Applications of Neutron-Star Universal Relations to Gravitational Wave Observations

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July 3rd 2014
Universal Relations: Isolated NSs

Introduction

[Lattimer & Prakash (2001)]
[Urbanec+ (2013)]

Many others

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Universal Relations: Binary NSs

Introduction

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Roadmap

(I) I-Love-Q & Multipole Love Relations

(II) Applications to GW Observations

(III) Other Related Topics

(IV) Open Questions
I-Love-Q & Multipole Love Relations
I-Love-Q Relations

(i) small/static tidal deformation
(ii) unmagnetized
(iii) uniform/slow-rotation
(iv) barotropic

Maselli+ (2013)
Haskell+ (2014)
Martinon+ (2014)

\[ I \equiv \frac{I}{M_*^3} \quad \tilde{\lambda}_2 \equiv \frac{\lambda_2}{M_*^5} \]

\[ Q \equiv -Q/\left(\frac{M_*^3}{\chi^2}\right) \quad \chi \equiv \frac{S_1}{M_*^2} \]

I-Love-Q

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Universal I-Q Plane

Universality still holds for a fixed $\chi = S_1/M^2$

QS plane is almost identical to the NS one

[Pappas & Apostolatos (2014)]
[KY+ (2014)]
Similar relation holds for \( \lambda_4 - \lambda_2 \)
Applications to GW Observations

(I) GW Astrophysics
(II) Gravitational Physics
(III) Nuclear Physics
(I) GW Astrophysics

NS/NS gravitational waveform phase

\[ \Psi(f) \approx \Psi_0 f^{-5/3} \left[ 1 + \ldots + \Psi_3(\beta)f + \Psi_4(\sigma)f^{4/3} + \ldots + \Psi_{10}(\lambda)f^{10/3} \right] \]

\[ \sigma = \sigma_{SS} + \sigma_Q \]

Q-Love relation breaks the degeneracy between spins and Q

→ Allows us, in principle, to measure independent spins
(II) Gravitational Physics

- double binary pulsar $\Delta \bar{I}/\bar{I} = 10\%$
- GWs $\Delta \bar{\lambda}_2/\bar{\lambda}_2 = 60\%$

Gravitational Physics
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Strong-field Tests of Gravity

[KY & Yunes, Science, PRD (2013)]

Dynamical Chern-Simons

$M_\ast$ (Shen) [$M_\odot$]

$10^6$ times stronger than the current solar system bound.

Chern-Simons

GR

APR

SLy

LS220

Shen

PS

PCL2

Polytrope (n=1)
(iii) Nuclear Physics

\[ \Psi \tilde{\lambda}_\ell \sim \tilde{\lambda}_\ell^{(1)} X_1^{2\ell-1} x^{2\ell-3/2} + (1 \leftrightarrow 2) \]

\[ (2\ell + 1) \text{ PN} \]

Useful number of cycles

\[ D_L = 100 \text{Mpc} \quad m_1 = m_2 \]

\[ \begin{align*}
\tilde{\lambda}_\ell &\equiv \frac{\lambda_\ell}{m^{2\ell+1}} \\
X_A &\equiv \frac{m_A}{M} \\
x &\equiv (\pi M f)^{2/3} \\
M &\equiv m_1 + m_2
\end{align*} \]
Impact of Multipole Love Relations

$D_L = 100\text{Mpc}$

$m_1 = m_2$

Shen EoS

LIGO III

![Graph showing the impact of universality relations on statistical errors. Red line represents statistical error without universality relation, green line with universality relation, and magenta dots indicate systematic error from fit.](image)

- **Without Univ. Rel.**
- **With Univ. Rel.**
- **Error from fit**

Univ. rel. can reduce the stat. error by a factor of 4-5.

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[KY (2014)]
(I) Applications to X-ray Observations
(II) 3-Hair Relations for Newtonian Polytropes
(III) Why I-Love-Q
(I) Applications to X-ray Observations

Parameters:

\( (M, R, I, Q, \cdots) \)

Strong degeneracies among parameters

\[ \Downarrow \text{universal relations} \]

Reduce the number of parameters

\[ \Downarrow \text{Allows one to measure mass and radius accurately!} \]
(II) Newtonian 3-Hair Relations

- Newtonian
- rigid rotation
- unmagnetized
- $p = K \rho^{1+1/n}$
- elliptical isodensity

\[
M_\ell + i \frac{q}{a} S_\ell = \bar{B}_n, \left[ \frac{\ell-1}{2} \right] M (iq)^\ell
\]

\[
\begin{aligned}
S_1 &= aM \\
M_2 &= -q^2M
\end{aligned}
\]

Black Hole No-hair Relation

[Hansen (1974)]

\[
M_\ell + iS_\ell = M (ia)^\ell
\]

Once the polytropic index $n$ is specified, all the higher moments can be expressed in terms of the first three.

Newtonian 3-hair

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Equation of State Dependence

\[ M_\ell + \frac{q}{a} S_\ell = \overline{B}_{n,\ell} \left( \frac{\ell - 1}{2} \right) M(iq)^\ell \]

Coefficient is equation of state insensitive within \(~5\%\) for low-\(l\) modes

Effective NS No-hair Property

[KY+ (2014)]

-NS lower multipole moments can be given in terms of the first three.
-confirmed up to \(M_4\)
Near self-similarity of isodensity contours plays a crucial role in the universality.
Open Questions
Open Questions

(i) Improving parameter estimation:
- Fisher → Bayesian

(iii) Universal relations in differentially-rotating NSs
- Naturally breaks the isodensity self-similarity
- 4-hair relation?

(ii) Universal relations & Tests of GR in other non-GR theories:
e.g. - Lorentz-violation in gravity
     Einstein-Aether, Horava-Lifshitz
     - curvature correction
     Einstein-dilaton Gauss-Bonnet f(R)

(iv) Newtonian analysis
- multipole Love relation
- NS oscillation modes?
  Other universal relations?

(v) Love numbers for spinning NSs?
-Can we construct a hybrid inspiral-merger-ringdown waveform with just one tidal parameter?
  
  Universal relations between the damping time and compactness?
  Universal relations among other parameters and compactness?

-How much post-merger oscillations further help constrain the EoS on top of the inspiral?
  
  Which is better?
  (i) Rewrite $\Lambda$ and $f_2$ into a single parameter ($R$?) and constrain it strongly.
  (ii) Treat $\Lambda$ and $f_2$ independently and constrain different part of EoS.