extract $R_{AA}$

$$R_{AA}(x_\perp, p_T, \varsigma, b) = \exp(-\bar{\gamma}(x_\perp, p_T, \varsigma, b))$$

- Using the overlap density as the probability distribution function for quarkonium production vertices and geometrically averaging

$$\langle R_{AA}(p_T, \varsigma, b) \rangle \equiv \frac{\int_{x_\perp} dx_\perp T_{AA}(x_\perp) R_{AA}(x_\perp, p_T, \varsigma, b)}{\int_{x_\perp} dx_\perp T_{AA}(x_\perp)}$$
State Suppression Factors, $R_{AA}^i$

$\sqrt{s_{NN}} = 2.76$ TeV

$\tau_0 = 0.3$ fm/c

$T_0 = 580$ MeV

$\eta \frac{S}{S} = \frac{1}{4\pi}$
Inclusive Bottomonium Suppression


Computed inclusive $\Upsilon$(1s) and $\Upsilon$(2s) suppression including effects of feed-down, finite formation time, and aHydro evolution with anisotropic complex-valued quarkonium potential.
Conflict with ALICE data

- Thermal suppression model has $R_{AA}$ approaching 1 at forward/backward rapidity ($T \to 0$)
- Using a Gaussian rapidity profile (Landau hydro) does not come close to the data
- Using a boost-invariant rapidity profile (Bjorken hydro) gives enhanced suppression, **but it also doesn’t describe what was seen by ALICE!**
  - IS effect?
  - Assumption of small anisotropy breaking down?
  - Poor/limited hydro modeling?
  - Recombination?
(Some of) the problems with my calculation

- Small anisotropy expansion used for the imaginary part of the potential
- Dynamics was effectively 1+1d and used smooth initial conditions
- No regeneration
- No IS/CNM effects
- No singlet/octet transition $\text{Im}[V]$
- Simplistic model of how the anisotropy affects the long range part of the potential
- ...
What am I working on now?

• We now have a 3+1d AHYDRO code that can handle fluctuating initial conditions ✔
• Using this code, we can have two fluids: the bulk can be ~ ideal hydro, while quarkonium states can be ~ free streaming; keep track of their full spatial distribution ✔
• We have generated our first 3d bottomonium RAA results ✔
• The main difference so far: rapidity-dependence of RAA gets slightly flatter but it still seems to be above the ALICE data?
• Full anisotropy ($ξ$) dependence of the imaginary part of the potential (in progress)
• Include regeneration effects; density dependent local recombination
• Take initial $R_{AA}$ from independent IS/CNM calculation; effects from IS/CNM and QGP suppression are multiplicative
Conclusions

• All signs point to an anisotropic QGP $\rightarrow$ need to self-consistently calculate rates including this effect
• At central rapidities, the model seems to work reasonably well
• For the 1s state, there is a large dependence on assumed value of $\eta/s$
• This offers the possibility to constrain $\eta/s$ using bottomonium $R_{AA}$
• The strong suppression seen at forward rapidities is a challenge for the “thermal” model, but there is substantial room for improvement
- Backup slides -
1st Order Hydro – 0+1d

Additionally one finds for the first order distribution function

\[
f(x, p) = f_{eq} \left( \frac{p^\mu u_\mu}{T} \right) \left[ 1 + \frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2(E + P)T^2} \right] \rightarrow f_{eq} \left( \frac{E}{T} \right) \left[ 1 + \frac{\eta}{S} \frac{p_x^2 + p_y^2 - 2p_z^2}{3\tau T^3} \right]
\]

- Distribution function becomes anisotropic in momentum space
- There are also regions where \( f(x, p) < 0 \)
- Anisotropy and regions of negativity increase as \( \tau \) or \( T \) decrease OR \( \eta/S \) increases

\[ T = 1 \text{ GeV} \]
\[ \tau = 0.1 \text{ GeV}^{-1} \]
\[ \eta/S = 1/4\pi \]
1st Order Hydro – 0+1d

Additionally one finds for the first order distribution function

\[ f(x, p) = f_{eq}\left(\frac{p^\mu u_\mu}{T}\right) \left[ 1 + \frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2(E + \mathcal{P})T^2} \right] \rightarrow f_{eq}\left(\frac{E}{T}\right) \left[ 1 + \frac{\eta}{S} \frac{p_x^2 + p_y^2 - 2p_z^2}{3\tau T^3} \right] \]

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