The running of couplings and anomalous thermodynamics in Bose gases near resonance

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Quantum Bose Gas near Feshbach Resonance
(Upper Branch)

…. Papp + Pino et. al, (Wieman, Jin, Cornell, 2009); Pollack+ Dries et al., (Hulet, 2009); Navon+Piatecki et al.,
( Chevy and Salomon, 2011); Wild +Makotyn et al., (Cornell, Jin, 2012); Ha+Hung et al., (Chin, 2012); Makotyn et al., (Cornell, Jin, 2013)…
Dilute Bose Gases

Lee-Yang-Huang (56; 57-58) and Beliaev (58)
And is valid for small scattering lengths.

\[ \sqrt{n a^3} \alpha \frac{a}{\xi} \ll 1 \]

There have been efforts to improve LYH-Beliaev theory by taking into the higher order contributions. (Wu, Sawada, 59; ..... Braatten et al, 02...) At infinity a, each term diverges.

\[ E = \frac{2\pi \hbar^2 n^2 a}{m} (1 + \frac{128}{15\sqrt{\pi}} \sqrt{n a^3} + \frac{8(4\pi - 3\sqrt{3})}{3}\times[\ln(n a^3) + 4.72 + 2B]n a^3 + \ldots.) \]
Approaches to unitary Bose gases

1) Variational approaches (numerical):

Cowell et al. (Pethick group), 2002; Song & Zhou, 2009…
Diederix et al. (Stoof group), 2011; Yin & Radzihovsky, 2013. Sykes et al. (JILA +Greene), 2014 ….
2D: Pilati et al., (Giorgini group), 2005.

2) Effective potential via Loop summation (“single shot renormalization method”)
Borzov, Mashayekhi, Song, Bernier, Jiang, Liu, Semenoff, Maki, FZ (2011---now)

.......... 

Q: Can one have an analogue of BEC-BCS theory but for upper branch Bose gases?
Outline

1) Ideas/Cartoons: the running of coupling or scale dependent interactions;

2) Implementation: Theory frame work for unitary Bose gases (an analogue of BEC-BCS CO theory for bosons beyond the dilute limit.);

3) Results

Ref:

a) Borzov, Mashayekhi, Zhang, Song and FZ, PRA 85, 023620 (2012).
Physics for Short range interactions

\[ \frac{2 \mu V_0 r_0^2}{\hbar^2} \]

\[ u(r) \]

\[ r_0 \]

\[ r \]
Positive scattering lengths = Effective repulsive interactions?

\[ V_{pseudo}(r) = \frac{4\pi a}{m} \delta(r) \frac{\partial}{\partial r} (r...) \sim \frac{4\pi a}{m} \delta(r) \]

Low energy scattering physics of a short range 2-body attractive potential is equivalent to a repulsive interaction.
Scale dependence and deviation from “$4\pi a$”:
Energy for two atoms in a box of size $L$  (kinetic energy $\sim 1/L^2$.)

$$E_{2-b}(L) \sim \frac{a}{mL^3}(1 + C \frac{a}{2L} + ...), a << L;$$

$$\Rightarrow g_2(\Lambda \sim 0) = 4\pi a(1 + C\Lambda a + ....)$$
Scale dependence II: A cartoon in real Space at short distance $L \ll a$

$$r_0 \rightarrow \lambda r_0, V_0 \rightarrow V_0 \frac{1}{\lambda^2},$$

or $$g_2 \sim V_0 r_0^3 \rightarrow \lambda V_0 r_0^3$$

$$\Lambda = \frac{1}{r_0} \rightarrow \frac{\Lambda}{\lambda}, \ g_2(\Lambda) \rightarrow g\left(\frac{\Lambda}{\lambda}\right) = \lambda g_2(\Lambda)$$

$$\Rightarrow g_2(\Lambda) \sim -\frac{1}{\Lambda} + o\left(\frac{1}{a\Lambda}\right)$$
Wilsonian renormalization of interaction constant
The running of 2-body Coupling Constant:

\[
g_2(\Lambda \sim 0) = g_2(0)(1 + C\Lambda a...), \quad \Lambda a \Rightarrow \sqrt{2\mu a} \sim \sqrt{na^3}
\]
Patching: Flow and boundary condition

\[ g = -2\pi^2 \]

\[ \mu \sim g_2(\Lambda \sim \sqrt{\mu})n \]

\[ [MF : \mu = g_2(\Lambda = 0)n] \]
1) Nearly fermionized near resonance (analogue of 1D TG gases);
2) Chemical potential $\mu$ reaches a maximum;
3) Maximum accompanied by an instability.

$\mu$ calculated using the Running coupling at $\mu$. 

Dashed line---LHY dilute gas theory.
Dotted line --- imaginary part of chemical potential
Theory Frame Work

$\mu_c(n_0, \mu) = \frac{\partial E(n_0, \mu)}{\partial n_0}$, $n = n_0 - \frac{\partial E(n_0, \mu)}{\partial \mu}$,

$\mu = \mu_c(n_0, \mu)$,

Condensate \hspace{0.5cm} Non-condensed Chem. potential

Self-consistent Equilibrium Cond.

E is the total interaction energy of condensate at fixed $n_0$
Is also the effective potential for the quantum bosonic field
(Coleman-Weinberg type but calculated at a finite $\mu$).
A typical L-loop diagram for $E_{\text{Diag}(L,N)}$

2D and 3D: arbitrary $L, N$ but with up to 3-body irreducible diags;

4D: epsilon expansion---systematic.
\[
\frac{g_2(\Lambda_{\mu}) - g_2(0)}{LHY} = \frac{(c)}{(c) + (d)} = \frac{9\pi\sqrt{2}}{40} = 99.96\%
\]
Blue: resummation predicts a critical point.
Red: with Efimov physics; Dashed line: Lee-Yang-Huang
three-body physics in BECs at varying $\mu$

Consistent with RGE in Bedaque, Hammer, van Klock, 1999.
Anomalous Compressibility
2D, Mashayekhi+ Bernier et al, 2013. Consistent with the variational QMC simulations by Giorgini et al., 2005.
Implications of epsilon expansion

\[ \mu = \epsilon^{\frac{2}{4-\epsilon}} \epsilon_F \sqrt{\frac{2}{3}} (1 + 0.474\epsilon - i1.217\epsilon + \cdots), \]

\[ n_0 = \frac{2}{3} n (1 + 0.0877\epsilon + \cdots). \]
Relation to the estimate of liquid Helium

\[ V < 0 \quad \text{and} \quad V > 0 \]

\[ \text{C.F.}\approx 8\% \quad \text{(Onsager and Penrose, 56)} \]

\[ \text{C.F.}\approx 50\% - 80\% \]

R/d \quad a/d

~,1 \quad ~1

??
Conclusions

1) 3D Bose gases near resonance (Beyond Lee-Huang-Yang limit)
   a) are nearly fermionized;
   b) chemical potential reaches a maximum;
   c) an onset of instability near resonance---unexpected in the dilute theory;
   d) Efimov effects play a role though not significant near instability.

2) However 3-body effects significant in 2D gases.

3) Generally, the rigorous solution suggests that near 4D are a collection of independent scattering pairs.
   Moving away from 4D, 3, 4-body etc. effects get stronger and stronger, while the life time gets shorter. Consistent with 1) And 2).