Recent Lattice NRQCD Studies of Bottomonium at Non-Zero Temperature

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in collaboration with
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Outline

1. Challenges
2. LNRQCD
3. $T = 0$
4. $T \neq 0$
5. Systematics
6. Conclusion
Scales at $T = 0$

- Lattice QCD is based on

$$\langle O \rangle = \frac{\int D\phi O e^{-\int d^4 x L_E}}{\int D\phi e^{-\int d^4 x L_E}}$$  \hspace{1cm} (1)

- Lattice QCD is defined on discrete space-time lattices

$\rightarrow$ various scales

$a_\tau, a_s$ (UV cutoff)

$\frac{1}{M_q}$ (Compton wavelength)

$N_s a_s$ (spatial IR cutoff)

$N_\tau a_\tau$ (temporal IR cutoff)
Scales at $T = 0$

$$a_\tau \ll \frac{1}{M_q} \ll (N_S a_s, N_\tau a_\tau)$$

- for bottomonium, $M_q = M_b (\sim 4.65 \text{GeV})$,

$$\frac{1}{M_q} \sim 0.04 \text{ fm and spatial size } \sim 1 \text{ fm}.$$  

if $a_s \sim 0.01 \text{ fm}, N_S \sim 100$
Scales at $T = 0$

- bound state dynamics in quarkonium $\sim O(100) \text{ MeV}$

<table>
<thead>
<tr>
<th>$n^{S+1}L_J$</th>
<th>State</th>
<th>$a_T M$</th>
<th>$E_0 + M$ (MeV)</th>
<th>$M_{\text{expt}}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^1S_0$</td>
<td>$\eta_b$</td>
<td>0.20549(4)</td>
<td>9409(12)</td>
<td>9398.0(3.2)</td>
</tr>
<tr>
<td>$2^1S_0$</td>
<td>$\eta'_b$</td>
<td>0.311(3)</td>
<td>10004(21)</td>
<td>9999(4)</td>
</tr>
<tr>
<td>$1^3S_1$</td>
<td>$\gamma$</td>
<td>0.21460(5)</td>
<td>9460*</td>
<td>9460.30(26)</td>
</tr>
<tr>
<td>$2^3S_1$</td>
<td>$\gamma'$</td>
<td>0.318(3)</td>
<td>10043(22)</td>
<td>10023.26(31)</td>
</tr>
<tr>
<td>$1^1P_1$</td>
<td>$h_b$</td>
<td>0.2963(4)</td>
<td>9920(15)</td>
<td>9899.3(1.0)</td>
</tr>
<tr>
<td>$1^3P_0$</td>
<td>$\chi_{b0}$</td>
<td>0.2921(4)</td>
<td>9896(15)</td>
<td>9859.44(52)</td>
</tr>
<tr>
<td>$1^3P_1$</td>
<td>$\chi_{b1}$</td>
<td>0.2964(4)</td>
<td>9921(15)</td>
<td>9892.78(40)</td>
</tr>
<tr>
<td>$1^3P_2$</td>
<td>$\chi_{b2}$</td>
<td>0.2978(4)</td>
<td>9928(15)</td>
<td>9912.21(40)</td>
</tr>
</tbody>
</table>

**Table:** comparison from FASTSUM

- large energy scale separation between $M_b$ and binding energy

- sub-percent level accuracy required
Scales at $T = 0$

- Effective Field Theory (EFT) : $M_b, M_b \nu, M_b \nu^2$
- NRQCD : $M_b$ scale is “integrated away”
  - bottom quark is “point-like” ($M_b a \sim 1$)
- pNRQCD : $M_b, M_b \nu$ scales are “integrated away”
  - bottom quark is “point-like”
  - and bottomonium is also “point-like”
    (“Bohr radius” is also an expansion parameter)
- our choice is lattice NRQCD
Scales at $T \neq 0$

• temperature is an additional scale

$$T = \frac{1}{N_{\tau} a_{\tau}}$$  \hspace{1cm} (1)

• for consistent lattice NRQCD, $M_b a_{\tau} \sim 1$

• to keep NRQCD remain valid as an effective field theory, $T << M_b$

• in summary, a consistent lattice NRQCD for bottomonium ($M_b = 4.65$ GeV) requires

$$a_{\tau} \sim \frac{1}{4.65} (\text{GeV}^{-1})$$  \hspace{1cm} (2)

and

$$T = \frac{1}{N_{\tau} a_{\tau}} \sim \frac{4.65 \text{GeV}^{-1}}{N_{\tau}}$$  \hspace{1cm} (3)

• if we are interested upto $\sim 2T_c (\sim 300$ MeV for $N_f = 2 + 1$),

$N_{\tau} \sim O(10)$
Scales at $T \neq 0$

- for the study of EoS (entropy density, pressure, energy density etc), $N_\tau \sim O(10)$ doesn’t pose a problem

- for the study of in-medium bottomonium, bottomonium correlator is important

$$G(\tau) = \sum_{\vec{x}} \langle \phi^\dagger(\vec{x}, \tau; 0, 0) \phi(\vec{x}, \tau; 0, 0) \rangle$$  \hspace{1cm} (1)

- spectral information (mass shift, thermal broadening etc) needs to be obtained from $G(\tau)$ evaluated at $N_\tau \sim O(10)$ of $\tau$ position

Bottomonium correlator, $G(\tau, x)$
Scales at $T \neq 0$

$$G(\tau) = \sum_n e^{-E_n \tau} |\langle 0 | \phi(0) | n \rangle|^2$$  \hfill (1)

- if the states are well defined stationary states,

$$\rightarrow G(\tau) \sim a_0 e^{-E_0 \tau} + a_1 e^{-E_1 \tau} + a_2 e^{-E_2 \tau} + \cdots$$  \hfill (2)

usual $\chi^2$ fitting is sufficient

- for in-medium bottomonium, the states are no longer narrow

$$\rightarrow$$ spectral information is needed unless the functional form is known
Scales at $T \neq 0$

- spectral representation

$$G_{\Lambda}(\tau) = \sum_{\vec{x}} \langle \bar{\psi}(\tau, \vec{x}) \Lambda \psi(\tau, \vec{x}) \bar{\psi}(0, \vec{0}) \Lambda \psi(0, \vec{0}) \rangle$$  \hspace{1em} (1)

$$= \int \frac{d^3p}{(2\pi)^3} \int_{0}^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega) \rho_{\Lambda}(\omega, \vec{p})$$ \hspace{1em} (2)

and

$$K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}.$$ \hspace{1em} (3)

- the spectral function of Euclidean correlator has all the information on the finite temperature behavior of a propagator

- numerically ill-posed problem

- Maximum Entropy Method is used (cf. M. Asakawa, T. Hatsuda, Y. Nakahara, PPNP46 (2001) 459)
Scales at $T \neq 0$

\[ G_\Lambda(\tau) = \sum_{\vec{x}} \langle \bar{\psi}(\tau, \vec{x}) \Lambda \psi(\tau, \vec{x}) \bar{\psi}(0, \vec{0}) \Lambda \psi(0, \vec{0}) \rangle \]  

\[ = \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \rho_\Lambda(\omega, \vec{p}) \]  

and

\[ K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}. \]


- both the kernel($K(\tau, \omega)$) and the spectral density($\rho_\Lambda(\omega, \vec{p})$) depend on temperature

- constant contribution
Scales at $T \neq 0$

- In NRQCD, with $\omega = 2M + \omega'$ and $T/M \ll 1$, $K(\tau, \omega) \rightarrow e^{-\omega \tau}$

$$G(\tau) = \int_{-2M}^{\infty} \frac{d\omega'}{2\pi} \exp(-\omega' \tau) \rho(\omega')$$ (1)

- inverse Laplace transform problem

- new improved Bayesian method (Burnier-Rothkopf, PRL111 (2013) 182003, Alexander's talk)
Lattice data used for NRQCD

- FASTSUM anisotropic lattice on $24^3 \times N_t$ (ref. G. Aarts et al, JHEP1407 (2014) 097)

<table>
<thead>
<tr>
<th>$N_s$</th>
<th>$N_t$</th>
<th>$a_{\tau}^{-1}$</th>
<th>T(MeV)</th>
<th>$T/T_c$</th>
<th>No. of Conf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>128</td>
<td>5.63(4)GeV</td>
<td>44</td>
<td>0.24</td>
<td>499</td>
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<tr>
<td>24</td>
<td>40</td>
<td>5.63(4)GeV</td>
<td>141</td>
<td>0.76</td>
<td>502</td>
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<tr>
<td>24</td>
<td>36</td>
<td>5.63(4)GeV</td>
<td>156</td>
<td>0.84</td>
<td>503</td>
</tr>
<tr>
<td>24</td>
<td>32</td>
<td>5.63(4)GeV</td>
<td>176</td>
<td>0.95</td>
<td>998</td>
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<tr>
<td>24</td>
<td>28</td>
<td>5.63(4)GeV</td>
<td>201</td>
<td>1.09</td>
<td>1001</td>
</tr>
<tr>
<td>24</td>
<td>24</td>
<td>5.63(4)GeV</td>
<td>235</td>
<td>1.27</td>
<td>1002</td>
</tr>
<tr>
<td>24</td>
<td>20</td>
<td>5.63(4)GeV</td>
<td>281</td>
<td>1.52</td>
<td>1000</td>
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<tr>
<td>24</td>
<td>16</td>
<td>5.63(4)GeV</td>
<td>352</td>
<td>1.90</td>
<td>1042</td>
</tr>
</tbody>
</table>

Table: summary for the FASTSUM lattice data set, $M_b a_s = 2.92, M_b a_{\tau} = 0.834$

- tadpole- and Symanzik- improved gauge action, tapole-improved Wilson clover quark action ($N_f = 2 + 1$)
### Lattice data used for NRQCD

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$T$</th>
<th>$T/T_c$</th>
<th>$a$(fm)</th>
<th>$u_0$</th>
<th>$M_{ba}$</th>
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<tbody>
<tr>
<td>6.664</td>
<td>140</td>
<td>0.911</td>
<td>0.117</td>
<td>0.87025</td>
<td>2.76</td>
</tr>
<tr>
<td>6.700</td>
<td>145</td>
<td>0.944</td>
<td>0.113</td>
<td>0.87151</td>
<td>2.67</td>
</tr>
<tr>
<td>6.740</td>
<td>151</td>
<td>0.980</td>
<td>0.109</td>
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<td>2.57</td>
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<tr>
<td>6.770</td>
<td>155</td>
<td>1.01</td>
<td>0.106</td>
<td>0.87388</td>
<td>2.50</td>
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<tr>
<td>6.800</td>
<td>160</td>
<td>1.04</td>
<td>0.103</td>
<td>0.87485</td>
<td>2.42</td>
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<tr>
<td>6.840</td>
<td>166</td>
<td>1.08</td>
<td>0.0989</td>
<td>0.87612</td>
<td>2.34</td>
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<tr>
<td>6.880</td>
<td>172</td>
<td>1.12</td>
<td>0.0953</td>
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<tr>
<td>6.910</td>
<td>177</td>
<td>1.15</td>
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<td>0.0893</td>
<td>0.87945</td>
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<tr>
<td>6.990</td>
<td>191</td>
<td>1.24</td>
<td>0.086</td>
<td>0.88060</td>
<td>2.03</td>
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<td>7.030</td>
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<td>1.29</td>
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<td>7.100</td>
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<td>7.150</td>
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<td>1.44</td>
<td>0.0743</td>
<td>0.88493</td>
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<td>7.280</td>
<td>249</td>
<td>1.61</td>
<td>0.0660</td>
<td>0.88817</td>
<td>1.56</td>
</tr>
</tbody>
</table>

**Table:** summary for $N_f = 2 + 1$ HotQCD $48^3 \times 12$ lattice (A.Bazavov et al, PRD85 (2012) 054503)
FASTSUM NRQCD and KPR NRQCD

- fixed lattice scale vs. variable lattice scale
- anisotropic lattices vs. isotropic lattices
- MEM (and BR, cf. T. Harris, Lat2014) vs. BR and MEM
- tuned $M_b$ using kinetic mass vs. $M_b = 4.65$ GeV
- different lattice actions (in lattice NRQCD, we can’t take continuum limit)
Lattice NRQCD Method

- Non-relativistic QCD in FT

\[
\begin{align*}
G(\vec{x}, t = 0) &= S(x) \\
G(\vec{x}, t = 1) &= \left[ 1 + \frac{1}{2n} \frac{\vec{D}^2}{2m_b^0} \right]^n U_4^+(\vec{x}, t) \left[ 1 + \frac{1}{2n} \frac{\vec{D}^2}{2m_b^0} \right]^n G(\vec{x}, 0) \\
G(\vec{x}, t + 1) &= \left[ 1 + \frac{1}{2n} \frac{\vec{D}^2}{2m_b^0} \right]^n U_4^+(\vec{x}, t) \left[ 1 + \frac{1}{2n} \frac{\vec{D}^2}{2m_b^0} \right]^n \left[ 1 - \delta H \right] G(\vec{x}, t)
\end{align*}
\]

(2)

where \(S(x)\) is the random source (not smearing) and
Lattice NRQCD Method

\[
\delta H = -\frac{(\vec{D}^{(2)})^2}{8(m_b^0)^3}
\]

\[
+ \frac{ig}{8(m_b^0)^2} (\vec{D} \cdot \vec{E} - \vec{E} \cdot \vec{D}) - \frac{g}{8(m_b^0)^2} \vec{\sigma} \cdot (\vec{D} \times \vec{E} - \vec{E} \times \vec{D}) - \frac{g}{2m_b^0} \vec{\sigma} \cdot \vec{B}
\]

\[
+ \frac{a^2 \vec{D}^{(4)}}{24m_b^0} - \frac{a(\vec{D}^{(2)})^2}{16n(m_b^0)^2}
\]
Lattice NRQCD Method

bottomonium correlator, $G(\tau, x)$

- NRQCD dispersion relation has undetermined zero point energy

$$E_q = \sqrt{M_q^2 + p^2} \sim M_q + \frac{p^2}{2M_q} - \frac{p^4}{8M_q^3} + \cdots$$

- simulation at zero temperature is required to determine the zero point energy

- FASTSUM NRQCD requires just one $T = 0$ calibration to fix the zero point energy. KPR NRQCD requires $T = 0$ calibration for each lattice spacing.
FASTSUM $T = 0$ correlator

\[
a_\tau M(\Upsilon) = 0.21460(5)
\]
\[
a_\tau M(\chi_{b1}) = 0.2964(4)
\]
KPR $T = 0$ correlator
FASTSUM and KPR $T = 0$ spectral function
FASTSUM S-wave spectral functions

\( \Upsilon \) channel spectral function
KPR S-wave spectral functions

\( \Upsilon \) channel spectral function
FASTSUM P-wave spectral functions

$\chi_{b1}$ channel spectral function
KPR P-wave spectral functions

$\chi_{b1}$ channel spectral function
Systematics study in FASTSUM

- $\omega_{\text{min}}, \omega_{\text{rmax}}$ range
- default model dependency
- statistical error dependency
- $\tau$ range dependency
- comparison with free NRQCD spectral function
- high momentum stability
Systematics study in FASTSUM

\[ \gamma, N_T = 28 \]

\[ \gamma, N_T = 20 \]

\[ \chi_{b1}, N_T = 28 \]

\[ \chi_{b1}, N_T = 20 \]

\[ \rho(\omega)/m_b^2 \]

\[ \rho(\omega)/m_b^4 \]
Systematics study in KPR

- $\omega_{\text{min}}, \omega_{r\text{max}}$ range
- default model dependency
- statistical error dependency
- $\tau$ range dependency
- comparison with free NRQCD spectral function
- high momentum stability
- spectral function reconstruction method dependency
Systematics study in KPR

- Challenges LNRQCD
- $T = 0$ $T \neq 0$
- Systematics
- Conclusion
Conclusion

• on $T = 0$ and $T \neq 0$, lattice NRQCD + new Bayesian Reconstruction (BR) of spectral function on bottomonium, which is systematically improvable and is based on the first principle of quantum field theory (not a model)

• free from known problem in QCD (constant contribution problem) and improvement from MEM

• from both BR and MEM, the ground state of $\Upsilon$ survives but the excited states are suppressed as the temperature increases above $T_c$. In FASTSUM study, 1S peak of $\Upsilon$ channel remains upto $T = 1.9 T_c$ and in KPR study it remains upto $T = 1.6 T_c$

• in FASTSUM study, the ground state of $\chi_{b1}$ melts above $T_c$. In KPR study, the ground state of $\chi_{b1}$ from BR spectral function retains peak structure even at $1.6 T_c$ but that from MEM spectral function shows melting around $1.3 T_c$

• further studies are in progress