In-medium heavy quarkonium from a Bayesian point of view

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References:
Y. Burnier, O. Kaczmarek (Bielefeld-U.), AR.: in preparation
Outline

- **Physics Motivation:** Relativistic heavy-ion collisions and heavy quarkonium

- **Technical progress:** Bayesian spectral function reconstruction in lattice QCD

- **Project I:** The static in-medium heavy quark potential

- **Project II:** Bottomonium spectral functions from lattice NRQCD

- **Conclusion**
Relativistic Heavy-Ion collisions

- Probes that are susceptible to medium but distinguishable from it: $Q_{\text{probe}} \gg T_{\text{med}}$

Bound states of $c\bar{c}$ or $b\bar{b}$: Heavy quarkonium $m_Q \gg T_{\text{med}}$

- $b\bar{b}$ produced in the early stages of the collision ($M_b=4.65\text{GeV}$)
- Rapid bound state formation expected
- Long lifetime due to OZI rule ($\Gamma^\gamma=54\text{keV}$)
**Bottomonium as QGP probe**

- **QGP is strongly interacting at** $T \sim 220-305 \text{MeV}$
  
  *see e.g.* ALICE NPA 904-905 (2013) and PHENIX PRL 104 (2010) 132301

- **Goal:** Non-perturbative understanding of in-medium Bottomonium via lattice QCD
Two distinct paths, One common challenge

<table>
<thead>
<tr>
<th>The T&gt;0 static interquark potential</th>
<th>In-medium Bottomonium spectra</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Simplification: Infinitely heavy quarks</td>
<td>- Realistic heavy quark masses</td>
</tr>
<tr>
<td>- Allows a real-time description of the approach towards equilibrium</td>
<td>- Determination of melting/survival</td>
</tr>
<tr>
<td>- Also describes in-medium spectra of thermalized QQbar</td>
<td>- Full kinetic equilibration</td>
</tr>
<tr>
<td></td>
<td>- Based on the effective field theory NRQCD, also used in T=0 lattice QCD</td>
</tr>
</tbody>
</table>

- Lattice simulations in Euclidean time, no direct access to dynamical information
  - Analytic continuation from a finite and noisy dataset necessary: ill-defined problem
    M. Jarrell, J. Gubernatis, Physics Reports 269 (3) (1996)

  Approach via lattice spectral functions: Improve on the Maximum Entropy Method
Technical Progress

Bayesian spectral function reconstruction in lattice QCD
Novel Bayesian Spectral Reconstruction

- Inversion of Laplace transform required to obtain spectra from correlators

\[ D_i = \sum_{l=1}^{N_{\omega}} \exp[-\omega_l \tau_i] \rho_l \Delta \omega_l \]

1. \( N_{\omega} \) parameters \( \rho_l \gg N_\tau \) datapoints
2. data \( D_i \) has finite precision

- Give meaning to problem by incorporating prior knowledge: Bayesian approach

- Bayes theorem: Regularize the naïve \( \chi^2 \) functional \( P[D|\rho] \) through a prior \( P[\rho|I] \)

\[ P[\rho|D, I] \propto P[D|\rho] P[\rho|I] \]

- New prior enforces: \( \rho \) positive definite, smoothness of \( \rho \), result independent of units

\[ P[\rho|I] \propto e^S \quad S = \alpha \sum_{l=1}^{N_{\omega}} \Delta \omega_l \left( 1 - \frac{\rho_l}{m_l} + \log \left[ \frac{\rho_l}{m_l} \right] \right) \]

- Different from Maximum Entropy Method: \( S \) not entropy, no more flat directions

\[ \frac{\delta}{\delta \rho} P[\rho|D, I] \bigg|_{\rho=\rho^BR} = 0 \]

- No apriori restriction on the search space
- Convergence to unique global extremum

References:
- Y. Burnier, A.R. PRL 111 (2013) 18, 182003
A first mock data test

Mock analysis:
three delta peaks in the spectrum

MEM Reconstruction of 3 \( \delta \) peaks

Bayesian Reconstruction of 3 \( \delta \) peaks

New method
The static in-medium interquark potential
The static inter-quark potential at $T>0$

- A lot of intuition has been accumulated over the years:
  - Lattice QCD at $T=0$: Confining linear rise + string breaking
  - Analogy with an Abelian plasma: Debye screening
  - Modeling: Color singlet free energies or internal energies from lattice QCD

- For static quarks a clean definition of the potential from QCD is available
  - Use heavy meson operators: $M(x,y,t) = Q(x,t)\Gamma U(x,y)\bar{Q}(y,t)$
    
    $$D^>(R,t) = \langle M(x,y,t) M^\dagger(x,y,0) \rangle_{\text{med}}$$
  - In the static limit: $D^>$ becomes the **real-time Wilson loop**
    
    $$D^>(R,t) \xrightarrow{m\to\infty} W_{\square}(R,t) = \langle \text{Tr} \left( \exp \left[ -i g \int_{\square} dz^\mu A_\mu(z) \right] \right) \rangle$$
  - Potential emerges at late times: QQbar timescale much slower than gluons
    
    $$i\partial_t W_{\square}(R,t) \xrightarrow{t\to\infty} V^{QCD}(R)W_{\square}(R,t)$$

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{i\partial_t W_{\square}(R,t)}{W_{\square}(R,t)}$$

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Heavy Flavor and Electromagnetic Probes in Heavy-Ion Collisions
The high temperature potential

- T >> T_C: Asymptotic freedom of QCD allows weak coupling evaluation

\[
W_{\Box}(R, t) = \begin{bmatrix}
\text{Debye screening} & \text{Landau damping}
\end{bmatrix}
\]

\[
V_{QCD}^{HTL}(R) = -\frac{gC_F}{4\pi} \left[ m_D + \frac{e^{-m_D R}}{R} \right] - i \frac{g^2 T C_F}{4\pi} \phi(m_D R)
\]

- Re[V] from Debye screening: presence of deconfined color charges
- Im[V] from gluon scattering (Landau damping) and absorption (singlet octet transition)

Presence of Im[V] is a QCD result not a model assumption

Im[V_{QCD}(R)]

Re[V_{QCD}(R)]

T = 0.5 - 2.2 GeV

\[
\phi(x) = \int_0^\infty dz \frac{z}{(z^2 + 1)^2} \left[ 1 - \frac{\sin(zx)}{zx} \right]
\]
How are the spectrum and the potential related?

Real-time not directly accessible!

How to connect to the Euclidean domain: spectral functions

\[
W_\square(R, t) = \int_{-\infty}^{\infty} d\omega \ e^{-i\omega t} \rho_\square(R, \omega) \quad \leftrightarrow \quad W_\square(R, \tau) = \int_{-\infty}^{\infty} d\omega \ e^{-\omega \tau} \rho_\square(R, \omega)
\]

\[
\mathcal{V}^{QCD}(R) = \lim_{t \to \infty} \frac{\int_{-\infty}^{\infty} d\omega \ e^{i\omega t} W_\square(R, \omega) \rho_\square(R, \omega)}{\int_{-\infty}^{\infty} d\omega \ e^{i\omega t} \rho_\square(R, \omega)}
\]

How are the spectrum and the potential related?

\[
\rho_\square(R, \omega) = \frac{1}{\pi} e^{\gamma_1(R)} \frac{\Gamma_0(R) \cos[\gamma_2(R)] - (\omega_0(R) - \omega) \sin[\gamma_2(R)]}{\Gamma_0^2(R) + (\omega_0(R) - \omega)^2} + \kappa_0(R) + \kappa_1(R)(\omega_0(R) - \omega) + \ldots
\]

\[
\lim_{t \to \infty} \frac{\int_{-\infty}^{\infty} d\omega \ \omega \ e^{-i\omega t} \rho_\square(R, \omega)}{\int_{-\infty}^{\infty} d\omega \ e^{-i\omega t} \rho_\square(R, \omega)} = \omega_0(R) + i\Gamma_0(R)
\]

The extraction strategy

- From lattice QCD Euclidean Wilson loops to the complex heavy quark potential

- Technical detail: avoid cusp divergences using Wilson line correlators in CG

- Quenched lattice QCD: anisotropic lattices with naïve Wilson action $32^3 \times N_\tau$

<table>
<thead>
<tr>
<th>$N_\tau$</th>
<th>24</th>
<th>32</th>
<th>40</th>
<th>48</th>
<th>56</th>
<th>64</th>
<th>72</th>
<th>80</th>
<th>96</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T/T_C$</td>
<td>3.11</td>
<td>2.33</td>
<td>1.86</td>
<td>1.55</td>
<td>1.33</td>
<td>1.17</td>
<td>1.04</td>
<td>0.93</td>
<td>0.78</td>
</tr>
<tr>
<td>$N_{meas}$</td>
<td>2750</td>
<td>1570</td>
<td>1680</td>
<td>1110</td>
<td>760</td>
<td>1110</td>
<td>700</td>
<td>940</td>
<td>690</td>
</tr>
</tbody>
</table>
Towards $V_{QQ}^{r}(r)$ on quenched lattices

Presence of Im[V] at high T already visible from curvature in the correlator data
The potential in quenched lattice QCD

- Transition from a confining to a Debye screened behavior
- \( \text{Re}[V] \) lies close to the color singlet free energies \( F^1(r) \)

\[
F^{(1)}(r) = -\frac{1}{\beta} \log \left[ W_{\parallel}(r, \tau = \beta) \right]_{\text{CG}}
\]

- For small \( r \): good agreement between \( \text{Im}[V] \) and HTL prediction down to \( 1.17T_C \)
In-medium Bottomonium spectral functions from lattice QCD
A Lattice QCD Challenge

- **PRACTICAL**: High cost if light and heavy d.o.f share the same spacetime grid

\[ \alpha \ll \frac{1}{2m_b} \approx 0.02 \text{fm} \quad \frac{1}{T} = N_\tau \alpha \sim 1 \text{fm} \]

Turn the separation of scales into an advantage: effective field theory NRQCD

Effective Field Theory: Lattice NRQCD

\[ L_{\text{NRQCD}} = \psi^\dagger (i D_t + \frac{D_i^2}{2 M_Q} + \ldots) \psi + \xi^\dagger (\ldots) \xi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \bar{q} (\ldots) q \]

Heavy quark \( \psi \) and antiquark \( \xi \) as separate non-relativistic Pauli spinors

Light medium d.o.f. from a fully relativistic lattice simulation

- Separation of scales \( T/M_Q \ll 1, \Lambda_{\text{QCD}}/M_Q \ll 1, p/M_Q \ll 1 \): systematic expansion in \( 1/M_Q a \)

- Individual \( Q \) or anti-\( Q \) in a medium background: Initial value problem \( G(\tau) = \langle \psi(\tau) \psi^\dagger(0) \rangle \)

\[ G(x, \tau + a) = U^\dagger_4(x, \tau) \left( 1 - \frac{p_{\text{lat}}^2}{4 M_Q a} + \ldots \right) G(x, \tau) \]

well behaved if \( M_Q a > 1.5 \)


- \( ^3S_1 (\Upsilon) \) and \( ^3P_1 (\chi_{b1}) \) channel correlators \( D(\tau) \) from products of heavy quark propagators \( G(\tau) \)

\[ D(\tau) = \sum_x \langle O(x, \tau) G_{x \tau} \rangle_{\text{med}} \quad O(^3S_1; x, \tau) = \sigma_i, \quad O(^3P_1; x, \tau) = \Delta_i \sigma_j - \Delta_j \sigma_i \]

IN-MEDIUM HEAVY QUARKONIUM FROM A BAYESIAN POINT OF VIEW

A Medium With Nf=2+1 Light HISQ Flavors

- Light d.o.f. (gluons, u d s quarks) represented by HotQCD configurations

- 48³×12 with relatively light pions $M_\pi \sim 161\text{MeV}$ and a $T_C=159\pm3\text{MeV}$

<table>
<thead>
<tr>
<th>HotQCD</th>
<th>HISQ/tree action</th>
<th>$48^3 \times N_\tau$</th>
<th>$m_{u,d}/m_s = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a[\text{fm}]$</td>
<td>0.1169</td>
<td>0.1130</td>
<td>0.1087</td>
</tr>
<tr>
<td>$M_\text{b/a}$</td>
<td>2.759</td>
<td>2.667</td>
<td>2.566</td>
</tr>
<tr>
<td>$T/T_C(N_\tau = 12)$</td>
<td>0.911</td>
<td>0.944</td>
<td>0.980</td>
</tr>
</tbody>
</table>

- Important property for the use with lattice NRQCD: $2.759 > M_\text{b/a} > 1.559 > 1.5$

- Temperature changed by variation of the lattice spacing $140\text{MeV} < T < 249\text{MeV}$
  For a study based on the fixed scale approach see: FASTSUM G. Aarts et. al. JHEP 1407 (2014) 097, JHEP 1111 (2011) 103

- Low temperature configurations available at $b=6.664$, 6.8, 6.95, 7.28
“Integrating out $M_b$” in setting up NRQCD introduces a scale dependent frequency shift.

**Drawback:** setting absolute frequency scale at $T>0$ requires additional $T=0$ calibration

**Advantage:** Correlator not periodic in $1/T$ and linked to spectra via simple $T=0$ Kernel,

$$D(\tau) = \int_{-2M_Q}^{\infty} d\omega e^{-\tau\omega} \rho(\omega)$$
Bottomonium Correlators Close To T=0

- Set absolute scale by comparison to experiment

\[
M_{\Upsilon(1S)}^{\text{exp}} = M_{\Upsilon(1S)}^{\text{NRQCD}} + 2(Z_{M_b} M_b - E_0) \]

\[
M_{\Upsilon(1S)}^{\text{exp}} = 9.46030(26) \text{ GeV} 
\]

\[
C_{\text{shift}}(\beta) = (-0.779\pm0.006) \times \beta + (13.765\pm0.041) 
\]

- Linear dependence: interpolated values to calibrate mass shift at intermediate \( \beta \)

\[
M_{\Upsilon(1S)}^{\text{exp}} = M_{\Upsilon(1S)}^{\text{NRQCD}} + 2(Z_{M_b} M_b - E_0) 
\]

\[
M_{\Upsilon(1S)}^{\text{exp}} = 9.46030(26) \text{ GeV} 
\]
IN-MEDIUM HEAVY QUARKONIUM FROM A BAYESIAN POINT OF VIEW

Spectral Functions Close To T=0

- Bayesian reconstruction:
  - $N_\omega=1200$, $l_\omega=[-0.5,30]$, $\beta^{\text{num}}=20$, $N_{\text{jack}}=10$
  - $m_l=\text{const}$, 512 bit precision, $\Delta\text{tol}=10^{-60}$

- S-wave ground state peak very well resolved, next peak mostly from $Y(2S)$
- P-wave ground state broader: worse s/n ratio and smaller physical peak size

\[ M_{\chi_1^{1P}} = M_{\chi_1^{\text{NRQCD}}} + C(\beta) = 9.917(3) \text{GeV} > M_{\chi_1^{1P}}^{\exp} = 9.89278(26)(31) \text{GeV} \]
Reconstruction Accuracy: S-wave

- High precision of the improved Bayesian reconstruction (narrow width resolved)
- How does accuracy suffer from limited available information at $T>0$ ($N\tau=12$)?
- One of the tests we ran: truncate $T=0$ dataset ($N\tau=32/64$) to $N\tau=12$

Overall Limits:

\[ \beta = 6.664 : \quad \Delta m_T < 2\text{MeV}, \quad \Delta \Gamma_T < 5\text{MeV} \]
\[ \beta = 7.280 : \quad \Delta m_T < 40\text{MeV}, \quad \Delta \Gamma_T < 21\text{MeV} \]
Reconstruction Accuracy: P-wave

- Estimate systematics: truncate $T=0$ dataset ($N_\tau=32/64$) to $N_\tau=12$
- Due to a worse signal-to-noise ratio, effect in P-wave is larger than for S-wave

Overall Limits:

$\beta = 6.664 : \Delta m_T < 60\text{MeV}, \quad \Delta \Gamma_T < 20\text{MeV}$

$\beta = 7.280 : \Delta m_T < 200\text{MeV}, \quad \Delta \Gamma_T < 40\text{MeV}$
Bottomonium Correlators At Finite T

- **S-wave at most 1% change**
  - Statistically significant in-medium modification above $T=160\text{MeV}$
- **P-wave at most 5% change**
  - Side remark: similar qualitative and quantitative behavior for $\eta_b$ and $h_b$ (scalar)
**S-wave Spectral Functions At T>0**

![Graph showing S-wave Spectral Functions At T>0](image)

- Bayesian reconstruction:
  - New Bayesian method resolves peaks much better than MEM
  - Observed broadening and peak shifts at finite T smaller than accuracy limits
- Well defined **ground state peak present up to 1.61T\(_C\)***

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**In-medium Heavy Quarkonium from a Bayesian Point of View**

- **N\(_\omega\)=1200 \(l_\omega=[-1.25]\)** \(\beta^{\text{num}}=20\) \(N_{\text{jack}}=10\)
- \(m_1=\text{const}\) \(512\) bit precision, \(\Delta\text{tol}=10^{-60}\)
**P-wave Spectral Functions At T>0**

**Ground state peak well defined up to T=1.61T_c**

- Worse signal to noise ratio leads to larger Jackknife errors than for S-wave
  - observed broadening and peak shifts also smaller than accuracy limits

- New approach finds well defined peak up to highest T investigated 249 MeV

MEM result similar to FASTSUM G. Aarts et. al. JHEP 1407 (2014) 097
How To Verify Survival Of A Bound State?

- Inspection by eye insufficient: systematic comparison to non-interacting spectra

- **Analytically**: From free NRQCD dispersion relation:

  $$ a_T E_p = -\log \left(1 - \frac{p^2_{\text{lat}}}{8 M_b a_s} \right) $$

  $$ \rho_s(\omega) = \frac{4\pi N_c}{N_s^2} \sum_P \delta(\omega - 2E_P) \quad \rho_P(\omega) = \frac{4\pi N_c}{N_s^2} \sum_P p^2 \delta(\omega - 2E_P) $$

  G.Aarts et al., JHEP 1111 (2011) 103

- **Numerically**: Reconstruct from free NRQCD correlator ($U_\mu = 1$)

- Expectation: Presence of peaked features due to numerical **Gibbs ringing**
S-wave And P-wave Survival At T=249MeV

- At T=140MeV clear difference between ground state peak and numerical ringing

- At T=249 MeV: Ground state peak still stronger than numerical ringing
Conclusion

- Improved Bayesian approach to spectral function reconstruction is promising
  - Outperforms MEM consistently: higher resolution on same datasets
  - No restricted search space: accuracy suffers from loss of information alone

- The in-medium potential between static quarks can be accessed in lattice QCD
  - Re[V] lies close to color singlet free energies in Coulomb gauge at all T
  - Im[V] in quenched QCD: same order of magnitude as HTL perturbation theory at $T > T_C$

- Bottomonium spectra on HotQCD lattices with $N_f=2+1$ light HISQ flavors
  - In-medium modification of correlators above $T=160\text{MeV}$  \[\text{[up to }1\% (\Upsilon) \text{ and } 5\% (\chi_{b1})\] \]
  - $N_{\tau}=12$ datapoints allow us to set upper bounds on in-medium modification
  - A systematic comparison between free and interacting spectra show:
    S-wave and P-wave ground state survive up to at least $T=249\text{MeV}$

Thank you for your attention
Dependence On The NRQCD Discretization

- Reduce the effective temporal step size for NRQCD propagator E.O.M.

- As expected: high momentum behavior changes but IR unaffected
Default Model Dependence

\( \beta = 6.664, T = 140\text{MeV} \)

**S-wave**

\( \beta = 7.280, T = 249\text{MeV} \)

**S-wave**

\( \beta = 6.664, T = 140\text{MeV} \)

**P-wave**

\( \beta = 7.280, T = 249\text{MeV} \)

**P-wave**
Free Spectra: Default Model Dependence

**S-wave**

- $M=2.76$
- $T=140\text{MeV}$
- $N_c=400$
- $\rho^{\Upsilon,\text{BR}}(\omega)$
- $m=\text{norm}$
- $m=\text{norm} \times 0.05$
- $m=\text{norm} \times 0.1$
- $m=\text{norm} \times 0.2$
- $m=\text{norm} \times 0.5$
- $m=\text{norm} \times 5$
- $m=\omega$
- $m=\omega^2$

**P-wave**

- $M=2.76$
- $T=140\text{MeV}$
- $N_c=400$
- $\rho^{\chi_{b1},\text{BR}}(\omega)$
- $m=\text{norm}$
- $m=\text{norm} \times 0.05$
- $m=\text{norm} \times 0.1$
- $m=\text{norm} \times 0.2$
- $m=\text{norm} \times 0.5$
- $m=\text{norm} \times 5$
- $m=\omega$
- $m=\omega^2$