Heating up QGP: towards charm quark chemical equilibration

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What is it?
Melting / recombination:

Leptonic annihilation:

Chemical equilibration: (in either direction)
Why is it of wider interest?
Weakly Interacting Massive Particles as dark matter

The system thermalizes after inflation, but then chemically decouples when pair annihilation is not fast enough to track the equilibrium distribution, which is $\sim \left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T}$ at $T \ll M$. 

Dark Matter  
Visible Matter  

"freeze-out" $\sim 1/T^2$
Back of the envelope estimate

Equate Hubble rate ($H$) with annihilation rate ($\Gamma \sim n\sigma v$):

\[ H \sim n\sigma v \]

\[ \Leftrightarrow \quad \frac{T^2}{m_{Pl}} \sim \left( \frac{MT}{2\pi} \right)^{3/2} e^{-M/T} \frac{\alpha_w^2}{m_W^2} \left( \frac{T}{M} \right)^{1/2} \]

\[ \Rightarrow \quad \frac{M}{T} \sim \ln \left[ \frac{\alpha_w^2 m_{Pl} M}{m_W^2 (2\pi)^{3/2}} \right] \sim 30. \]

(A real computation gives $M/T \sim 25$.)

"WIMP miracle": the order of magnitude of the resulting $n$ and energy density $e = M n$ is correct for $M \sim 1$ TeV.
Can we “simulate” this in QCD? 1

(i) Initial production

Initial state is out-of-equilibrium, with a non-thermal abundance of heavy quarks with hard momenta:\(^2\)

If nothing happens afterwards, heavy quarks and antiquarks constitute separate conserved charges.\(^3\)

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\(^3\) e.g. A. Andronic et al, Nucl. Phys. A 789 (2007) 334 [nucl-th/0611023].
(ii) Kinetic equilibration

Charm (and even bottom) do equilibrate kinetically: jets get quenched,\(^4\) quarks adjust their velocities to hydrodynamic flow.\(^5\)

\(^4\) e.g. A. Dainese [ALICE Collaboration], 1106.4042.
\(^5\) e.g. G. Ortona [ALICE Collaboration], 1207.7239.
(iii) Chemical equilibration: how fast does pair creation or annihilation take place?

The computation is in principle the same as for strangeness,\textsuperscript{6} and near equilibrium the answer can be expressed as:

\[
\Gamma_{\text{chem}} = \frac{g^4 C_F}{8\pi M^2} \left( N_f + 2C_F - \frac{N_c}{2} \right) \left( \frac{TM}{2\pi} \right)^{\frac{3}{2}} e^{-M/T}.
\]

Numerical estimates:

\[ \Gamma_{\text{chem}} \approx \frac{2\pi \alpha_s^2 T^3}{9 M^2} \left( \frac{7}{6} + N_f \right) \frac{\chi_f}{\chi_0}, \]

where \( \chi_f, \chi_0 \) are massive and massless quark number susceptibilities. For \( N_f = 3, \alpha_s \sim 0.3, M \sim 1.5 \) GeV, and \( \chi_f/\chi_0 \) from lattice,\(^7\) yields:

\[ \Gamma_{\text{chem}}^{-1} \gtrsim 60 \text{ fm/c}, \quad \text{for } T \sim 400 \text{ MeV}, \]
\[ \Gamma_{\text{chem}}^{-1} \sim 10 \text{ fm/c}, \quad \text{for } T \sim 600 \text{ MeV}. \]

In the current LHC setup: \( \Delta t \lesssim 20 \text{ fm/c at } T_{\text{initial}} \lesssim 500 \text{ MeV}. \)

Goal for HIC@FCC: \( \Delta t \lesssim 50 \text{ fm/c at } T_{\text{initial}} \lesssim 1 \text{ GeV (?).} \)

\(^7\) H.-T. Ding et al, 1011.0695; S. Borsanyi et al, 1204.0995.
Open questions

- Validity of the weak-coupling expansion?
- Validity of the non-relativistic expansion?
- Non-equilibrium effects beyond linear response?
- Geometry, asymmetries, ...
Issues with perturbation theory (staying non-relativistic)
Sommerfeld effect (i)

Pair-annihilating particles have strong “initial state” interactions; pair-created particles have strong “final state” interactions.

The methods have been elucidated in cosmology, where the “Sommerfeld effect” may also play an important role.\(^8\)

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**Sommerfeld effect (ii)**

Consider two heavy particles of mass $M$, interacting through an attractive Coulomb-like potential

$$V(r) = -\frac{g^2 C_F}{4\pi r},$$

where $r = |r_1 - r_2|$ is the relative distance. Recalling that the reduced mass is $M/2$, and denoting by $v$ the velocity with respect to the center-of-mass frame ($v = v_{\text{rel}}/2$), the stationary Schrödinger equation takes the form

$$\left( -\frac{\nabla^2}{M} + V(r) \right) \psi = M v^2 \psi.$$

The probability that the two particles meet, allowing them to co-annihilate, is proportional to $|\psi|^2(0)$. 
Sommerfeld effect (iii)

Now, we could first solve the problem with free particles, obtaining a plane-wave solution, and an $r$-independent $|\psi|^2(g_0)$.

However, because of the attractive force, there is an increased probability for the particles to meet.

This increase constitutes the Sommerfeld effect, and is characterized by the coefficient

$$S_1 \equiv \frac{|\psi|^2(g^2)(0)}{|\psi|^2(g^0)(0)}.$$

[This can be defined separately for $s$-wave, $p$-wave, ...]
Sommerfeld effect (iv)

Remarkably, the value of $S_1$ can be determined in closed form for the $s$-wave case:\(^9\)

$$S_1 = \frac{X_1}{1 - e^{-X_1}}, \quad X_1 = \frac{g^2C_F}{4v}.$$ 

If we then consider a thermal environment, the factor needs to be averaged over the thermal ensemble:

$$\bar{S}_1 \equiv \frac{4}{\sqrt{\pi}} \left( \frac{M}{T} \right)^{3/2} \int_0^\infty dv \, v^2 e^{-Mv^2/T} S_1.$$

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Sommerfeld effect ($v$)

Typical values, obtained for QCD-like parameters (here $b!$):

$T = 250 \text{ MeV}, M = 4 \text{ GeV}, \alpha_s = 0.34$
Sommerfeld effect (vi)

As it happens, in pQCD the process splits up into two parts, the “colour-singlet” discussed here as well as a “colour-octet” one, in which case the interaction is repulsive, and $\bar{S}_8 < 1$.

$$\Gamma_{\text{chem}} = \frac{g^4 C_F}{8\pi M^2} \left( \frac{MT}{2\pi} \right)^{3/2} e^{-M/T}$$

$$\times \left[ \frac{1}{N_c} \bar{S}_1 + \left( \frac{N_c^2 - 4}{2N_c} + N_f \right) \bar{S}_8 \right] .$$

The colour-octet channel is weighted more than the colour-singlet channel (with $\bar{S}_1 \approx 3.4$). So, accidentally, the numerical effect on charm equilibration in QCD is small.

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10 Virtuality $\sim MT \gg k_0 \times$ (width for colour decoherence) $\sim M \times g^2 T/\pi$. 
Beyond perturbation theory?
Recall scales:

Extent of imaginary time coordinate: \( \frac{1}{T} \).

Expected physical time scale: \( \frac{1}{\Gamma_{\text{chem}}} \sim \frac{M^{1/2}}{T^{3/2}} e^{M/T} \gg \frac{1}{T} \).

So even if managed to shift away the exponential factor, the dynamical time scale is still much larger than the lattice extent, and naive Wick rotation is insufficient.
The ideal theoretical probe for charm is the trace anomaly.

\[ T^{\mu \mu} = c_\theta g_B^2 F^{\alpha \mu \nu} F^\alpha_{\mu \nu} \equiv \theta + \bar{\psi} M_B \psi , \quad c_\theta = -\frac{b_0}{2} - \frac{b_1 g^2}{4} + \ldots . \]

The contribution from \( S \) should be small (i) in the chiral limit \( M \ll T \), and (ii) for \( M \gg T \) when the charm decouples.

Is the relevant comparison \( M \leftrightarrow T, M \leftrightarrow 3T, M \leftrightarrow 2\pi T \), and which mass to use for \( M \) (pole, \( \overline{\text{MS}} \), \( D^0 \))?
How severe is the exponential suppression?

Measure $\langle T^\mu_\mu \rangle_T$ assuming chemical equilibration.$^{11}$

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**pQCD**

- $O(g^6 \ln \frac{1}{g}) N_f = 3 + O(g^2)$ charm
- $O(g^6 \ln \frac{1}{g}) N_f = 3$

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**lattice (w/o extrapolations)**

black: 2+1+1 flavors
red: 2+1 flavors

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For dynamics: the 2-point correlator of the trace anomaly.

Trace of the energy-momentum tensor yields “bulk viscosity”:

\[
\zeta = \frac{1}{9} \lim_{\omega \to 0^+} \left\{ \frac{1}{\omega} \int_X e^{i\omega t} \left\langle \frac{1}{2} \left[ T^\mu_{\mu}(x), T^\mu_{\mu}(0) \right] \right\rangle T \right\}.
\]

Heavy-quark contribution:

\[
\delta \zeta = \frac{1}{18T} \lim_{\omega \to 0^+} \left\{ \frac{2M^2 \chi_f \Gamma_{\text{chem}}}{\omega^2 + \Gamma_{\text{chem}}^2} \right\} = \frac{M^2 \chi_f}{9T \Gamma_{\text{chem}}}.
\]

Measure:

\[
G_S(\tau) = \left\langle \int_X S(\tau, x) S(0) \right\rangle_T.
\]
⇒ Here charm has 25-30% influence even at $T \sim 300$ MeV. There is a strong mass dependence.

\footnote{12} H.-T. Ding et al, 1204.4945 (quenched). In the simulations, $m_c(\bar{\mu}_{\text{ref}}) \approx 0.97$ GeV. In the plot, $Q \sim M/m_c(\bar{\mu}_{\text{ref}}) = 1 + 4g^2(\bar{\mu}_{\text{ref}})C_F/(4\pi)^2 + \mathcal{O}(g^4) \simeq 1.2.$
Summary
With increasing energy, it may become possible to “simulate” the non-equilibrium thermodynamics of WIMP freeze-out in future Heavy Ion Collision experiments.

In this case the weak interactions of WIMPs are replaced by the strong interactions of charm quarks, but this change is compensated for by the much faster expansion rate.

For a quantitative determination of the charm quark chemical equilibration rate as a function of temperature, further work is needed both in perturbation theory (e.g. NLO) and on the lattice.